CAHIERS DE RECHERCHE / WORKING PAPERS

04-04

Poverty-Reducing and Welfare-Improving Marginal Public Price Cap Reforms.

Paul MAKDISSI
et
Quentin WODON

UNIVERSITÉ DE SHERBROOKE
Faculté des lettres et sciences humaines
Département d'économique

Poverty-Reducing and Welfare-Improving Marginal Public Price and Price Cap Reforms*

Paul Makdissi† Quentin Wodon‡

May 2004

Abstract

This paper extends familiar results on the optimal pricing of publicly provided goods and price cap regulations in a stochastic dominance framework. The key advantage is that the assessment as to whether pricing or price cap reforms are poverty reducing or welfare improving is not contingent on any given social welfare function. Rather, robust assessments of the impact of reforms can be made for wide classes of ethical judgments.

Keywords: Poverty, social welfare, public prices, price caps
JEL Codes: I31, I32, L43

---

*This paper was funded by a grant from the ESMAP Program on Low Income Energy Assistance in Latin America. The opinions expressed here are those of the authors and need not represent those of the World Bank, its Executive Directors, or the countries they represent. The first author also benifitted from the support of FQRSC and SSHRC. We thank Martin Boyer for usefull comments.

† Département d'économique and GRAP, Université de Sherbrooke, 2500, boulevard de l'Université, Sherbrooke, Québec, Canada, J1K 2R1; Email: paul.makdissi@USherbrooke.ca.

‡ AFTPM, World Bank, 1818 H Street, NW, Washington, DC 20433, USA, Email: qwodon@worldbank.org.

1 Introduction

The issue of how to price publicly provided goods or goods subject to price regulation by public authorities is important, especially given the fact that in both developed and developing countries, many households living in poverty have difficulties in purchasing these goods (see for example Estache, Foster and Wodon, 2002, on the affordability of utilities in developing countries). The standard reference for the optimal structure of public prices in the presence of distributional considerations is a paper by Feldstein (1972), who proposed to adapt the traditional Ramsey-Boiteux pricing rules in order to take into account the relationship between consumption patterns and the distribution of income (see also Yang, 1993). Laffont and Tirole (2000) later showed that Ramsey prices could be implemented using price cap regulation instead of direct price interventions, and the possibility of relying on price caps to implement Feldstein pricing was demonstrated by Iozzi, Poritz and Valentini (2002).

Unfortunately, one of the limitations of standard Ramsey and Feldstein pricing rules is that in order to find the optimal prices (or price cap regulations), the analyst must rely on a specific social welfare function. It is likely, however, that disagreements will be observed regarding the choice of this function. If what might have been optimal for a certain social welfare function is not optimal for another such function, then the issue of finding "piecemeal" or marginal pricing or price cap reforms for improving social welfare becomes crucial.

The objective of this paper is to show how marginal changes in prices, or marginal changes in price cap regulations, can be shown to be welfare improving for wide classes of ethical judgments on which many analysts could agree in principle. To do so, we follow the approach used in the social welfare literature for assessing the impact of indirect tax reforms on social welfare. The seminal work in this area was conducted by Yitzhaki and Slemrod (1991; see also Yitzhaki and Thirsk, 1990; Yitzhaki and Lewis, 1996; and Mayshar and Yitzhaki, 1995, 1996). Building on this work, Makdissi and Wodon (2002), as well as Duclos, Makdissi, and Wodon (2002, 2004) provided extensions to deal with the impact of marginal indirect tax reforms not only on social welfare, but also on poverty, and they also considered higher orders of stochastic dominance (i.e., higher order classes of ethical judgments) than was feasible in the original papers on this topic.

The structure of the paper runs as follows. In section 2, our objective is essentially to establish our notation and vocabulary by restating in a social welfare and stochastic dominance framework the key results obtained by Ramsey and Feldtsein regarding the optimal pricing rules that should be used by a social planner in order to maximize welfare. The only new bit of information provided in that section relates to how these pricing rules must be adapted when the social planner wishes to minimize poverty rather than to maximize utility.

Section 3 extends the results provided for the impact on poverty and welfare of indirect tax reforms in Makdissi and Wodon (2002) and Duclos, Makdissi, and Wodon (2002, 2004) to the issue of the pricing of publicly provided goods. The key difference versus previous work is that the so-called Marginal Efficiency Cost of Fund parameter which may affect whether marginal price reforms are indeed welfare improving or poverty reducing must now take into account not only the price (and cross-price) elasticities of demand for various goods, but also the structure of the production costs for these goods as captured through price markups.

Section 4 then shows how to assess in a robust way the impact of marginal changes in price cap regulations on social welfare and poverty, showing that all the results obtained through direct marginal price reforms in Section 3 also apply for marginal price cap reforms. A brief conclusion follows.

