Restricted Inequality and Relative Poverty*

Jean-Yves Duclos[†] and Paul Makdissi[‡]

August 2001

Abstract

This paper develops criteria for a new concept of restricted inequality dominance and show how they relate to criteria for comparing relative poverty. The results warn against the use of some popular indices of inequality. They do, however, suggest an interesting extension of the Schutz coefficient as well as a use of Lorenz curves for the analysis of relative poverty and restricted inequality. We also illustrate graphically how the new criteria of restricted inequality dominance extend the ranking power of previously proposed inequality dominance criteria.

Keywords: Inequality, Poverty, Stochastic Dominance

JEL number: I32, D63

* We thank Peter Lambert, Yves Richelle and Cyril Tézédo for many useful comments. We are grateful for the financial support of the FCAR, CRSH, MIMAP programme of the IDRC and Bureau de la recherche of Université de Sherbrooke.

[†] Département d'économie and CRÉFA, Pavillon De Sève, Université Laval, Sainte-Foy, Québec, Canada, G1K 7P4; Email: jyves@ecn.ulaval.ca

[‡] Département d'économie and CEREF, Université de Sherbrooke; 2550, boulevard de l'Université; Sherbrooke, Québec; Canada J1K 2R1; Email: paul.makdissi@courrier.usherb.ca

1. Introduction

Comparisons of inequality and, in particular, the ethical robustness of such comparisons, have been of considerable scientific and policy interest in the last decades. Early influential work (see Kolm (1969), Atkinson (1970), and Dasgupta, Sen and Starrett (1973)) showed how comparisons of the now widely-used Lorenz curves could establish whether inequality orderings obeyed the Pigou-Dalton principle of transfers. This principle is closely linked to the well-known risk aversion assumption that underlies the search for second-order stochastic dominance in the risk-taking literature. Subsequent work showed how third-order inequality dominance could be checked (see *inter alia* Kolm (1976), Shorrocks and Foster (1987), Foster and Shorrocks (1988), or Davies and Hoy (1994)), and to higher orders of Lorenz dominance (see Zoli (1999) and Aaberge (2001)) using rank-order ethical principles. Recent extensions to arbitrary orders of inequality dominance can be found in Foster and Sen (1997), Zheng, Formby, Smith and Chow (2000), and Duclos and Makdissi (2001).

This paper follows a different route to these recent developments by proposing and investigating instead the concept of restricted inequality comparisons. In doing this, we formalise the links between restricted inequality comparisons and relative poverty comparisons (such as comparisons of the proportions of individuals below a certain proportion of the mean). This is particularly relevant in the light of the increasing popularity of relative poverty comparisons, and the suspicion often expressed that these are really comparisons of inequality. This procedure also leads to the derivation of restricted inequality dominance criteria.

Restricted inequality comparisons have in fact often been (implicitly) applied for some time in the empirical literature on the distribution of well-being. Restricted inequality indices that are frequently used include ratios of quantiles over means, ratios of quantiles, and interquantile differences normalised by the mean. We show, however, that the latter two indices are not consistent with inequality dominance orderings (of any order), and can thus conflict with generally accepted ethical criteria for comparing inequality in distributions of well-being.

Our results also suggest a useful extension of the Schutz inequality index. This extension measures the distance between some hypothetical and the actual income shares for individuals underneath various proportions of the mean, and is equivalent to measuring the average poverty gap ratio with proportions of the mean as relative poverty lines. We also describe an interesting use of Lorenz curves for the analysis of relative poverty and restricted second-order inequality, which

gives as a “critical” relative poverty line the slope of the tangent to crossing Lorenz curves. We complete our analysis by illustrating geometrically how the ordinal ranking power of inequality dominance criteria is increased by focussing on comparisons of relative poverty and restricted inequality.

The rest of the paper is organized as follows. Section 2 defines classes of inequality and relative poverty indices. Section 3 briefly recalls previously proposed dominance criteria for inequality and relative poverty comparisons. Section 4 derives dominance criteria for restricted inequality comparisons, and links them to criteria for relative poverty dominance. Section 5 discusses the links between restricted inequality orderings and some of the (implicit) restricted inequality indices often used in the empirical literature on the distribution of well-being. Section 6 presents a geometric analysis of the ordinal ranking power provided by the new restricted inequality dominance criteria, in comparison to some of the existing inequality dominance criteria. Section 7 concludes. The proofs of the propositions appear in the Appendix.

2. Measuring Inequality and Relative Poverty

Formally speaking, we wish to compare inequality and poverty across two distributions of well-being y (income, for short), each of which is represented by a distribution function drawn from the set $\mathfrak{S}$:

$$\mathfrak{S} := \{F : [0, a] \rightarrow [0, 1] \mid F \text{ is nondecreasing, continuous and onto}\},$$

where F is an income distribution function and a is some value equal to (or exceeding) the maximum possible income. To compare relative poverty, the analyst must first select a function, $\xi(F)$, that sets the poverty line as a function of the income distribution F . In this paper, we fix this relative threshold as a proportion of the mean of the distribution¹. Thus, we have:

$$\xi(F) = \gamma\mu_F, \tag{1}$$

where μ_F is the mean of the distribution F and $\gamma \in [0, \infty)$. Note that $\gamma\mu_F$ need not be less than a .

¹This is often done in the empirical literature. For example, the EUROMOD simulation package uses this definition for the poverty line for poverty comparisons across countries of the European Union (cf. Immervoll *et al.* (1999)).

In order to aggregate the various incomes' contribution to relative poverty, we consider additive poverty indices, which are defined as:

$$P_F^R(\gamma) = \int_0^a p_R(y, \gamma\mu_F) dF(y), \quad (2)$$

where

$$\left. \begin{aligned} p_R(y, \gamma\mu_F) &\geq 0, & \text{if } y &\leq \gamma\mu_F, \\ p_R(y, \gamma\mu_F) &= 0, & \text{if } y &> \gamma\mu_F, \end{aligned} \right\} \quad (3)$$

and where

$$p_R(y, \gamma\mu_F) \text{ is homogeneous of degree zero in } y \text{ and } \mu_F. \quad (4)$$

The function $p_R(y, \gamma\mu_F)$ measures the contribution an individual with income y makes to total poverty when the income distribution function is F . Equation (3) says that this contribution is zero if y exceeds $\gamma\mu_F$. Equation (4) essentially assumes that a scaling up or down of all income levels does not affect individual or total poverty; these indices of poverty are called relative by Blackorby and Donaldson (1980). We will use the classes of relative poverty indices Π_R^s , $s = 1, 2, \dots$, defined as:

$$\Pi_R^s(\gamma) := \left\{ P^R | p_R(y, \gamma\mu_F) \in \hat{C}^s \quad \text{and} \quad (-1)^i \frac{\partial^i p_R(y, \gamma\mu_F)}{\partial y^i} \geq 0 \quad \forall i = 1, 2, \dots, s \right\}, \quad (5)$$

where $\hat{C}^s$ represents the set of functions that are s times piecewise differentiable on $[0, \infty)^2$. We will return later on page 5 to the interpretation of the conditions on the derivatives in (5).

A popular example of additive relative poverty indices is provided by the class of FGT (Foster, Greer and Thorbecke (1984)) indices, defined as³

$$\overline{FGT}_F(\alpha, z) = \int_0^z \left(\frac{z-y}{z} \right)^\alpha dF(y), \quad (6)$$

and to which we will revert later.

