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Monotonicity and Bounds for Cost Shares under the Path Serial Rule*

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Preliminary
For comments only

Abstract

We pursue the analysis of the Path Serial Cost Sharing Rule by examining how the cost share of an agent varies with respect to its own demand and the one of other agents. We also provide bounds for cost shares under an appropriate assumption on the cost function.

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1 Introduction

The Serial Cost Sharing Rule has received much attention since its introduction by Shenker (1990) and its extensive analysis by Moulin and Shenker (1992, 1994). It was originally conceived for problems where n agents request different quantities of a private good, the sum of which is produced by a single facility. This rule can be constructed from two ethical axioms: Equal Treatment of Equals (in terms of demands) and Independence of Larger Demands (a protection of small demanders against larger ones). It satisfies other interesting properties and has other characterizations as well.

In Téjédo and Truchon (2001b), we introduced the Path Serial Cost Sharing Rule to deal with situations where each agent requests a list of goods that may be private, public, or specific to some agents and where aggregate demand is not necessarily the sum of individual demands. This rule admits general paths along which demands may be scaled down to construct intermediate demands. We showed that the Path Serial Rule is characterized by the Equal Treatment of Equivalent Demands (in terms of stand alone costs) and the Path Serial Principle (a weaker form of Independence of Larger Demands). It also satisfies a general scale invariance condition defined and called Ordinality by Sprumont (1998).

In the present paper, we pursue the analysis of this rule by examining how the cost share of an agent varies with respect to its own demand and the one of other agents. We also provide bounds for cost shares under an appropriate assumption on the cost function. Moulin and Shenker (1994) prove that, under appropriate assumptions on the cost function, the original Serial Rule produces cost shares that are monotone with respect to own and others' demands and that lay between reasonable bounds. We transpose their results to the Path Serial Rule by restricting Monotonicity and Cross Monotonicity to hold along paths. Moulin (1996) shows that the original Serial Rule satisfies the Stand Alone Test, i.e. under increasing returns, no agent and no subset of agents pay more than their stand alone cost. We also extend this result to the Path Serial Rule.

The paper is organized as follows. For the sake of completeness, we give again the complete formulation of the problem and the main definitions in Section 2. In particular, we add a definition of "diminishing and increasing incremental cost", which will be used to define Path Cross Monotonicity and the Stand Alone Test. The Path Serial Cost Sharing Rule is defined in Section 3. Monotonicity is the object of Section 4 while the bounds for cost shares are dealt with in Section 5. A brief conclusion follows as Section 6. Most proofs are collected in the last section.

2 The Cost Sharing Problem

A cost sharing problem starts with a profile of demands, to which a cost function is applied. In some cases, as with serial cost sharing, demands may have to be scaled down to meet certain conditions. The cost sharing problem must thus be completed by a description of how this should be made. We address each of these elements in the next three subsections.

2.1 The demands

Throughout this paper, there is a fixed set of divisible *commodities* $K = \{1, \ldots, k\}$ and a fixed set of *agents* $N = \{1, \ldots, n\}$. The commodities may be goods, characteristics serving to describe needs, or specifications of a certain facility. A commodity may be specific to a particular agent or a subset of agents. This means that these agents are the only ones to be able to consume, use, or enjoy the commodity in question. Hence, they will be the only ones to demand positive quantities of this commodity. As for non specific commodities, they may be private or public or anything in between.

For each agent $i \in N$, let there be a positive integer $m_i \leq k$ and a one-to-one function $\ell_i : \{1, \ldots, m_i\} \to K$, specifying the list of commodities that may be requested by this agent. Next, let M_i be the range of ℓ_i, i.e. $M_i = \{\ell_i(1), \ldots, \ell_i(m_i)\}$. In plain words, M_i is the subset of commodities for which agent i may request a positive quantity. We assume that $K = \cup_{i=1}^n M_i$. Thus, for each commodity, there is at least one agent concerned by this commodity.

The demand of agent i is described by a vector $q_i \in \mathbb{R}_+^{m_i}$. The scalar q_{ih} is the demand of commodity $\ell_i(h) \in M_i$ by agent i. Let $M = \{M_1, \ldots, M_n\}$ with cardinality $m = \sum_{i=1}^n m_i \leq nk$. A *profile of demands* is an element $Q \in \mathbb{R}_+^m = \prod_{i=1}^n \mathbb{R}_+^{m_i}$. Given a subset $S \subset N$ and $Q \in \mathbb{R}_+^m$, let $Q^S \in \mathbb{R}_+^m$ be the vector obtained from Q by changing all components q_j, $j \in N \backslash S$, for components of 0.

2.2 The cost function

To complete the description of the problem, we assume that the agents jointly own a facility to jointly produce any list of commodities that are requested. The cost of producing a bundle $Y \in \mathbb{R}_+^m$ is $C(Y)$. A special case is $M_i = K$ $\forall i$ and $C(Y) = c(\sum_i y_i)$ with $c : \mathbb{R}_+^k \to \mathbb{R}_+$. In this case, all commodities are homogeneous and private. Following Moulin and Shenker (1994) and Sprumont (1998), we call these functions and the resulting problems *homogeneous.*

A *cost function* $C:\mathbb{R}_{+}^{m}\rightarrow\mathbb{R}_{+}$ also induces n *stand alone cost functions* $c_i:\mathbb{R}_{+}^{m_i}\rightarrow\mathbb{R}_{+}$ defined by:

$$c_i\left(y_i\right)=C\left(Y^{\{i\}}\right)\ \forall i\in N$$

We shall say that $c_i:\mathbb{R}_{+}^{m_i}\rightarrow\mathbb{R}_{+}$ is increasing if $y_i\ll y_i^{\prime}$ implies $c_i\left(y_i\right)<c_i\left(y_i^{\prime}\right)$. Thus, c_i is increasing if an increase in all components of y_i yields a cost increase.