2 Ramsey and Feldstein Pricing: A Restatement

Consider a public firm that produces n goods and sells them at prices $p = (p_1, p_2, ..., p_n)$ to I consumers. Throughout this paper, we will assume that the level of public prices p does not affect prices in the private sector. Let y be the nominal income indicator for the consumers of the firm, and let $f(y)$ be its density function defined over $[0, y^{\max}]$. Denote by $v(y, p)$ the indirect utility function. Following King (1983), we will use a vector of reference public prices, p^R, to assess the consumers' well-being in the presence of varying public prices. Let $y^E\left(y, p, p^R\right)$ be the equivalent income function for the consumers, so that y^E is implicitly defined by

$$v\left(y^E\left(y,p,p^R\right),p^R\right)\equiv v\left(y,p\right). \tag{1}$$

By definition, y^R gives the level of income that provides consumers with the same level of utility under prices p^R as yielded by income y under prices p.

Starting with social welfare, we may assume that a social planner maximizes an utilitarian social welfare function U such that:

$$U=\int_0^{y^{\max}} u\left(y^E\left(y,p,p^R\right)\right)f\left(y\right)dy. \tag{2}$$

We will focus on social welfare indices belonging to classes Ω^s, $s=1,2,\ldots,$ defined as

$$\Omega^s=\left\{U\left|u\left(y^E\right)\in C^s(\infty),\,(-1)^{i+1}u^{(i)}\left(y^E\right)\geq 0 \text{ for } i=1,2,...,s,\right.\right\}. \tag{3}$$

Every class Ω^s has a specific normative interpretation. When $s=1$, the welfare indices weakly increase ($u^{(1)}\left(y^E\right)\geq 0$) when a consumer's equivalent income increases. These indices are thus Paretian but they also obey the well-known symmetry or anonymity axiom: interchanging any two consumers' incomes leaves unchanged the poverty and social welfare indices. When $s=2$, the welfare indices are concave. They respect the Pigou-Dalton principle of transfers, which postulates that a mean-preserving transfer of income from a higher-income consumer to a lower-income consumer increases social welfare. By their third-order derivative, the social welfare indices that belong to Ω^3 increase with favorable composite transfers. These transfers are such that a beneficial Pigou-Dalton transfer within the lower part of the distribution, accompanied by an adverse Pigou-Dalton transfer within the upper part of the distribution, will add to social welfare, provided that the variance of the distribution is not increased [1]. For the interpretation of $s>3$, we can use the generalized transfer principles of Fishburn and Willig (1984). For $s=4$, for instance, consider a combination of two exactly opposite and symmetric composite transfers, the first one being favorable and occurring within the lower part of the distribution,

[1] Kolm (1976) was the first to introduce this condition into the inequality literature. See also Shorrocks and Foster (1987) for a characterization of the composite transfer principle and Davies and Hoy (1994) for a description on the normative implications of this principle.

and the second one being unfavorable and occurring within the higher part of the distribution. Because the favorable composite transfer occurs lower down in the income distribution, indices that are members of the $s = 4$ class will respond favorably to this combination of composite transfers. Generalized higher-order transfer principles essentially postulate that, as s increases, the weight assigned to the effect of transfers occurring at the bottom of the distribution also increases.

Consider now a social planner who whishes to reduce poverty instead of maximizing social welfare. As shown in Duclos and Makdissi (2004), a poverty index is essentially a social welfare index censored at a poverty line. We will follow most of the literature in focusing for simplicity on additive poverty indices expressed as:

$$P(z) = \int_0^{y^{\max}} \pi\left(y^E\left(y, p, p^R\right), z\right) f(y)\, dy \tag{4}$$

where $P(z)$ is an additive poverty index, z is a poverty line defined in the equivalent income space[2] and $\pi\left(y^E\left(y, p, p^R\right), z\right)$ is the contribution to total poverty of a consumer with equivalent income y^E (with $\pi\left(y^E\left(y, p, p^R\right), z\right) = 0$ for all $y^E > z$). We define various classes of poverty indices $P(z) \in \Pi^s$ such that

$$\Pi^s(z) = \left\{ P(z) \;\middle|\; \begin{array}{c} \pi\left(y^E, z\right) = 0 \text{ if } y > z,\ \pi(y, z) \in \widehat{C}^s(z), \\ (-1)^i \pi^{(i)}\left(y^E, z\right) \geq 0 \text{ for } i = 0, 1, ..., s, \\ \pi^{(t)}(z, z) = 0 \text{ for } t = 0, 1, ..., s-2 \text{ when } s \geq 2 \end{array} \right\} \tag{5}$$

where $\widehat{C}^s(z)$ is the set of functions that are s-time piecewise differentiable[3] over $[0, z]$, and where the subscript $^{(s)}$ stands for an s^{th}-order derivative with respect to y^E. Classes $\Pi^s(z)$ have essentially the same normative interpretation that classes Ω^s except that the normative principles are only concerned with changes that occur under the poverty line, z. We will return very shortly to the interpretation of the derivative assumptions.