²See Duclos and Makdissi (2001) for further discussion of this assumption, which could be relaxed.

³Other examples of additive indices are the Ckavrarty (1983) poverty indices and the Watts (1968) index.

Turning now to the measurement of inequality, we first consider utilitarian social welfare functions U , such that

$$U_F = \int_0^a u(y) dF(y), \quad (7)$$

and we focus on social welfare indices that belong to the classes Ω^s , $s = 1, 2, \dots$, defined by:

$$\Omega^s = \left\{ U \mid u(y) \in \hat{C}^s \quad \text{and} \quad (-1)^i \frac{d^i u(y)}{dy^i} \leq 0 \quad \forall i = 1, 2, \dots, s \right\}. \quad (8)$$

As Blackorby and Donaldson (1978) demonstrate, each of these social welfare functions can be linked to a traditional Atkinson-Kolm-Sen index of inequality of the form

$$I_F^U = 1 - \frac{y_{e,F}}{\mu_F}, \quad (9)$$

where the equally-distributed-equivalent income $y_{e,F}$ is implicitly defined as:

$$u(y_{e,F}) = U_F = \int_0^a u(y) dF(y). \quad (10)$$

The indices I_F^U are relative inequality indices⁴ if and only if the function $u(y)$ is isoelastic. Given the subset of the class Ω^s for which $u(y)$ is isoelastic, it is possible to define corresponding classes of relative inequality indices Υ^s . We have:

$$\Upsilon^s := \{ I^U \mid U \in \Omega^s \text{ and } u \text{ is isoelastic} \}. \quad (11)$$

At this point it is useful to interpret the ethical contents of the different classes Π_R^s and Υ^s of poverty and inequality indices. Indices that are members of $\Pi_R^1(\gamma)$ and Υ^1 are such that relative poverty and inequality weakly decrease when an individual's income increases and when mean income is held artificially constant. In addition to being symmetric in income, these indices are thus in some loose sense of the Pareto type. Indices in $\Pi_R^2(\gamma)$ and Υ^2 obey the Pigou-Dalton principle of transfers, which postulates that a mean-preserving transfer of income from a higher-income person to a lower-income person weakly decreases inequality and poverty. Indices in $\Pi_R^3(\gamma)$ and Υ^3 decrease following a favourable composite transfer, which combines a beneficial Pigou-Dalton transfer within a lower part of the

⁴An inequality index is relative if $I_F = I_G$ whenever $F(y) = G(\gamma y)$ for all y and for any given $\gamma > 0$.

distribution with a reverse Pigou-Dalton transfer higher up in the income distribution, and which does not increase the variance of the distribution (see for instance Kolm (1976), Kakwani (1980), and Shorrocks and Foster (1987)). Interpreting the ethical content of higher-order classes $\Pi_R^s(\gamma)$ and Υ^s of indices can be done using the generalised transfer principles of Fishburn and Willig (1984). These principles essentially postulate that, as s increases, the relative weight assigned to the importance of income changes occurring lower down in the distribution also increases (see also Blackorby and Donaldson (1978)).

3. Comparing Inequality and Relative Poverty

Now consider two income distribution functions, F and G , both drawn from the set $\mathfrak{S}$. In order to simplify the exposition, we follow Davidson and Duclos (2000) and define stochastic dominance curves $D_F^1(y) = F(y)$ and $D_F^s(y) = \int_0^y D_F^{s-1}(u) du$ for all integers $s \geq 2$. It can be shown that these curves can be expressed as

$$D_F^{s+1}(z) = s!^{-1} z^s \overline{FGT}_F(s, z). \quad (12)$$

$D_G^s(y)$ is defined similarly.

Moving from distribution F to distribution G entails no increase in inequality if:

$$\Delta I_{FG}^U = I_G^U - I_F^U \leq 0. \quad (13)$$

Similarly, if there is no increase in poverty subsequent to the passage from distribution F to distribution G , it must be that:

$$\Delta P_{FG}^R(\gamma) = \int_0^a p_R(y, \gamma \mu_G) dG(y) - \int_0^a p_R(y, \gamma \mu_F) dF(y) \leq 0. \quad (14)$$

Suppose that we are interested in relative poverty lines up to some maximum proportion of the mean, γ^+ . To establish relative poverty dominance criteria, we first define transformations of the distribution functions F and G , $\hat{F}$ and $\hat{G}$, obtained by dividing each income in the original distribution by the average income in that distribution. The transformed incomes are thus $\hat{y}_F = y/\mu_F$ for $\hat{F}$ and $\hat{y}_G = y/\mu_G$ for $\hat{G}$. For F , this yields

$$D_{\hat{F}}^{s+1}(\gamma) = s!^{-1} \int_0^{\gamma \mu_F} \left(\frac{\gamma \mu_F - y}{\gamma \mu_F} \right)^{s-1} dF(y) = s!^{-1} \overline{FGT}(s, \gamma \mu_F). \quad (15)$$

Duclos and Makdissi (2001) show that a necessary and sufficient condition for $\Delta P_{FG}^R(\gamma) \leq 0$, for relative poverty lines $\gamma\mu_F$ and $\gamma\mu_G$ with a common $\gamma \in [0, \gamma^+]$ and for all relative poverty indices $P_{FG}^R(\gamma) \in \Pi_R^s(\gamma)$, is that $D_{\hat{F}}^s(\gamma) - D_{\hat{G}}^s(\gamma) \geq 0$ for all $\gamma \leq \gamma^+$. If the latter condition holds, we are therefore assured of a very robust ordering of relative poverty indices across F and G since $P_F^R(\gamma)$ is then necessarily no lower than $P_G^R(\gamma)$ for a wide class of indices and choice of relative poverty lines.

An analogous result holds for ranking inequality indices. Zheng et. al. (2000) and Duclos and Makdissi (2001) show that a sufficient condition for $\Delta I_{FG}^U \leq 0$, and this, for all relative inequality indices $I_{FG}^U \in \Upsilon^s$, is that $D_{\hat{F}}^s(\gamma) - D_{\hat{G}}^s(\gamma) \geq 0$ for all $\gamma \in [0, \infty)$. If the latter condition holds, inequality in F is necessarily not lower than in G for all inequality indices that belong to Υ^s .

Inspection of the above results indicates that when $\gamma^+ = \infty$, robust relative poverty comparisons for the $\Pi_R^s(\gamma^+)$ classes of indices are in fact robust comparisons of inequality for the Υ^s classes of indices. It then becomes possible to treat a ranking of relative poverty indices as a ranking of inequality indices. Generally, however, γ^+ is likely not to extend to infinity and, therefore, *unrestricted* inequality dominance orderings will generally be more difficult to obtain than relative poverty orderings.

One outcome of this is that ordering inequality for all indices that are members of Υ^1 is not possible: first-order (unrestricted) inequality dominance is not feasible. To see this, consider Figure 1, where $D_{\hat{F}}^1(\gamma) = \hat{F}(\gamma)$. Since $\int_0^\infty \gamma d\hat{F}(\gamma) = \int_0^\infty \gamma d\hat{G}(\gamma) = 1$ by definition of the mean of the normalised distributions $\hat{F}(\gamma)$ and $\hat{G}(\gamma)$, the areas between the dominance curves $D_{\hat{F}}^1(\gamma)$ and $D_{\hat{G}}^1(\gamma)$ and the vertical axis must be equal to 1. This is shown on Figure 1, where $A + B = B + C = 1$. It follows from this that, when they are not everywhere the same, the inequality dominance curves $D_{\hat{F}}^1(\gamma)$ and $D_{\hat{G}}^1(\gamma)$ must cross at some point γ^{++} (shown on Figure 1), and that it therefore cannot be that one of the curves always lies above the other for all γ .