Let $\mathbb{C}\left(m\right)$ be the class of continuous and non-decreasing functions $C:\mathbb{R}_{+}^{m}\rightarrow\mathbb{R}_{+}$ satisfying:

- $C\left(0\right)=0$,
- the functions $c_i,\ i=1,\ldots,n$, induced by C are increasing,
- $\forall Y\in\mathbb{R}_{+}^{m},\ \forall i\in N:c_i\left(y_i\right)=0\Rightarrow C\left(Y\right)=C\left(Y^{N\backslash\{i\}}\right)$,
- $\forall i\in N,\ \forall Y\in\mathbb{R}_{+}^{m}:C\left(Y^{N\backslash\{i\}}\right)=C_{-i}\left(Y_{-i}\right)$, where C_{-i} is the restriction of C to the reduced profile $Y_{-i}=\left(y_1,\ldots,y_{i-1},y_{i+1},\ldots,y_n\right)$

We shall work with this class of functions throughout the paper. Whereas we need the mild assumption that each c_i be increasing, we do not want to impose and we do not need that C be increasing. In other words, $Y\leq Y^{\prime}\in\mathbb{R}_{+}^{m}$ and $y_i\ll y_i^{\prime}$ for some i do not necessarily imply $C\left(Y\right)<C\left(Y^{\prime}\right)$. Indeed, C may be the result of a more or less complex aggregation and optimization procedure. Thus, it is not necessarily increasing in all its components as, for example, when some public goods are involved. The last two conditions defining $\mathbb{C}\left(m\right)$ are natural. A demand from an agent with null stand alone cost has the same impact on total cost than a null demand and removing an agent with a null demand from a problem should have no impact on total cost.

In certain circumstances, the shape of the cost function may be of some importance. In particular, the behavior of the incremental cost, i.e. the change in cost following an increase in the level of production, may matter. These incremental costs may increase or diminish with the level of production. We now give a formal content to these concepts. In the following definition, we treat two increments in two different components of Y as equivalent if their impacts on their respective stand alone costs are the same. In the case of a single private good, this would imply identical increments.

Definition 1 A cost function $C \in \mathbb{C}(m)$ satisfies Diminishing Incremental Cost (DIC) if for any triple $(Y, Y', Z) \in \mathbb{R}_+^{3m}$ such that $Y \leq Y'$ and any pair $(i, j) \in N^2$ such that $c_i(y_i) \geq c_j(y_j)$ and $c_i(y_i + z_i) - c_i(y_i) = c_j(y_j + z_j) - c_j(y_j)$, the following holds:

$$C\left(Y + Z^{\{i\}}\right) - C(Y) \geq C\left(Y' + Z^{\{j\}}\right) - C(Y') \tag{1}$$

It satisfies Increasing Incremental Cost (IIC) if $-C$ satisfies (DIC).

Remark 1 Why should we insist on (1) to declare C as being a (DIC) function? Note that the condition $c_i(y_i + z_i) - c_i(y_i) = c_j(y_j + z_j) - c_j(y_j)$ may be written as:

$$C\left(Y^{\{i\}} + Z^{\{i\}}\right) - C\left(Y^{\{i\}}\right) = C\left(Y^{\{j\}} + Z^{\{j\}}\right) - C\left(Y^{\{j\}}\right) \tag{2}$$

Adding $Y^{\{j\}}$ and $Y^{\{i\}}$ to the arguments of the left and right members respectively should bring a lower value for both. However, the condition $c_i(y_i) \geq c_j(y_j)$ means that y_i is in a sense "larger" than y_j. Thus $Y^{\{i\}}$ is "larger" than $Y^{\{j\}}$ and if (DIC) holds, we should expect the value of the right member of (2) to decrease more than the left one, i.e. $C\left(Y^{\{i,j\}} + Z^{\{i\}}\right) - C\left(Y^{\{i,j\}}\right) \geq C\left(Y^{\{i,j\}} + Z^{\{j\}}\right) - C\left(Y^{\{i,j\}}\right)$. From this inequality, we may say that $Z^{\{i\}}$ is "larger" than $Z^{\{j\}}$. Thus, we should have $C\left(Y + Z^{\{i\}}\right) - C(Y) \geq C\left(Y + Z^{\{j\}}\right) - C(Y)$. Finally, changing Y for Y' in the right member can just reinforce this inequality to meet the claim that C is a (DIC) function. This is precisely what (1) says.

Note that we may have $i = j$. Actually, for homogeneous C^2 functions, (DIC) is merely an implication of a property of concavity, namely the second order directional derivatives are non-positive. (DIC) has itself several implications, which are recorded in the following lemma.

Lemma 1 *Let $C \in \mathbb{C}(m)$ satisfies (DIC), then:*

1. *For any triple $(Y, Y', Z) \in \mathbb{R}_+^{3m}$ such that $Y \leq Y'$, the following must hold:*

$$C(Y + Z) - C(Y) \geq C(Y' + Z) - C(Y') \tag{3}$$

2. *For any $Z \in \mathbb{R}_+^m$, let $I(Z) = \{i \in N : z_i \neq 0\}$. Then, for any triple $(Y, Y', Z) \in \mathbb{R}_+^{3m}$ such that $Y \leq Y'$, $Y + Z \leq Y' + Z^{\{h\}}$ for some $h \in I(Z)$, $c_i(y_i) \geq c_h(y_h)$ and $c_i(y_i + z_i) - c_i(y_i) = c_h(y_h + z_h) - c_h(y_h)$ $\forall i \in I(Z)$, the following must hold:*

$$C(Y + Z) - C(Y) \geq \#I(Z)\left(C\left(Y' + Z^{\{h\}}\right) - C(Y')\right) \tag{4}$$

3. For any pair $(Y,Y') \in \mathbb{R}_+^m \times \mathbb{R}_+^m$ such that $Y \leq Y'$ we have:

$$\sum_{i=1}^{n} c_i\left(y_i'\right) - \sum_{i=1}^{n} c_i\left(y_i\right) \geq C\left(Y'\right) - C\left(Y\right) \tag{5}$$

The above propositions hold with the reverse inequality if C satisfies (IIC).