The maximization of social welfare or the minimization of poverty must obey the public firm's budget constraint, so that total revenue minus production cost must be

[2]Defining the poverty line in the equivalent income space rather than in the nominal income space is convenient since the poverty line is then invariant to price reforms.

[3]When the $(s-1)^{th}$ derivative is a piecewise differentiable function, the function and its $(s-2)$ first derivatives are differentiable everywhere. The $\widehat{C}^s(z)$ continuity assumption is made for analytical simplicity since it could be relaxed to include indices whose $(s-1)^{th}$ derivative is discontinuous (and which are therefore not s-time piecewise differentiable).

equal to B. If $x_i(p,y)$ is the Marshallian demand of a consumer with income y, the total quantity of good i purchased by all consumers is given by

$$q_i(p) = I \int_0^{y^{\max}} x_i(p,y) f(y)\, dy. \tag{6}$$

Let $c(q)$ be the total cost of producing $q = (q_1, q_2, ..., q_n)$. The objective of the public firm is to maximize (2) or minimize (4) under the budget constraint

$$\sum_{i=1}^{n} p_i q_i(p) - c(q) = B. \tag{7}$$

To derive the first order conditions of the problem, we must use Roy's identity and set the optimal prices as reference prices. We find:

$$\left.\frac{\partial y^E(y,p,p^R)}{\partial p_i}\right|_{p^R=p} = -x_i(p,y). \tag{8}$$

Using this result and the identity $y = y^E(y,p,p)$, we can now state the first order conditions for utility maximization as:

$$-\int_0^{y^{\max}} x_i(p,y) u^{(1)}(y) f(y)\, dy + \lambda^U \left\{ q_i(p) + \sum_{j=1}^{n} (p_j - m_j) \frac{\partial q_j(p)}{\partial p_i} \right\} = 0, \text{ for } i = 1 \text{ to } n, \tag{9}$$

where $m_j = \partial c(q)/\partial q_j$ is the marginal cost of producing j and λ^U is the shadow price of the budget constraint in the case of utility maximization.

In the case of a poverty minimizing planner, the minimization of (4) is equivalent to maximizing $-\int_0^a \pi\left(y^E\left(y,p,p^R\right), z\right) f(y)\, dy$ so that if we denote by λ^P the shadow price of the budget constraint in the case of poverty minimization, (9) becomes

$$\int_0^{y^{\max}} x_i(p,y) \pi^{(1)}(y,z) f(y)\, dy + \lambda^P \left\{ q_i(p) + \sum_{j=1}^{n} (p_j - m_j) \frac{\partial q_j(p)}{\partial p_i} \right\} = 0, \text{ for } i = 1 \text{ to } n, \tag{10}$$

Now, let $\varepsilon_{ij} = (\partial q_i(p)/\partial p_j)(p_j/q_i)$ be the price elasticity of good i with respect to the price of good j. Following Feldstein (1972), we assume that the shares of good i and j in the consumers' total expenditure is small so that we can use $\varepsilon_{ij} \simeq \varepsilon_{ji} p_j q_j / p_i q_i$ as an approximation.

The first order conditions (9) and (10) can be rewritten as

$$R_i^{U,P} = \lambda^{U,P} \left\{ 1 + \sum_{j=1}^{n} \left(\frac{p_j - m_j}{p_j} \right) \varepsilon_{ij} \right\}, \text{ for } i = 1 \text{ to } n, \tag{11}$$

where $R_i^U = -\frac{1}{q_i(p)} \int_0^{y^{\max}} x_i(p,y)\, u^{(1)}(y)\, f(y)\, dy$ in the case of utility maximization, and $R_i^P = \frac{1}{q_i(p)} \int_0^{y^{\max}} x_i(p,y)\, \pi^{(1)}(y,z)\, f(y)\, dy$ in the case of poverty minimization. $R_i^{U,P}$ is simply Feldstein's distributional characteristic of good i. If there is no concern for redistribution in the utility maximization, R_i^U becomes irrelevant since $u^{(1)}(y)$ is constant for all y which in turn implies $R_i^U = \frac{1}{q_i(p)} \int_0^{y^{\max}} x_i(p,y)\, f(y)\, dy = 1$ for all i. In this particular case, equation (11) is equivalent to Ramsey-Boiteux pricing. The same is true (namely, $R_i^P = 1$) in the poverty reduction case if there is no special concern for poverty reduction.