An obvious question, then, is this: can we interpret relative poverty comparisons with a finite γ^+ as some sort of restricted inequality comparisons? We consider this issue in the next section.

4. Relative Poverty and Restricted Inequality Dominance

Consider the classes $\Upsilon^{*s}(\gamma^+) \subset \Upsilon^s$ of restricted inequality indices, $I^U(\gamma^+)$, defined by (9) and (10) and with an isoelastic function $u(y) = g(y; \mu)$ such that:

$$g(y; \mu) = \begin{cases} y, & \text{if } y \leq \gamma^+ \mu, \\ \gamma^+ \mu, & \text{if } y > \gamma^+ \mu. \end{cases} \quad (16)$$

For a given mean income, the variable γ^+ represents the proportion of that mean income up to which we consider incomes to be relevant in capturing inequality. An index $I^U(\gamma^+)$ of the class $\Upsilon^{*s}(\gamma^+)$ is thus not sensitive to (marginal) changes in the value of those incomes that lie above $\gamma^+ \mu$, so long as mean income is unaffected. This property effectively restricts the applicability of Pareto, Pigou-Dalton and other ethical principles to the bottom part of the distribution. For instance, as for Υ^2 , indices in Υ^{*2} are such that inequality must increase if a mean-preserving transfer occurs from an individual whose income lies below $\gamma^+ \mu$ to someone with a higher income. If, however, such a Pigou-Dalton transfer occurs among persons whose incomes are (and remain) greater than $\gamma^+ \mu$, the index necessarily remains unchanged. Since these new indices do not respect strictly the Pigou-Dalton principle of transfers everywhere across the income distribution, we can treat them as indices of restricted or censored inequality (in analogy to absolute poverty indices, which are sometimes described as indices of censored social welfare; see also Ravallion (1994) where welfare and absolute poverty are depicted as inclusive and exclusive well-being measures, respectively). The classes of restricted inequality indices $\Upsilon^{*s}(\gamma^+)$, $s = 1, 2, \dots$, are thus defined as:

$$\Upsilon^{*s}(\gamma^+) := \{I^U(\gamma^+) \mid U \in \Omega^s, u(y) = v[g(y; \mu)], \text{ and } v \text{ is isoelastic}\}. \quad (17)$$

We may now state dominance criteria for these new classes of restricted inequality indices, $\Upsilon^{*s}(\gamma^+)$.

Proposition 1. $\Delta I_{FG}^U(\gamma^+) \leq 0$ for all $I^U(\gamma^+) \in \Upsilon^{*s}(\gamma)$ and for a given $s = 1, 2, \dots$, if:

$$\begin{aligned} D_{\hat{F}}^s(\gamma) - D_{\hat{G}}^s(\gamma) &\geq 0 \quad \forall \gamma \in [0, \gamma^+], \\ &\text{and, if } s \geq 3, \\ D_{\hat{F}}^i(\gamma^+) - D_{\hat{G}}^i(\gamma^+) &\geq 0 \quad \forall i \in \{2, 3, \dots, s-1\}. \end{aligned} \quad (\text{DSI}^*)$$

Note that, unlike for the class Υ^1 , it is sometimes feasible to rank distributions unambiguously over the class $\Upsilon^{*1}(\gamma^+)$; we revert to this point later. In general,

given that restricted inequality indices focus only on the bottom part of the distribution, ranking them for $s = 1, 2$ is easier than ranking unrestricted inequality indices, for which we require $D_{\hat{F}}^s(\gamma) - D_{\hat{G}}^s(\gamma) \geq 0$ for all positive γ .

Given the results stated below (15), Proposition 1 leads straightforwardly for $s = 1, 2$ to the following link between the measurement of relative poverty and restricted inequality:

Proposition 2. *If $\Delta P_{FG}^R(\gamma) \leq 0$, for any index $P^R(\gamma) \in \Pi_R^s(\gamma)$, $s = 1$ or 2 , and for any threshold $\gamma \in [0, \gamma^+]$, then $\Delta I_{FG}^U(\gamma^+) \leq 0$ for any index $I(\gamma^+) \in \Upsilon^{*s}(\gamma^+)$.*

Proposition 2 implies that we can always treat a ranking of relative poverty at order $s = 1, 2$ as a ranking of restricted or censored inequality, that is, for inequality judgements that focus only on the lower portions of incomes. For higher orders, $\Pi_R^s(\gamma^+)$ regroups indices that effectively also act as indices of restricted inequality since they restrict attention to the value of those incomes that belong to $[0, \gamma^+\mu]$. In contrast to $\Upsilon^{*s}(\gamma^+)$, the indices in $\Pi_R^s(\gamma)$ are also continuous at $\gamma^+\mu$ (due to the $\hat{C}^s$ continuity assumption in (8)). $\Pi_R^s(\gamma^+)$ dominance rankings for $s \geq 3$ cannot therefore be made equivalent to restricted inequality rankings of the same order s .

5. Restricted Inequality Comparisons

Restricted inequality comparisons have in fact been (implicitly) performed for a long time in the empirical literature on the distribution of income. Until now, the ethical foundations of some of these comparisons did not, however, appear to be well understood. And, as we will discuss shortly, the comparisons have also often been done in a manner which is inconsistent with the dominance criteria derived above.

The first way in which restricted inequality comparisons are effectively found in the literature is through relative poverty comparisons. The commonest example of this is through comparisons of the proportions of individuals found underneath various proportions of the mean (as a relative poverty line). This proportion is given by $D_{\hat{F}}^1(\gamma)$ for distribution F , and is therefore an instance of a test of first-order restricted inequality dominance (recall Proposition 1). For comparisons at higher orders of dominance, the criteria simply involve the (common) use of the normalised FGT indices combined with various proportions of the mean as relative poverty lines.

The second way in which restricted inequality comparisons implicitly arise in the empirical literature is through the use of the dual (or " p -approach") to stochastic dominance (for more on this, see for instance Davidson and Duclos (2000)). For first-order restricted inequality dominance, this involves the use of ratios of quantiles over the mean (normalised quantiles), ratios of quantiles (*e.g.*, the 25th-percentile over the median, or the ratio of the 75th-percentile over the 25th percentile), or differences between normalised quantiles ("normalised interquantile distances").

To see how such comparisons relate to the first-order inequality dominance criteria derived above, consider Figure 2. On the horizontal axis, we find the proportion γ of the mean used as a relative poverty line to compare relative poverty in distributions F and G . On the vertical axis, we recall the proportion of individuals below those relative poverty lines in F and in G . Let $Q_F(p)$ be the inverse of the distribution function F , such that $Q_F(p) = F^{-1}(p)$. $Q_F(p)$ is thus the p -quantile of distribution F . Let $Q_G(p)$ be defined analogously, and denote by $\Gamma(p) = Q(p)/\mu$ the normalised quantiles. By definition, we thus have that $F(\Gamma_F(p)\mu_F) = F(Q_F(p)) = p$. For first-order relative poverty dominance of G over F , we thus require that $F(\gamma\mu_F) \geq G(\gamma\mu_G)$ for a range of γ , which in Figure 2 is obtained for all positive γ until γ^{++} . Equivalently, this can be tested by comparing normalised quantiles and by checking that $\Gamma_F(p) \leq \Gamma_G(p)$ for a range of p , which in Figure 2 is obtained for all positive p until p^{++} . Clearly, $F(\gamma^{++}\mu_F) = G(\gamma^{++}\mu_G) = p^{++}$. Hence, we have restricted first-order inequality dominance of G over F up to γ^{++} and p^{++} .