The proof is given in subsection 7.1.

Remark 2 Condition (3) by itself could be viewed as a (DIC) condition. However, while (5) follows from (3), we need the stronger Definition 1 to get (4), which will be needed to prove path cross monotonicity. The definition is stronger in that it involves the variation in incremental costs with respect to different increments in the demand while (3) involves the same increment. Since condition (5) can be written as $\sum_{i=1}^{n} c_i\left(y_i'\right) - C\left(Y'\right) \geq \sum_{i=1}^{n} c_i\left(y_i\right) - C\left(Y\right)$, it may be called "increasing benefit from cooperation".

2.3 The paths

Serial cost sharing requires that larger demands be initially scaled down to a level equivalent to smaller ones. In some circumstances, it may be natural to adjust all components of the demand of an agent along the ray to which it belongs, i.e. proportionally. This is the method used in the Radial Serial Rule. In other circumstances, this may not be appropriate. As pointed out by Koster et al. (1998) in their Remark 3.7, one can envisage other extensions of the serial rule using more general paths to scale the demands. This is the idea behind the Path Serial Rule. This approach requires that we add the rules according to which demands must be scaled to Q and C in the definition of a cost sharing problem.

For each $i \in N$, consider functions $h_i : \mathbb{R}_+^{m_i+1} \to \mathbb{R}_+^{m_i}$, which map each $y \in \mathbb{R}_+^{m_i}$ and $\tau \in \mathbb{R}_+$ onto a vector $h_i\left(y,\tau\right) \in \mathbb{R}_+^{m_i}$. Assume that $h_i\left(y,\cdot\right)$ is non-decreasing, increasing without bound in at least one component, and that for each $y \in \mathbb{R}_+^{m_i}$, there exists a $\tau' \in \mathbb{R}_+$ (necessarily unique) such that $h_i\left(y,\tau'\right) = y$. Let $\mathcal{H}_i$ be the class of these functions. Then, $h_i\left(y,\mathbb{R}_+\right)$ is the path through y defined by $h_i\left(y,\cdot\right)$. Clearly, the class $\left\{h_i\left(y,\mathbb{R}_+\right) : y \in \mathbb{R}_+^{m_i}\right\}$ scans $\mathbb{R}_+^{m_i}$ since h_i is defined for each $y \in \mathbb{R}_+^{m_i}$. Finally, let $h_i^R : \mathbb{R}_+^{m_i} \backslash \{0\} \times \mathbb{R}_+ \to \mathbb{R}_+^{m_i}$ be defined by $h_i^R\left(y,\tau\right) = \tau y$. This function defines the ray through a $y \neq 0$.

We do not impose that $h_i\left(y,0\right) = 0$ and that $h_i\left(y,\cdot\right)$ be continuous and increasing in all components. However, given a function $C \in \mathbb{C}\left(m\right)$, we restrict ourselves to the class of functions $\mathcal{H}_i\left(c_i\right) \subset \mathcal{H}_i$ for which $c_i\left(h_i\left(y,\cdot\right)\right)$ is continuous and increasing, with $c_i\left(h_i\left(y,0\right)\right) = 0$. Since $c_i\left(0\right) = 0$ and since c_i is increasing, this implies that there is at least one null component in $h_i(y,0)$. In words, a path starts on an axis but not necessarily at the origin. The cost of the bundle at the starting point is null and increasing thereafter. This definition of $\mathcal{H}_i(c_i)$ insures that for any $\alpha \in \mathbb{R}_+$, there is a unique τ_α such that $c_i(h_i(y,\tau_\alpha)) = \alpha$.

Let $\mathcal{H}(C) = \mathcal{H}_1(c_1) \times \cdots \times \mathcal{H}_n(c_n)$, $H(Y,\tau) = (h_1(y_1,\tau_1), \ldots, h_n(y_n,\tau_n))$, and $\mathbb{C}(m) \times \mathcal{H} = \{(C,H) : C \in \mathbb{C}(m) \text{ and } H \in \mathcal{H}(C)\}$. A *cost sharing problem* is a triple $(Q,C,H) \in \mathbb{R}_+^m \times \mathbb{C}(m) \times \mathcal{H}(C)$. Accordingly, a *cost sharing rule* is a mapping $\xi : \mathbb{R}_+^m \times \mathbb{C}(m) \times \mathcal{H} \to \mathbb{R}_+^n$ satisfying the budget balance condition:

$$\sum_{i=1}^{n} \xi_i(Q,C,H) = C(Q)$$

The vector $\xi(Q,C,H)$ is the list of cost shares for the problem (Q,C,H).

We assume that H is exogenous as is the case of Q. The choice of h_i may come from agent i, be imposed by the planner or be negotiated between all those concerned. The criteria leading to the adoption of a particular h_i may include technological considerations or preferences. For example, the different components of q_i may pertain to different technical characteristics of a facility and for technological reasons that only agent i knows, any change in q_i should be done according to a function h_i (not necessarily linear) supplied by the agent. Alternatively, h_i may be the expression of a preference by the agent. In the example given below, each agent has a two-component demand, gas in summer and gas in winter. If these demands are to be reduced, some agents may prefer a reduction of gas available in summer rather than a proportional reduction of both. Others may have different desiderata.

3 The Path Serial Cost Sharing Rule

In essence, it consists in first ordering individual demands according to their stand alone costs. Next, a first intermediate demand is constructed by reducing demands of agents 2 to n along the respective paths specified by the h_i, down to the points where their stand alone costs are the same as for agent 1 and the cost of this intermediate demand is shared equally among all agents. A second intermediate demand is constructed by reducing demands of agents 3 to n along the same paths down to the point where their stand alone costs are the same as for agent 2, etc., and the incremental cost of this intermediate demand as compared to the first one is shared equally among agents 2 to n.

Definition 2 (The Path Serial Rule) Given a problem $(Q, C, H) \in \mathbb{R}_+^m \times \mathbb{C}(m) \times \mathcal{H}(C)$, suppose, without loss of generality, that agents are ranked according to their $c_i(q_i)$:

$$c_1(q_1) \leq c_2(q_2) \leq \ldots \leq c_n(q_n).$$

Then, for each i, consider the *intermediate demand* $Q^i = (q_1^i, \ldots, q_n^i) \in \mathbb{R}_+^m$ defined by:

$$\begin{cases} q_j^i = q_j & \text{if} \quad c_j(q_j) \leq c_i(q_i) \\ q_j^i \in h_j(q_j, \mathbb{R}_+) \text{ and } c_j\left(q_j^i\right) = c_i(q_i) & \text{if} \quad c_j(q_j) > c_i(q_i) \end{cases}$$

By definition of $\mathcal{H}(C)$, these intermediate demands are uniquely defined. Finally, the cost allocation of the *Path Serial Rule* is given by the following formula:

$$\xi_i^{PS}(Q, C, H) = \sum_{j=1}^{i} \frac{C(Q^j) - C(Q^{j-1})}{n+1-j}, \ i = 1, \ldots, n.$$

Remark 3 The Radial Serial Rule ξ^{RS} of Koster et al. (1998) may be seen as the Path Serial Rule ξ^{PS} with the use of h_i^R as the scaling function for all i, and any pair $(Q, C) \in \mathbb{R}_+^m \times \mathbb{C}(m)$. In short, $\xi^{RS}(Q, C) = \xi^{PS}\left(Q, C, H^R\right)$. Both ξ^{PS} and ξ^{RS} reduce to the Axial Rule ξ^A of Sprumont (1998) when $M_i = \{i\}$ $\forall i$ and all three reduce to the Moulin-Shenker rule in the context of the single private good. They are Serial Extensions of the original Serial Rule.

4 Monotonicity

An ethical condition that has received much attention in the literature on cost sharing is monotonicity of the cost shares with respect to own demands. The Path Serial Rule does not satisfy the original monotonicity condition in the general context but we show that it satisfies a weaker form of this condition, called Path Demand Monotonicity. We also examine the behavior of the cost shares with respect to others' demands.

Definition 3 A cost sharing rule $\xi : \mathbb{R}_+^m \times \mathbb{C}(m) \times \mathcal{H} \to \mathbb{R}_+^n$ satisfies *Demand Monotonicity* (DM) if for two problems (Q, C, H) and $(Q', C, H') \in \mathbb{R}_+^m \times \mathbb{C}(m) \times \mathcal{H}(C)$, and any $i \in N$ such that $q_i \leq q_i'$ and $q_j = q_j'$ $\forall j \in N \backslash \{i\}$, we have $\xi_i(Q, C, H) \leq \xi_i(Q', C, H')$.

(DM) says that an agent should expect to pay more if he increases his demand. This does not imply that the other agents will not pay more as we shall see. Sprumont (1998) proves that the Axial Rule ξ^A satisfies (DM) in the context where $M_i = \{i\}$ $\forall i$. We show in Téjédo-Truchon (2001a) that this is not the case of the Radial Serial Rule ξ^{RS} even in the homogeneous context. A fortiori, the Path Serial Rule ξ^{PS} does not satisfy (DM) in the general context. This motivates the next definition.

Definition 4 A cost sharing rule $\xi : \mathbb{R}_+^m \times \mathbb{C}(m) \times \mathcal{H} \rightarrow \mathbb{R}_+^n$ satisfies *Path Demand Monotonicity* (PDM) if for two problems (Q, C, H) and $(Q', C, H) \in \mathbb{R}_+^m \times \mathbb{C}(m) \times \mathcal{H}(C)$, for any $i \in N$ such that $q_i \in h_i(q_i', \mathbb{R}_+)$, $q_i \leq q_i'$, and $q_j = q_j'$ $\forall j \in N \backslash \{i\}$, we have $\xi_i(Q, C, H) \leq \xi_i(Q', C, H)$.

Ideally if the cost function satisfies decreasing incremental cost (DIC), then the cost share of an agent i should not increase when the demand of another agent k increases and it should not decrease when the cost function satisfies increasing incremental cost (IIC). We shall show that this is the case with the Path Serial Rule.

Definition 5 A cost sharing rule $\xi : \mathbb{R}_+^m \times \mathbb{C}(m) \times \mathcal{H} \rightarrow \mathbb{R}_+^n$ satisfies *Path Cross Monotonicity* (PCM) if for two problems (Q, C, H) and $(Q', C, H) \in \mathbb{R}_+^m \times \mathbb{C}(m) \times \mathcal{H}(C)$ such that $q_k \in h_k(q_k', \mathbb{R}_+)$ and $q_k \leq q_k'$ for some k and $q_j = q_j'$ $\forall j \neq k$, we have:

- $\xi_i(Q, C, H) \geq \xi_i(Q', C, H)$ $\forall i \in N/\{k\}$ whenever C is a DIC cost function;
- $\xi_i(Q, C, H) \leq \xi_i(Q', C, H)$ $\forall i \in N/\{k\}$ whenever C is an IIC cost function, i.e. ξ_i is a non-decreasing function along the path $h_k(q_k, \mathbb{R}_+)$.

Theorem 1 *ξ^{PS} satisfies (PDM) and (PCM).*

The proof is given in subsection 7.2. Of course, we should expect the relation between $\xi_i^{PS}(Q, C, H)$ and $\xi_i^{PS}(Q', C, H)$ to hold when more than one component of Q is increased to give Q'. This is recorded in the following corollary.