The comparison of normalised quantiles in the empirical income distribution literature is thus consistent with the first-order restricted inequality dominance criteria developed above. To lead to a robust ranking of restricted inequality, such comparisons must, however, be done over a range of p values, starting from zero.

Ratios of quantiles and differences between normalised quantiles, are not, however, consistent with the restricted and unrestricted inequality dominance orderings derived in Sections 3 and 4 above. This is because these ratios do not obey the derivative (and implicit ethical) assumptions stated in (8) (see also (11) and (17)), and this, for any value of $s = 1, 2, \dots$. For an illustration of this, note in Figure 2 that although distribution F has robustly more restricted inequality than G , the ratio of the median over the 25th-percentile is *less* in F than in G : $\Gamma_F(0.25)/\Gamma_F(0.5) > \Gamma_G(0.25)/\Gamma_G(0.5)$. This wrongly suggests that there is *less* inequality in F than in G . Similarly, the normalised interquantile distance $\Gamma(0.5) - \Gamma(0.25)$ is also lower in F than in G , suggesting again wrongly that in-

equality is lower in F . Moreover, since first-order restricted inequality dominance (up to γ^{++}) implies higher-order restricted inequality dominance (at least up to γ^{++}), quantile ratios and normalised interquantile distances are inconsistent not only with first-order restricted inequality dominance, but also with any higher-order concept of restricted *or* unrestricted inequality dominance (and hence with Lorenz dominance). Such measures should therefore be best avoided.

Note, moreover, that for relative poverty comparisons to have a meaningful interpretation in terms of restricted inequality comparisons, the relative poverty line has been set as a proportion of the mean, not as a proportion of the median or of another quantile, as is sometimes done. Figure 2 can help us see why comparing proportions of individuals underneath proportions of the median would not be consistent with restricted or unrestricted inequality dominance. Assume for simplicity that $\mu_F = \mu_G = 1$. Although distribution F in Figure 2 has more restricted inequality than distribution G , its median is significantly lower than that of G : $\Gamma_F(0.5) < \Gamma_G(0.5)$. It can then be seen that for many common proportions π of the medians, $\pi\Gamma_F(0.5)$ and $\pi\Gamma_G(0.5)$, the proportion of individuals in F underneath $\pi\Gamma_F(0.5)$ is lower than the proportion of individuals in G underneath $\pi\Gamma_G(0.5)$. This use of proportions of the median may wrongly suggest that there is *less* inequality in F than in G .

Equation (15) and Proposition 1 showed how restricted inequality orderings can be checked by comparing normalised FGT indices using various proportions of the mean as relative poverty lines. For second-order dominance, this test has an interesting link with an extension of a well-known index of inequality. To see this, recall that the Lorenz curve for a distribution F at a percentile p is defined as $L_F(p) = \mu_F^{-1} \int_0^p Q_F(q) dq$ and indicates the proportion of total income held by those with rank p and below in the income distribution. The Schutz index measures the maximum vertical distance between the Lorenz curve and the 45° line (see *e.g.* Lambert (1993, p.35)) for a distribution F . It can thus be expressed as

$$S_F = F(\mu) - L_F(F(\mu)). \quad (18)$$

An extension of the Schutz index shows how this is linked to indices of relative poverty. Let a generalised Schutz index be defined as:

$$S_F(\gamma) = F(\gamma\mu) - \gamma^{-1} L_F(F(\gamma\mu)). \quad (19)$$

The reason for the presence of γ^{-1} in (19) will shortly become apparent. Rewriting

(19) leads to

$$S_F(\gamma) = \gamma^{-1} \underbrace{[\gamma F(\gamma\mu) - L_F(F(\gamma\mu))]}_{D(\gamma)} \quad (20)$$

where $D(\gamma)$ can be visualised as the maximum (positive) distance between a straight line of slope γ that passes through the origin and the Lorenz curve. This distance is illustrated in Figure 3 with $\gamma = 0.5$ and $\gamma = 1$. $\gamma F(\gamma\mu)$ is the share of total income that would accrue to those with income under $\gamma\mu$ if they all had income $\gamma\mu$. $D(\gamma)$ shows the distance between that hypothetical share and the actual share in total income, which is $L_F(F(\gamma\mu))$. $S_F(\gamma)$ is simply that distance normalised by γ . We can show that $S_F(\gamma)$ varies between 0 and 1, and that it increases with γ . When $\gamma = 1$, we obtain the above-mentioned interpretation of the usual Schutz coefficient as the maximum distance between the line of perfect equality and the Lorenz curve⁵. A simple transformation of (20) also shows that

$$S_F(\gamma) = \int_0^{\gamma\mu_F} \left(\frac{\gamma\mu_F - y}{\gamma\mu_F} \right)^{s-1} dF(y) = \overline{FGT}_F(1, \gamma\mu_F). \quad (21)$$

$S_F(\gamma)$ is thus simply the common average poverty gap ratio using a relative poverty line $\gamma\mu_F$. Comparing (20) and (21) (also recall Figure 3), note also that $D(\gamma) = \gamma S_F(\gamma)$ is the proportion of total income that would be required to bring all those underneath $\gamma\mu_F$ up to $\gamma\mu_F$.

Hence, by Proposition 1, an alternative test of second-order restricted inequality dominance up to γ^+ is to compare the generalised Schutz coefficients of two distributions for all values of γ between 0 and γ^+ . By (20), this test is equivalent to comparing the maximum distance between straight lines of slopes γ and the Lorenz curves for the two distributions, for γ between 0 and γ^+ .

This last result leads to an interesting link between Lorenz curves and restricted second-order inequality dominance. By Propositions 1 and 2, a distribution F will show more inequality than distribution G for all indices $I^U(\gamma^+) \in \Upsilon^{*s}(\gamma^+)$ if $L_G(p)$ lies initially above $L_F(p)$ and if γ^+ is no greater than the slope of the (first) tangent to the Lorenz curves of both distributions. This is illustrated in Figure 4, where the tangent to both Lorenz curves has slope γ^{++} . If such a tangent does not exist, it is because $L_G(p)$ lies everywhere above $L_F(p)$, and F will

⁵Lambert (1993, p. 43) also provides an interpretation of the usual Schutz coefficient as the proportion of total income that would need to be transferred from those above the mean to those below it for inequality to disappear completely. $S(1)$ is further equal to half the mean deviation (p.117).

then show more inequality than distribution G for all indices $I^U(\infty) \in \Upsilon^{*s}(\infty)$; in other words, we then have unrestricted second-order inequality dominance. If Lorenz curves cross n times, there will exist n tangents to both Lorenz curves: it is then the lowest slope of these tangents that is of principal interest for characterizing second-order restricted inequality dominance. The slope of the first tangent to two Lorenz curves thus has a valuable and informative ethical content:

1. it shows the maximum proportion of the mean (as a relative poverty line) up to which we can order second-order relative poverty robustly across distributions;
2. it shows the proportion of mean income up to which we may consider incomes to be relevant if we wish to rank second-order restricted inequality robustly;
3. it shows the bound on the range of γ for which the generalized Schutz index is larger in F than in G ;
4. and, by equation (12), it gives the proportion of the mean (as a relative poverty line) at which relative poverty using the average poverty gap ratio is exactly the same in both distributions, below which it is larger in F than in G , and above which it becomes larger in G than in F .