Corollary 1 *Consider two problems (Q, C, H) and $(Q', C, H) \in \mathbb{R}_+^m \times \mathbb{C}(m) \times \mathcal{H}(C)$ such that $\exists i \in N : \forall j \neq i : q_j \in h_j\left(q_j', \mathbb{R}_+\right)$ and $q_j \leq q_j'$. Then:*

- *$\xi_i^{PS}(Q, C, H) \geq \xi_i^{PS}(Q', C, H)$ whenever C is a DIC cost function;*
- *$\xi_i^{PS}(Q, C, H) \leq \xi_i^{PS}(Q', C, H)$ whenever C is an IIC cost function.*

The proof is given in subsection 7.3.

5 Bounds for Cost Shares

If they are free to decide, agents will choose to participate in a cost sharing problem only if they are guaranteed that their share of the cost will not be larger than their stand alone cost. This is a condition that Moulin and Shenker (1992) calls *participation*. We show that the Path Serial Rule meets this condition whenever C satisfies *increasing benefit from cooperation*, defined in Remark 2. Actually, under (DIC), the Path Serial Rule satisfies a stronger property. Any coalition of agents is guaranteed that their total share of the cost will not be larger than their stand alone cost as a coalition, a condition called the *Stand Alone Test* by Faulhaber (1975) and similar to the core property in cooperative game theory. We show that these properties, established by Moulin (1996) in the single private good context, extend to the general context of this paper under increasing benefit from cooperation or diminishing incremental cost.

If the cost function satisfies (IIC) instead of (DIC), we should not expect all agents to be willing to cooperate since at least one of them will have to pay more than its stand alone cost. However, there may be circumstances where agents are forced to cooperate even if the cost function satisfies (IIC). In such a case, we could impose that each agent or coalition pays at least its stand alone cost, the reverse of the previous conditions.[1] The Path Serial rule satisfies these reverse conditions.

Definition 6 A cost sharing rule $\xi : \mathbb{R}_+^m \times \mathbb{C}(m) \times \mathcal{H} \to \mathbb{R}_+^n$ satisfies *Participation* if $\forall (Q, C, H) \in \mathbb{R}_+^m \times \mathbb{C}(m) \times \mathcal{H}(C)$:

- $\xi_i(Q, C, H) \leq c_i(q_i)$ $\forall i \in N$ whenever C satisfies increasing benefit from cooperation;
- $\xi_i(Q, C, H) \geq c_i(q_i)$ $\forall i \in N$ whenever C satisfies decreasing benefit from cooperation.

Theorem 2 *The Path Serial Rule ξ^{PS} satisfies Participation.*

The proof is given in subsection 7.4.

Definition 7 A cost sharing rule $\xi : \mathbb{R}_+^m \times \mathbb{C}(m) \times \mathcal{H} \to \mathbb{R}_+^n$ satisfies the *Stand Alone Test* (SAT) if $\forall (Q, C, H) \in \mathbb{R}_+^m \times \mathbb{C}(m) \times \mathcal{H}(C)$:

- $\sum_{i \in S} \xi_i(Q, C, H) \leq C\left(Q^S\right)$ for any subset $S \subset N$ whenever C is a DIC cost function;
- $\sum_{i \in S} \xi_i(Q, C, H) \geq C\left(Q^S\right)$ for any subset $S \subset N$ whenever C is an IIC cost function.

Theorem 3 *Any cost sharing rule $\xi : \mathbb{R}_+^m \times \mathbb{C}(m) \times \mathcal{H} \to \mathbb{R}_+^n$ that satisfies (PCM) also meets the Stand Alone Test (SAT).*

[1] This can be seen as a fairness condition, to which Moulin and Shenker (1992) gave the name of Stand Alone Test. However, Faulhaber (1975), Moulin (1996), and others reserve this name for the condition defined above. In this paper, we cover both conditions under the same name.

The proof is given in subsection 7.5. The following corollary is immediate since ξ^{PS} satisfies (PCM).

Corollary 2 *The Path Serial Rule ξ^{PS} satisfies (SAT).*

Remark 4 Participation follows obviously from (SAT). However, we have been able to establish this property under the weaker *increasing benefit from cooperation.*

The other question of interest in the case of an IIC cost function is whether there is a reasonable upper bound on the contribution of each agent. In order to introduce such a bound, we first define for each i an *equal cost demand* $\tilde{Q}^i \in \mathbb{R}_+^m$ by scaling the demands up or down so as to satisfy:

$$\begin{array}{lll} q_j \in h_j\left(\tilde{q}_j^i, \mathbb{R}_+\right) & \text{and } c_j\left(\tilde{q}_j^i\right) = c_i\left(q_i\right) & \text{if } c_j\left(q_j\right) < c_i\left(q_i\right) \\ \tilde{q}_j^i \in h_j\left(q_j, \mathbb{R}_+\right) & \text{and } c_j\left(\tilde{q}_j^i\right) = c_i\left(q_i\right) & \text{if } c_j\left(q_j\right) \geq c_i\left(q_i\right) \end{array}$$

We then have the following condition.

Definition 8 A cost sharing rule $\xi : \mathbb{R}_+^m \times \mathbb{C}(m) \times \mathcal{H} \rightarrow \mathbb{R}_+^n$ satisfies the *Equal Cost Bound* (ECB) for a problem $(Q, C, H) \in \mathbb{R}_+^m \times \mathbb{C}(m) \times \mathcal{H}(C)$ if:

$$\xi_i(Q, C, H) \leq \frac{C\left(\tilde{Q}^i\right)}{n} \; \forall i \in N$$

For a homogeneous problem, $C\left(\tilde{Q}^i\right) = c\left(n\, q_i\right)$. Thus, the condition generalizes the *Unanimity Bound* of Moulin and Shenker (1992). Clearly, ξ^{PS} satisfies (ECB) for DIC cost functions. We shall now show that this bound is also satisfied for IIC cost functions.

Theorem 4 *The Path Serial Rule ξ^{PS} satisfies (ECB) for any class of problems $(Q, C, H) \in \mathbb{R}_+^m \times \mathbb{C}(m) \times \mathcal{H}(C)$ such that C is an IIC cost function.*

Proof. Since $\tilde{q}_j^i \geq q_j^i = q_j \; \forall j < i$ and $\tilde{q}_j^i = q_j^i \; \forall j \geq i$, applying Corollary 1, we have:

$$\xi_i^{PS}(Q, C, H) = \xi_i^{PS}\left(Q^i, C, H\right) \leq \xi_i^{PS}\left(\tilde{Q}^i, C, H\right) = \frac{C\left(\tilde{Q}^i\right)}{n} \; \forall i \in N$$

■

6 Conclusion

In Téjédo and Truchon (2001b), we have have defined the Path Serial Cost Sharing Rule to deal with problems where agents request several commodities that can be public, private, or specific to some of them and where aggregation may be very general. As it names implies, it consists in scaling down the demands along paths, which are part of the specification of the problem, in order to construct the intermediate demands that are at the root of serial cost sharing. Put differently, the Path Serial Rule consists in applying the original Serial Cost Sharing Rule to a projection of each demand onto the specified path. The rule is characterized by the Equal Treatment of Equivalents (demands) and the Path Serial Principle, and it satisfies other properties such as Independence of Null Agents, Rank Independence of Irrelevant Agents, and Ordinality.

In the present paper, we have extended the analysis of this rule by examining how the cost share of an agent varies with respect to its own demand and the one of other agents. We have shown that the Path Serial Rule satisfies Path Demand Monotonicity and Path Cross Monotonicity. The first says that the cost share of an agent does not decrease if the demand of this agent increases along the path specified with its demand. The second prescribes that under increasing returns, the cost share of an agent must not increase if some other agent increases its demand along the path specified with its demand ant that it must not decrease under decreasing returns.

We have also provided bounds for cost shares under increasing and decreasing returns. More precisely, under increasing returns, no agent and no subset of agents pay more than their stand alone cost. Under decreasing returns, no agent and no subset of agents pay less than their stand alone cost. These results generalize similar results of Moulin and Shenker (1994) and Moulin (1996) for the original Serial Rule.

7 Proofs

7.1 Proof of Lemma 1

1. Applying (1) iteratively to the triples $\left(Y+Z^{\{1,\dots,i-1\}}, Y'+Z^{\{1,\dots,i-1\}}, Z\right)$ and the pair $(i,i)\,,\ i=1,\dots,n$, with $Z^{\emptyset}=0$, to get

$$C\left(Y+Z^{\{1,\dots,i\}}\right)-C\left(Y+Z^{\{1,\dots,i-1\}}\right)\geq C\left(Y'+Z^{\{1,\dots,i\}}\right)-C\left(Y'+Z^{\{1,\dots,i-1\}}\right)$$

   and summing member-wise over i yields (3).

2. Consider any increasing sequence $S_1,S_2,\dots,S_{\#I(Z)}$ of proper subsets of $I(Z)$ such that h belongs only to $S_{\#I(Z)}$ and let $S_0=\emptyset$. Note that j is the cardinality of S_j and that $Y+Z^{S_j}\leq Y',\ j=1,\dots,\#I(Z)-1$. Thus, by (DIC), we can write

$$C\left(Y+Z^{S_j}\right)-C\left(Y+Z^{S_{j-1}}\right)\geq C\left(Y'+Z^{\{h\}}\right)-C\left(Y'\right),\ j=1,\dots,\#I(Z)$$

   with $Z^{S_0}=0$. Summing over all $j\in I(Z)$ yields (4).

3. Consider the triples $\left(Y^{\{i\}}, Y'^{\{i\}}, \hat{Z}^i\right)$ where $\hat{Z}^i=\left(y_1,\dots,y_{i-1},0,y'_{i+1},\dots,y'_n\right)$, $i=1,\dots,n$. Then, using (3) on each triple, we get

$$\begin{aligned}&C\left(y_1,\dots,y_{i-1},y_i,y'_{i+1},\dots,y'_n\right)-C\left(0,\dots,0,y_i,0,\dots,0\right)\\ \geq\ &C\left(y_1,\dots,y_{i-1},y'_i,y'_{i+1},\dots,y'_n\right)-C\left(0,\dots,0,y'_i,0,\dots,0\right)\end{aligned}$$

   from which:

$$\begin{aligned}c_i\left(y'_i\right)-c_i\left(y_i\right) &= C\left(0,\dots,0,y'_i,0,\dots,0\right)-C\left(0,\dots,0,y_i,0,\dots,0\right)\\ &\geq C\left(y_1,\dots,y_{i-1},y'_i,y'_{i+1},\dots,y'_n\right)-C\left(y_1,\dots,y_{i-1},y_i,y'_{i+1},\dots,y'_n\right)\end{aligned}$$

   Summing member-wise over i yields (5).

7.2 Proof of Theorem 1

Consider two problems (Q,C,H) and $(Q',C,H)\in\mathbb{R}_+^m\times\mathbb{C}(m)\times\mathcal{H}(C)$ such that C is an IIC cost function and such that $q_k\in h_k\left(q'_k,\mathbb{R}_+\right),\ q'_k\geq q_k$ and $q'_j=q_j\ \forall j\neq k$. We shall show that $\xi_i^{PS}(Q,C,H)\leq\xi_i^{PS}(Q',C,H)\ \forall i\in N$. We first suppose that $c_1(q_1)\leq c_2(q_2)\leq\ldots\leq c_n(q_n)$ and $c_1(q'_1)\leq c_2(q'_2)\leq\ldots\leq c_n(q'_n)$. We must distinguish four cases:

- $i<k$: In this case, $\xi_i^{PS}(Q',C,H)=\xi_i^{PS}(Q,C,H)$ by (PSP).