6. Relative Poverty and Restricted Inequality Dominance: A Geometric Analysis

6.1. Lorenz dominance

This section further develops the link between relative poverty and inequality by illustrating graphically how a focus on relative poverty and restricted inequality comparisons gives us a greater ranking power than a consideration of unrestricted inequality comparisons. To do this, suppose a society that comprises three individuals sharing a total income of 3μ . In what follows, when a distribution x is more unequal than a distribution y , for all $I \in \Upsilon^s$, we write $xI^s y$.

Refer now to Figure 5 and assume that the initial income distribution is given by point a in the interior of the simplex. Let us now determine those distributions for which there exists a robust ranking of inequality indices of class Υ^2 , *i.e.* distributions for which the Lorenz curves do not intersect those of distribution a . We know that permutations of the three individuals' incomes yield distributions

for which inequality is exactly the same as for distribution a . If we connect these points with line segments we obtain an irregular hexagon. All points on these line segments represent income distributions with less inequality than distribution a , except those points representing permutations of the incomes in a . The reason for this is relatively straightforward: at any point on a given line segment, a progressive transfer of income is occurring between two individuals while the income of the third individual is constant. The set of all points in the interior of the hexagon represents income distributions with less inequality than a . Indeed, moving into the interior of the hexagon, one approaches the point (μ, μ, μ) , which portrays a perfectly egalitarian distribution of incomes.

We have just provided a description of the set of distributions whose Lorenz curves dominate the Lorenz curve of distribution a (aI^2y). Let us now find distributions whose Lorenz curves are dominated by that of distribution a (yI^2a). These distributions are represented by the regions in the arrowheads situated at the extremities of the simplex. In fact, distribution a may be obtained from any one of these distributions through the simple implementation of progressive income transfers. Finally, there are distributions whose Lorenz curves intersect that of distribution a (?). We know that these distributions are dominated by distributions situated on the line segments forming the hexagon. However, since these distributions on the segments dominate distribution a , we cannot say anything concerning comparisons of inequality between distributions situated in the regions designated (?) and distribution a .

6.2. Third-order inequality dominance

There are two approaches by which we can increase the size of the set of distributions that can be ranked with a . The first is by following *inter alia* Foster and Shorrocks (1988b and c) and Davies and Hoy (1994) and by restricting the analysis to indices of class Υ^3 . Recall that inequality indices of the class Υ^3 are those which are sensitive to composite transfers. From our present purposes, we can think of a beneficial composite transfer as two income transfers between four individuals, i, j, k , and l , having incomes $y_i < y_j \leq y_k < y_l$. The first transfer is from j to i and the second is from k to l , such that the variance of the distribution does not change. Note that the first transfer is a progressive transfer, while the second one is regressive. This approach increases the number of distributions that can be subject to a robust ranking with respect to a through the addition of the regions denoted (+) and (−) in Figure 6.

To understand this diagram, refer to the definition of the variance for a distribution of income between three persons:

$$3 \cdot \sigma^2 = (y_1 - \mu)^2 + (y_2 - \mu)^2 + (y_3 - \mu)^2. \quad (22)$$

The set of points whose variance is equal to the variance of a is thus a sphere centred on (μ, μ, μ) and passing through a . The intersection of this sphere with the simplex gives us the circle drawn on Figure 6. The regions denoted $(+)$ depict distributions whose Lorenz curves intersect the Lorenz curve of distribution a and are such that aI^3y . The regions represented by $(-)$ depict distributions whose Lorenz curves intersect the Lorenz curve of distribution a and are such that yI^3a . To see this, notice that all distributions on the circle between points a and b , between points c and d , and between points e and f , are such that aI^3y . These loci are, in fact, obtained by redistributing some of the income of the person with the median income towards the richest and the poorest person without increasing the variance of the distribution. Conversely, the distributions located between points a and f , between points b and c , and between points d and e are such that yI^3a . These distributions arise from shifting some income from the wealthiest person, and some from the poorest person, towards the person in the middle and this, without increasing the variance. We thus know that distribution a can be obtained from these distributions through implementation of a beneficial composite transfer. Inequality is thus lower in a than in these other distributions for all indices of class Υ^3 .

6.3. Second-order restricted dominance

We now illustrate how restricting the analysis to measures of the class $\Upsilon^{*2}(\gamma^+)$ increases the number of distributions that can be robustly ranked with a . Though convex, social indifference curves corresponding to indices of the class $\Upsilon^{*2}(\gamma^+)$ are not strictly convex. Take the case of a total income 2μ being split between two persons. Figure 7 illustrates a social indifference curve corresponding to an inequality index sensitive to the entire distribution. As in Zheng (2001), Figure 8 illustrates the corresponding curve for an index belonging to $\Upsilon^{*2}(\gamma^+)$. Notice that in Figure 8 the set of income distributions for which each individual has an income greater than $\gamma^+\mu$ is equivalent, from a social perspective, to the egalitarian distribution in which each has income μ .

Returning to the case of three individuals, the flat segment of the social indifference curve now corresponds to the regions situated in the interior of the

dotted lines in Figure 9. These distributions are preferred to a because they are equivalent, from a social perspective, to the egalitarian distribution under social preferences corresponding to inequality indices of class $\Upsilon^{*2}(\gamma^+)$. As this triangle includes points (regions denoted $(+)$) corresponding to income distributions whose Lorenz curves intersect the Lorenz curve of distribution a , focussing on indices within the class $\Upsilon^{*2}(\gamma^+)$ increases the number of distributions that can be ranked with a .

The set of points located within the dotted triangle of Figure 9 is thus equivalent to the point (μ, μ, μ) from an ethical perspective. This triangle corresponds to the flat section on the indifference curve in Figure 8. Moreover, regions denoted $(-)$ correspond to income distributions such that $yI^{*2}a$, but whose Lorenz curves intersect a 's Lorenz curve. Given that inequality indices belonging to $\Upsilon^{*2}(\gamma^+)$ are sensitive only to changes in the bottom incomes of a distribution, we obtain a robust ranking even if the Lorenz curves intersect. To see this, notice that all points situated on the line segment passing through a and f , and located between the dotted lines, are equivalent to point a from a social perspective. Again, this is due to the fact that the social indifference surfaces are not strictly convex for indices in the class $\Upsilon^{*2}(\gamma^+)$. These points are located on a flat segment of this surface. Conversely, points located on any line segment parallel to that passing through a and f , situated to the north-west of it, and bracketed by the two dotted lines, constitute loci of distributions which are equivalent but which entail more inequality than a , since they belong to an indifference surface below that of a .

Note that for every distribution a there exists a range of sufficiently small γ^+ such that the power of ranking a over the class $\Upsilon^{*2}(\gamma^+)$ is greater than the power of ranking a over the class Υ^2 . This requires that γ^+ be small enough for the distribution a to be restricted to a flat segment of the social indifference surface. The maximal value of γ^+ for which this ranking power is greater for $\Upsilon^{*2}(\gamma^+)$ than for Υ^2 is obtained when the dotted lines that form a triangle in Figure 9 pass through a . This maximal value of γ^+ is illustrated in Figure 10. For greater values of γ^+ than the one shown in Figure 10, the ranking power of $\Upsilon^{*2}(\gamma^+)$ for a is identical to that of Υ^2 .

6.4. First-order restricted dominance

Finally, recall that it is not possible to rank distributions robustly over the class Υ^1 of inequality indices. Intuitively, this is because it is impossible for every normalised quantile to be greater in one distribution than in another: it cannot be

that everyone's share of total income be greater in one distribution than in another. Such a ranking may, however, be possible for class $\Upsilon^{*1}(\gamma^+)$. To see this graphically, refer to Figure 11 and consider the income distribution represented by point a . No other distribution in the simplex can dominate a or be dominated by a for a Υ^1 inequality dominance ranking. However, what of indices belonging to class $\Upsilon^{*1}(\gamma^+)$?

First, we know that distributions within the dotted triangle dominate distribution a , because under these distributions no one has an income below $\gamma^+\mu$. We also know that distributions represented by points on the line segments bc , ef , and hi are equivalent to distribution a for these indices. Indeed, points on the segment bc are generated from distribution a by redistributing income between individuals whose income is greater than $\gamma^+\mu$ before and after the redistribution. Since indices of the class $\Upsilon^{*1}(\gamma^+)$ are not sensitive to these transfers, these distributions are equivalent to a .

Points on the segments ef and hi simply represent distributions symmetric to those on segment bc . Points located between the segment bc and the dotted triangle (+) dominate distribution a , since they are obtained by increasing the income of a person situated below $\gamma^+\mu$ while maintaining the remaining incomes above that line. Points between the segment bc and the north-east border of the simplex (−) are dominated by distribution a , since they are obtained by redistributing some share of income from the person below the $\gamma^+\mu$ line to those whose income is already above $\gamma^+\mu$.

Points on the segment cd (excluding points c and d) cannot be ranked unambiguously relative to a , since they imply redistribution between two individuals whose income is below $\gamma^+\mu$. Points situated between the segment ce and the dotted triangle (?) dominate the distributions on the segment ce . However, they cannot be ranked relative to a , since points on the segment ce cannot be given a robust ranking relative to a . Points between ce and the edge of the triangle cde are dominated by distributions on the segment ce . However, they cannot be ranked with respect to a , since points on the segment ce cannot be ranked robustly with respect to a .

Finally, points on the segment cd , except the point c , are dominated by a (and c), since they imply a transfer from a person whose income is below $\gamma^+\mu$ to a person whose income is above $\gamma^+\mu$, while maintaining the third person's income constant. The set of points between the triangle cde and the border of the simplex can be reached through a transfer from a person whose income is below $\gamma^+\mu$ towards another whose income is above $\gamma^+\mu$. Since each of these points is

dominated by at least one point on the segment cd (or de), they are also dominated by a .

Thus, limiting ourselves to restricted indices of inequality can increase significantly the ranking power of ordinal inequality comparisons. This is most striking for first-order restricted inequality dominance, by which the set of distributions located in sections represented by the regions denoted $(+)$ or $(-)$ in Figure 11 are robustly ranked with respect to distribution a , but for which a Υ^1 ranking would not have been possible for any of the distributions in the simplex.

7. Conclusion

The paper develops links between relative poverty and restricted inequality comparisons at arbitrary orders of stochastic dominance. In particular, first-order relative poverty orderings can be usefully interpreted as first-order restricted inequality orderings. Such first-order restricted inequality comparisons are in fact frequently found in the empirical literature on the distribution of income. The indices used to perform such comparisons include, *inter alia*, proportions of individuals underneath various proportions of the mean or the median, quantiles normalised by the mean, ratios of quantiles, and mean-normalised interquantile distances. We find *inter alia* that comparisons of normalised quantiles and of proportions of individuals underneath proportions of the mean are consistent with and provide appropriate first-order inequality dominance criteria. The use of the other indices is not, however, consistent with the search for inequality dominance, restricted or unrestricted, and this, for *any* order of dominance.

There also exist interesting links between second-order restricted inequality dominance criteria and some commonly used inequality indices. For instance, the paper shows that second-order restricted inequality dominance obtains when an extended version of the Schutz coefficient can be ordered for a range of parameter values. This, in turn, is equivalent to ordering across distributions the maximum positive distance between Lorenz curves and straight lines of a given slope that pass through the origin. It is also equivalent to obtaining a second-order relative poverty ordering. Second-order relative poverty dominance of one distribution over another can be established when the proportions of the mean considered as relative poverty lines do not extend above the slope of the line that is tangent to the Lorenz curves of both distributions.

A geometric analysis finally illustrates how censoring inequality indices can increase significantly the ranking power of inequality comparisons, and how this

compares with other ways of extending the ranking power of inequality dominance criteria.

Appendix

Proof of proposition 1. In order to prove the DSI* condition we use the isoelasticity of the function $v(y)$. Given that $v(y)$ is isoelastic, given that $g(y)$ is homogenous of degree 1 for $y \leq \gamma^+ \mu_F$, and given that μ_F is also homogenous of degree 1 in y , we can write $u\left(\frac{y}{\mu_F}\right) = \left(\frac{1}{\mu_F}\right)^r u(y)$ where r represents the elasticity of the function $v(y)$. We then have

$$\int_0^a u(x) d\hat{F}(x) = \left(\frac{1}{\mu_F}\right)^r \int_0^a u(y) dF(y), \quad (23)$$

and thus

$$u(y_{e,\hat{F}}) = \left(\frac{1}{\mu_F}\right)^r u(y_{e,F}). \quad (24)$$

Again, using the isoelasticity of the function v , we conclude that:

$$y_{e,\hat{F}} = \frac{y_{e,F}}{\mu_F}. \quad (25)$$

Now, we know that $\mu_{\hat{F}} = 1$ by definition. We thus see that $I_{\hat{F}} = I_F$, and, similarly, $I_{\hat{G}} = I_G$. However, since, by definition, $\mu_{\hat{F}} = \mu_{\hat{G}} = 1$, it is sufficient to test whether $y_{e,\hat{G}} \geq y_{e,\hat{F}}$ in order to establish whether $\Delta I_{FG} = I_G - I_F \leq 0$. Given that $u(y)$ is an increasing function, this test is equivalent to verifying that

$$\int_0^a u(x) d\hat{G}(x) - \int_0^a u(x) d\hat{F}(x) \geq 0, \quad (26)$$

which is, in fact, a welfare comparison of the transformed functions $\hat{F}$ and $\hat{G}$. We can now rewrite the above expression as:

$$\begin{aligned} \int_0^a u(x) d\hat{G}(x) - \int_0^a u(x) d\hat{F}(x) &= \int_0^{\gamma^+} v(x) d\hat{G}(x) \\ &\quad - \int_0^{\gamma^+} v(x) d\hat{F}(x) \\ &\quad + v(\gamma^+) [D_{\hat{F}}^1(\gamma^+) - D_{\hat{G}}^1(\gamma^+)]. \end{aligned} \quad (27)$$

Integrating $\int_0^{\gamma^+} v(x) d\hat{F}(x)$ by parts yields:

$$\begin{aligned} \int_0^{\gamma^+} u(x) d\hat{F}(x) &= v(x) D_{\hat{F}}^1(x) \Big|_0^{\gamma^+} \\ &\quad - \int_0^{\gamma^+} \frac{dv(x)}{dx} D_{\hat{F}}^1(x) dx. \end{aligned} \quad (28)$$