- $i = k$: In this case, $\xi_i^{PS}(Q', C, H) \geq \xi_i^{PS}(Q, C, H)$ since $C(Q'^i) \geq C(Q^i)$, $C(Q'^{i-1}) = C(Q^{i-1})$, and $\xi_j^{PS}(Q', C, H) = \xi_j^{PS}(Q, C, H)$ $\forall j < k$. This, together with the complement given below for the case where the ranks of the agents are changed when going from Q to Q', establishes (PDM).

- $i = k+1$: Note that

$$\xi_{k+1}^{PS}(Q, C, H) = \sum_{j=1}^{k-1} \xi_j^{PS}(Q, C, H) + \frac{C\left(Q^k\right) - C\left(Q^{k-1}\right)}{n-k+1} + \frac{C\left(Q^{k+1}\right) - C\left(Q^k\right)}{n-k}$$

By (PSP), we know that $\xi_j^{PS}(Q', C, H) = \xi_j^{PS}(Q, C, H)$ $\forall j < k$. We shall show that

$$\begin{aligned} &\frac{C\left(Q^k\right) - C\left(Q^{k-1}\right)}{n-k+1} + \frac{C\left(Q^{k+1}\right) - C\left(Q^k\right)}{n-k} \\ \leq &\frac{C\left(Q'^k\right) - C\left(Q'^{k-1}\right)}{n-k+1} + \frac{C\left(Q'^{k+1}\right) - C\left(Q'^k\right)}{n-k}, \end{aligned} \tag{6}$$

which is equivalent to:

$$C\left(Q'^k\right) - C\left(Q^k\right) \leq (n-k+1)\left(C\left(Q'^{k+1}\right) - C\left(Q^{k+1}\right)\right)$$

This is a necessary condition for $\xi_{k+1}^{PS}(Q, C, H) \leq \xi_{k+1}^{PS}(Q', C, H)$ to hold.

Let $Y = Q^k$, $Y' = Q^{k+1}$, $Z = Q'^k - Q^k = \left(0, \ldots, 0, q_k'^k - q_k^k, \ldots, q_n'^k - q_n^k\right)$ and note that $Q'^{k+1} - Q^{k+1} = Z^{\{k\}}$. By definition, we have $c_j\left(q_j'^k\right) = c_k\left(q_k'\right)$ and $c_j\left(y_j\right) = c_j\left(q_j^k\right) = c_k\left(q_k\right) = c_k\left(y_k\right)$ $\forall j \geq k$, from which $c_j\left(q_j'^k\right) - c_j\left(q_j^k\right) = c_k\left(q_k'\right) - c_k\left(q_k\right)$, i.e. $c_j\left(y_j + z_j\right) - c_j\left(y_j\right) = c_k\left(y_k + z_k\right) - c_k\left(y_k\right)$ $\forall i, j \geq k$. Also note that $Y + Z = Q'^k \leq Q'^{k+1} = Y' + Z^{\{k\}}$. Thus, (6) follows from part 2 of Lemma 1 and more precisely from (4) with the reversed inequality.

- $i > k+1$: In this case, $Q'^i - Q^i = Q'^{i-1} - Q^{i-1}$, with $q_k' - q_k$ as the only positive component and $Q^{i-1} \leq Q^i$. Thus, $C\left(Q'^{i-1}\right) - C\left(Q^{i-1}\right) \leq C\left(Q'^i\right) - C\left(Q^i\right)$ by (IIC) or part 1 of Lemma 1, from which $C\left(Q^i\right) - C\left(Q^{i-1}\right) \leq C\left(Q'^i\right) - C\left(Q'^{i-1}\right)$. Combining this last inequality with $\xi_j^{PS}(Q, C, H) \leq \xi_j^{PS}(Q', C, H)$ $\forall j < i$ yields the result.

Next, suppose that the order of the stand alone costs is changed when going from Q to Q'. More precisely, suppose that $c_{k+p}\left(q_{k+p}'\right) < c_k\left(q_k'\right)$ for some $p \leq n-k$ and $c_k\left(q_k'\right) \leq c_{k+p+1}\left(q_{k+p+1}'\right)$ whenever $k+p+1 \leq n$. Then, consider a sequence $Q = \hat{Q}^0, \hat{Q}^1, \ldots, \hat{Q}^p = Q'$, where $\hat{q}_j^\ell = q_j$ $\forall j \neq k$ and where $\hat{q}_k^\ell$ is chosen so that $q_k \in h_k\left(\hat{q}_k^\ell, \mathbb{R}_+\right)$ and $c_k\left(\hat{q}_k^\ell\right) = c_{k+\ell}\left(q_{k+\ell}\right)$, $\ell = 1, \ldots, p-1$. In other words, $\hat{Q}^1$ is obtained by increasing q_k to get a $\hat{q}_k^1$ such that $q_k \in h_k\left(\hat{q}_k^1, \mathbb{R}_+\right)$ and $c\left(\hat{q}_k^1\right) = c_{k+1}\left(q_{k+1}\right)$. $\hat{Q}^2$

is obtained by further increasing q_k until its stand alone cost reaches $c_{k+2}(q_{k+2})$ and so on until $\hat{Q}^p$. Note that the ranks k and $k+\ell$ may be interchanged in each of the problems $\left(\hat{Q}^\ell, C, H\right)$, $\ell = 1, \ldots, p$, without changing the cost shares for each problem under ξ^{PS}. Therefore, ξ_i^{PS} is non-decreasing along this sequence of problems and thus:

$$\xi_i^{PS}(Q, C, H) \leq \xi_i^{PS}\left(\hat{Q}^1, C, H\right) \leq \xi_i^{PS}\left(\hat{Q}^p, C, H\right) = \xi_i^{PS}(Q', C, H)$$

It suffices to change the sense of the relevant inequalities when C is a DIC cost function.