We thus have:

$$\int_0^a u(x) d\hat{G}(x) - \int_0^a u(x) d\hat{F}(x) = \int_0^{\gamma^+} \frac{dv(x)}{dx} [D_{\hat{F}}^1(x) - D_{\hat{G}}^1(x)] dx. \quad (29)$$

We see that this condition is similar to condition DSU in Duclos and Makdissi (2001) for $s = 1$ and for $a = \gamma^+$ (see proof of their Proposition 3). For $s \geq 2$, we can apply condition DSU exactly in the same manner. ■

Proof of proposition 2. The proof of this proposition is trivial. It is simply a matter of noting that, for $s = 1$ and 2 the DSPR condition in Duclos and Makdissi (2001) is equivalent to the DSI* condition. ■

References

- [1] Aaberge, R. (2001), Ranking Intersecting Lorenz Curves, mimeo.
- [2] Atkinson, A.B. (1970), On the Measurement of Inequality, *Journal of Economic Theory*, 2, 244-263.
- [3] Blackorby, C. and D. Donaldson (1978), Measures of Relative Equality and Their Meaning in Terms of Social Welfare, *Journal of Economic Theory*, 18, 59-80.
- [4] Blackorby, C. and D. Donaldson (1980), Ethical Indices for the Measurement of Poverty, *Econometrica*, 48, 1053-1062.
- [5] Chakravarty, S.R. (1983), A New Index of Poverty, *Mathematical Social Sciences*, 6, 307-313.
- [6] Dasgupta, P., A.K. Sen and D. Starrett (1973), Notes on the Measurement of Inequality, *Journal of Economic Theory*, 6, 180-187.
- [7] Davidson, R. and J.Y. Duclos (2000), Statistical Inference for Stochastic Dominance and for Measurement of Poverty and Inequality, *Econometrica*, , 68, 1435-1465.
- [8] Davies, J. and M. Hoy (1994), The Normative Significance of Using Third-Degree Stochastic Dominance in Comparing Income Distributions, *Journal of Economic Theory*, 64, 520-530.
- [9] Duclos, J.Y. and P. Makdissi (2001), Restricted and Unrestricted Dominance for Welfare, Inequality and Poverty Orderings, mimeo.
- [10] Fishburn, P.C. and R.D. Willig (1984), Transfer Principles in Income Redistribution, *Journal of Public Economics*, 25, 323-328.
- [11] Foster, J.E., J. Greer and E. Torbecke (1984), A Class of Decomposable Poverty Measures, *Econometrica*, 52, 761-766.
- [12] Foster, J.E. and A. Sen (1997), On Economic Inequality after a Quarter Century, in *On Economic Inequality*, (expanded edition), Oxford: Clarendon Press.

- [13] Foster, J.E. and A.F. Shorrocks (1988), Inequality and Poverty Orderings, *European Economic Review*, 32, 654-662.
- [14] Immervoll, H., C. O'Donoghue and H. Sutherland (1999), An introduction to EUROMOD, EUROMOD Working Paper No. EM0/99.
- [15] Kakwani, N. (1980), On a Class of Poverty Measures, *Econometrica*, 48, 437-446.
- [16] Kolm, S.C. (1969), The Optimal Production of Social Justice, in J. Margolis and H. Guitton (eds.), *Public Economics*, Macmillan, London, United Kingdom.
- [17] Kolm, S.C. (1976), Unequal Inequality: I, *Journal of Economic Theory*, 12, 416-442.
- [18] Lambert, P.J. (1993), *The Distribution and Redistribution of Income: A Mathematical Analysis*, St-Martin Press.
- [19] Ravallion, M. (1994a), Measuring Social Welfare With and Without Poverty Lines, *American Economic Review*, 84, 359-364.
- [20] Shorrocks, A.F. and J. Foster (1987), Transfer Sensitive Inequality Measures, *Review of Economic Studies*, 54, 485-497.
- [21] Watts, H.W. (1968), An Economic Definition of Poverty, in D.P. Moynihan (ed.), *On Understanding Poverty*, New York: Basic Books.
- [22] Zheng, B. (2001), Poverty Orderings: A Graphical Illustration, *Social Choice and Welfare*, 18, 165-178.
- [23] Zheng, B., J.P. Formby, W.J. Smith and V.K. Chow (2000), Inequality Orderings, Stochastic Dominance and Statistical Inference, *Journal of Business and Economic Statistics*, 18, 479-488.
- [24] Zoli, C. (1999), Intersecting Generalized Lorenz Curves and the Gini index, *Social Choice and Welfare*, 16, 183-196.

Figure 1
First-order inequality dominance

Figure 2

Restricted first-order inequality dominance

Figure 3
Lorenz curves and the Schutz coefficient

Figure 4
Lorenz curves and restricted inequality dominance

Figure 5: Distribution of 3μ between 3 individuals.

Figure 6: Ordering with Υ^3 .

Figure 7: Ordering with Υ^{*2} .

Figure 8: Strictly convex social indifference curves.

Figure 9: Non-strictly convex social indifference curves.

Figure 10: Maximum γ^+ for which the power of ranking a is increased.

Figure 11: Ordering with Υ^{*1} .

- 91-01 HANEL, Petr, ***Standards in International Trade. A Canadian Perspective.*** (Paru dans *Revue Canadienne des Sciences de l'Administration - Canadian Journal of Administrative Sciences*, vol. 10.1, Mars 1993).
- 91-02 FORTIN, Mario, ***La réforme de l'impôt sur le revenu des particuliers: A-t-on vraiment abaissé les taux d'imposition?***
- 91-03 ASCAH, Louis, ***Attribution of Pension Fund Surpluses: An Economic Perspective.*** (Published by Canadian Centre for Policy Alternatives, Ottawa).
- 91-04 ASCAH, Louis, ***Attribution du surplus d'un régime de retraite - un regard économique.*** (Publié par Centre canadien de recherche en politiques de rechange, Ottawa).
- 91-05 ASCAH, Louis, ***Recent Federal and Provincial Private Pension Plan Reform Legislation: Missing, Misleading and Shrinking Proposals*** (Published by Canadian Centre for Policy Alternatives, Ottawa).
- 91-06 ASCAH, Louis, ***La comptabilité des déficits publics : des illusions à la réalité.***
- 91-07 ALLIE, E., R. DAUPHIN et M. FORTIN, ***Les aspirations de fertilité des étudiants de l'Université de Sherbrooke.***
- 92-01 HANEL, PETR, ***The Liberalization of International Trade in Czechoslovakia, Hungary, and Poland.*** (Paru dans Comparative Economic Studies, vol. 34, no 3-4, Fall-Winter 1992).
- 92-02 ASCAH, Louis, ***Public Pension Theory for the Real World.***
- 93-01 FORTIN, Mario, ***L'écart des taux de chômage canadien et américain.***
- 93-02 BILODEAU, Marc et AI SLIVINSKI, ***Rational Nonprofit Entrepreneurship.***
- 93-03 FORTIN, Mario, ***The Impact of Unemployment Insurance on the Unemployment Rate.***
- 93-04 GENTZOGLANIS, Anastassios, ***Innovation and Competition in the High - and Medium - Intensity R&D Industries.***
- 93-05 FORTIN, Mario et A. ABDELKRIM, ***Sectoral Shifts, Stock Market Dispersion and Unemployment in Canada.*** (Paru dans Applied Economics, volume 29, pp. 829-839, juin 1997.)
- 93-06 HANEL, Petr, ***Interindustry Flows of Technology: An Analysis of the Canadian Patent Matrix and Input-Output Matrix for 1978-1989.*** (Paru dans Technovation, vol 14, no. 8, October 1994).
- 94-01 BILODEAU, Marc et AI SLIVINSKI, ***Toilet Cleaning and Department Chairing: Volunteering a Public Service.*** (À paraître dans Journal of Public Economics)
- 94-02 ASCAH, Louis, ***Recent Retirement Income System Reform: Employer Plans, Public Plans and Tax Assisted Savings.***
- 94-03 BILODEAU, M. et AI SLIVINSKI, ***Volunteering Nonprofit Entrepreneurial Services.*** (À paraître dans Journal of Economic Behavior and Organization)
- 94-04 HANEL, Petr, ***R&D, Inter-Industry and International Spillovers of Technology and the Total Factor Productivity Growth of Manufacturing Industries in Canada, 1974-1989.***
- 94-05 KALULUMIA, Pene et Denis BOLDUC, ***Generalized Mixed Estimator for Nonlinear Models: A Maximum Likelihood Approach.***
- 95-01 FORTIN, Mario et Patrice Langevin, ***L'efficacité du marché boursier face à la politique monétaire.***
- 95-02 HANEL, Petr et Patrice Kayembe YATSHIBI, ***Analyse de la performance à exporter des industries manufacturières du Québec 1988.***
- 95-03 HANEL, Petr, ***The Czech Republic: Evolution and Structure of Foreign Trade in Industrial Goods in the Transition Period, 1989-1994.*** (Paru dans The Vienna Institute Monthly Report, numéro 7, juillet 1995)
- 95-04 KALULUMIA, Pene et Bernard DÉCALUWÉ, ***Surévaluation, ajustement et compétitivité externe : le cas des pays membres de la zone franc CFA.***
- 95-05 LATULIPPE, Jean-Guy, ***Accès aux marchés des pays en développement.***
- 96-01 ST-PIERRE, Alain et Petr HANEL, ***Les effets directs et indirects de l'activité de R&D sur la profitabilité de la firme.***
- 96-02 KALULUMIA, Pene et Alain MBAYA LUKUSA, ***Impact of budget deficits and international capital flows on money demand: Evidence From Cointegration and Error-Correction Model.***

- 96-03 KALULUMIA, Pene et Pierre YOUROUGOU, ***Money and Income Causality In Developing Economies: A Case Study Of Selected Countries In Sub-Saharan Africa.***
- 96-04 PARENT, Daniel, ***Survol des contributions théoriques et empiriques liées au capital humain (A Survey of Theoretical and Empirical Contributions to Human Capital).*** (Paru dans L'Actualité économique, volume 72, numéro 3, 1996)
- 96-05 PARENT, Daniel, ***Matching Human Capital and the Covariance Structure of Earnings***
- 96-06 PARENT, Daniel, ***Wages and Mobility : The Impact of Employer-Provided Training.*** (À paraître dans le Journal of Labor Economics)
- 97-01 PARENT, Daniel, ***Industry-Specific Capital and the Wage Profile : Evidence From the NLSY and the PSID.***
- 97-02 PARENT, Daniel, ***Methods of Pay and Earnings: A Longitudinal Analysis***
- 97-03 PARENT, Daniel, ***Job Characteristics and the Form of Compensation.***
- 97-04 FORTIN, Mario et Michel BERGERON, Jocelyn DUFORT et Pene KALULUMIA, ***Measuring The Impact of Swaps on the Interest Rate Risk of Financial Intermediaries Using Accounting Data.***
- 97-05 FORTIN, Mario, André LECLERC et Claude THIVIERGE, ***Testing For Scale and Scope Effects in Cooperative Banks: The Case of Les Caisses populaires et d'économie Desjardins.***
- 97-06 HANEL, Petr, ***The Pros and Cons of Central and Eastern Europe Joining the EU***
- 00-01 MAKDISSI, Paul et Jean-Yves DUCLOS, ***Restricted and Unrestricted Dominance Welfare, Inequality and Poverty Orderings***
- 00-02 HANEL, Petr, John BALDWIN et David SABOURIN, ***Les déterminants des activités d'innovation dans les entreprises de fabrication canadiennes : le rôle des droits de propriété intellectuelle***
- 00-03 KALULUMIA, Pene, ***Government Debt, Interest Rates and International Capital Flows: Evidence From Cointegration***
- 00-04 MAKDISSI, Paul et Cyril TÉJÉDO, ***Problèmes d'appariement et politique de l'emploi***
- 00-05 MAKDISSI, Paul et Quentin WODON, ***Consumption Dominance Curves: Testing for the Impact of Tax Reforms on Poverty***
- 00-06 FORTIN, Mario et André LECLERC, ***Demographic Changes and Real Housing Prices in Canada.***
- 00-07 HANEL, Petr et Sofiene ZORGATI, ***Technology Spillovers and Trade: Empirical Evidence for the G7 Industrial Countries.***
- 01-01 MAKDISSI, Paul et Quentin WODON, ***Migration, poverty, and housing: welfare comparisons using sequential stochastic dominance.*** Avril 2001. (23 p)
- 01-02 HUNG Nguyen Manh et Paul MAKDISSI, ***Infantile mortality and fertility decisions in a stochastic environment.*** Mars 2001. (12 p).
- 01-03 MAKDISSI, Paul et Quentin WODON, ***Fuel poverty and access to electricity: comparing households when they differ in needs.*** Juin 2001. (19 p)
- 01-04 MAKDISSI, Paul et Yves GROLEAU, ***Que pouvons-nous apprendre des profils de pauvreté canadiens ?*** Juillet 2001. (47 p)
- 01-05 MAKDISSI, Paul et Quentin WODON, ***Measuring poverty reduction and targeting performance under multiple government programs*** Août 2001. (16 p)
- 01-06 DUCLOS, Jean-Yves et Paul MAKDISSI, ***Restricted inequality and relative poverty.*** Août 2001. (31 p)
- 01-07 TÉJÉDO, Cyril et Michel TRUCHON, ***Serial cost sharing in multidimensional contexts*** Septembre 2001. (37 p)
- 01-08 TÉJÉDO, Cyril, ***Strategic analysis of the serial cost sharing rule with symmetric cost function.*** Février 2001. (25 p)
- 02-01 DUCLOS, Jean-Yves, Paul MAKDISSI et Quentin WODON, ***Socially-efficient tax reforms*** Janvier 2002. (47 p)

* Tous ces cahiers de recherche sont disponibles sur notre site WEB (www.usherb.ca/flsh/eco) ou au Centre de documentation de la FLSH A3-330 (UdeS).

Prrière d'adresser vos commentaires ou demandes d'exemplaires d'un cahier de recherche antérieur (1976 à 1990) à monsieur Pene KALULUMIA, coordonnateur des Cahiers de recherche du Département d'économique, Tél : (819) 821-7233 Télécopieur : (819) 821-7237 Courriel : pkalulum@courrier.usherb.ca

Comments or requests for copies of previous Working Papers (1976 to 1990) should be made to the Working Papers Coordinator at the Département d'économique, Mr. Pene KALULUMIA. Tel: (819) 821-7233 FAX:(819) 821-7237 E-mail: pkalulum@courrier.usherb.ca