7.3 Proof of Corollary 1

Suppose that C is an IIC cost function and consider a sequence $Q = \hat{Q}^0, \hat{Q}^1, \ldots, \hat{Q}^n = Q'$, in which each term $\hat{Q}^k$ is defined by:

$$\hat{q}_j^k = \begin{cases} q_j' & \text{if} \quad j \leq k \\ q_j & \text{if} \quad j > k \end{cases}$$

In plain word, each component is increased, if needed, one at a time along this sequence until Q' is reached. Then, by Theorem 1, ξ_i^{PS} is non-decreasing along the sequence of problems $\left(\hat{Q}^k, C, H\right)$, $k = 1, \ldots, n$, and thus:

$$\xi_i^{PS}(Q, C, H) \leq \xi_i^{PS}\left(\hat{Q}^1, C, H\right) \leq \cdots \leq \xi_i^{PS}\left(\hat{Q}^n, C, H\right) = \xi_i^{PS}(Q', C, H)$$

7.4 Proof of Theorem 2

Suppose that C satisfies increasing benefit from cooperation. We proceed by induction. With $Y = 0$ and $Y' = Q^1$ in (5), we get $C(Q^1) \leq n\, c_1(q_1)$, from which:

$$\xi_1(Q, C, H) = \frac{C(Q^1)}{n} \leq c_1(q_1)$$

Next, suppose that $\xi_{i-1} \leq c_{i-1}(q_{i-1})$ is true and note that:

$$\begin{aligned} \sum_{j=1}^{n} c_j\left(q_j^i\right) &= \sum_{j=1}^{i-1} c_j(q_j) + (n-i+1)\, c_i(q_i) \\ \sum_{j=1}^{n} c_j\left(q_j^{i-1}\right) &= \sum_{j=1}^{i-1} c_j(q_j) + (n-i+1)\, c_{i-1}(q_{i-1}) \end{aligned}$$

Then, (5) of Lemma 1 implies:

$$C\left(Q^i\right) - C\left(Q^{i-1}\right) \leq (n-i+1)\left[c_i(q_i) - c_{i-1}(q_{i-1})\right]$$

Collecting all the above yields:

$$\begin{aligned}\xi_i(Q,C,H) &= \xi_{i-1}(Q,C,H)+\frac{C(Q^i)-C(Q^{i-1})}{n-i+1}\\ &\leq c_{i-1}(q_{i-1})+\frac{C(Q^i)-C(Q^{i-1})}{n-i+1}\\ &\leq c_i(q_i)\end{aligned}$$

It suffice to reverse all inequalities when C satisfies decreasing benefit from cooperation.

7.5 Proof of Theorem 3

Consider a cost sharing rule ξ satisfying (PCM), a problem $(Q,C,H)\in\mathbb{R}_+^m\times\mathbb{C}(m)\times\mathcal{H}(C)$ where C is a (DIC) cost function, and any proper subset $S\subset N$. Let $\tilde{Q}$ be the profile of demands obtained by substituting $h_i(q_i,0)$ to q_i in Q for all $i\notin S$. By definition of $\mathcal{H}(C)$, we have $c_i(h_i(q_i,0))=0$ and by definition of $\mathbb{C}(m)$, $C\left(\tilde{Q}^{N\backslash S}\right)=0$. Since $\tilde{Q}_S^{N\backslash S}\leq\tilde{Q}_S$, using (PCM), more precisely Corollary 1, we get $\xi_i\left(\tilde{Q},C,H\right)\leq\xi_i\left(\tilde{Q}^{N\backslash S},C,H\right)=0\ \forall i\notin S$, from which $\xi_i\left(\tilde{Q},C,H\right)=0\ \forall i\notin S$. Thus,

$$\sum_{i\in S}\xi_i\left(\tilde{Q},C,H\right)=\sum_{i\in N}\xi_i\left(\tilde{Q},C,H\right)=C\left(\tilde{Q}\right)=C\left(Q^S\right)$$

where the last equality follows from the definition of $\mathbb{C}(m)$. Using the above, (PCM), and the fact that $\tilde{Q}_{N\backslash S}\leq Q_{N\backslash S}$, we finally get:

$$\sum_{i\in S}\xi_i(Q,C,H)\leq\sum_{i\in S}\xi_i\left(\tilde{Q},C,H\right)=C\left(Q^S\right)$$

The proof is similar when C is an (IIC) cost function.

References

[1] Faulhaber, G. (1975), "Cross Subsidization: Pricing in Public Enterprises," *American Economic Review*, 65, 966-977.

[2] Koster M., Tijs S., and Borm P. (1998), "Serial Cost Sharing Methods for Multicommodity Situations," *Mathematical Social Science*, **36,** 229-242.

[3] Moulin H. and Shenker S. (1992), "Serial Cost Sharing," *Econometrica*, **60**, 1009-1037.

[4] Moulin H. and Shenker S. (1994), "Average Cost Pricing versus Serial Cost Sharing: An Axiomatic Comparison," *Journal of Economic Theory*, **64**, 178-201.

[5] Moulin H. (1996), "Cost Sharing under Increasing Returns: A Comparison of Simple Mechanisms," *Games And Economic Behavior,* **13**, 225-251.

[6] Shenker S. (1990), "Making Greed Work in Networks: A Game-Theoretic Analysis of Gateway Service Disciplines," *Mimeo*, Xerox Research Center, Palo Alto.

[7] Sprumont Y. (1998), "Ordinal Cost Sharing," *Journal of Economic Theory*, **81**, 126-162.

[8] Téjédo C. and Truchon M. (2001a), "The Radial Serial Cost Sharing Rule with General Aggregation", mimeo.

[9] Téjédo C. and Truchon M. (2001b), "Serial Cost Sharing in Multidimensional Contexts", mimeo.

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