Note finally that when there is no cross-elasticity of demand (i.e. when $\varepsilon_{ij} = 0$ for all i and j), we have

$$\frac{p_i - m_i}{p_i} = \frac{R_i^{U,P} - \lambda^{U,P}}{\lambda^{U,P}} \frac{1}{\varepsilon_{ii}}. \tag{12}$$

3 Impact of Marginal Price Reforms

The results in the previous section rely on the fact that a specific utility function, or a specific poverty measure, is used by the social planner, so that an exact solution can be found in order to maximize welfare or minimize poverty. In practice however, there is often disagreement regarding the use of a given utility function or poverty measure. In addition, we may also start from a sub-optimal position, so that what is needed is to find the direction that marginal price reforms should take, i.e., which prices should be increased at the margin, and which prices should be reduced. In this section, we provide results that indicate how to test for the impact of marginal price reforms on poverty for wide classes of social welfare and poverty measures.

If we assume that prices are directly under the control of the social planner, we can derive a number of results for the impact on poverty and social welfare of balanced budget marginal price reforms, in a manner similar to that of Makdissi and Wodon (2002) and Duclos, Makdissi and Wodon (2002) for the impact of indirect tax reforms. The main difference relies in that the cost structure of the firm must be taken into consideration here, while this was not necessary in the case of indirect tax reforms (because producer prices were simply assumed to remain constant; in the case of public firms with potential monopolies in water or electricity production and distribution, this is clearly not realistic).

To present the results, it is handy to first introduce the concept of stochastic dominance curves. When $p = p^R$, and for orders of dominance $s = 1, 2, ...$, the curves are defined as follows

$$D^s(z) = \frac{1}{(s-1)!} \int_0^z [z-y]^{(s-1)} f(y) \, dy. \tag{13}$$

In the case of the comparisons of two relative density function f_a and f_b, Duclos and Makdissi (2004) have shown that for any poverty line $z \in [0, z^+]$ and any absolute poverty index $P \in \Pi^s$, a necessary and sufficient condition for reducing poverty, namely for $P_b(z) - P_a(z) \leq 0$, is:

$$D_a^s(y) - D_b^s(y) \geq 0 \quad \forall y \leq z^+. \tag{14}$$

Similarly, for all $U \in \Omega^s$, a sufficient condition for increasing utility, namely for $U_b - U_a \geq 0$, is:

$$\begin{array}{c} D_a^s(y) - D_b^s(y) \geq 0 \quad \forall y \in [0, a]. \\ \text{and, if } s \geq 3, \\ D_a^i(a) - D_b^i(a) \geq 0 \quad \forall i \in \{2, ..., s-1\}. \end{array} \tag{15}$$

Here, we are interested in assessing how a given income distribution is affected by a price reform instead of simply comparing two income distributions. Thus, we need to consider how stochastic dominance curves are affected by changes in prices. Using (8), we can show that:

$$\left.\frac{\partial D^s(z)}{\partial p_k}\right|_{q=q^R} = \begin{cases} x_k\left(z, q^R\right) f(z), & \text{if } s = 1 \\ \frac{1}{(s-2)!} \int_0^z x_k\left(y, q^R\right) (z-y)^{s-2} f(y) \, dy & \text{if } s = 2, 3, ... \end{cases} \tag{16}$$

Using the terminology introduced by Makdissi and Wodon (2002) and Duclos, Makdissi and Wodon (2002), we will define "consumption dominance" (CD) curves as:

$$CD_k^s(z) = \frac{\partial D^s(z)}{\partial p_k}, \; s = 1, 2, ... \tag{17}$$

Normalized CD curves, $\overline{CD}_k^s(z)$, are just the above CD curves for good k normalized by the average consumption of that good:

$$\overline{CD}_k^s(z) = I\frac{CD_k^s(z)}{q_k(p)}. \tag{18}$$

Let us now consider a marginal price reform that decreases marginally the price of good i and increase the price of good j in order to keep the firm's profits constant. Constant profits and (7) imply that

$$dB = 0 = \left[q_i(p) + \sum_{k=1}^{n}(p_k - m_k)\frac{\partial q_k(p)}{\partial p_i}\right]dp_i + \left[q_j(p) + \sum_{k=1}^{n}(p_k - m_k)\frac{\partial q_k(p)}{\partial p_j}\right]dp_j, \tag{19}$$

and

$$dp_j = -\gamma\left(\frac{q_i(p)}{q_j(p)}\right)dp_i. \tag{20}$$

Using the approximation $\varepsilon_{ij} \simeq \varepsilon_{ji}p_jq_j/p_iq_i$, we have

$$\gamma = \frac{1 + \sum_{k=1}^{n}\left(\frac{p_k - m_k}{p_k}\right)\varepsilon_{ik}}{1 + \sum_{k=1}^{n}\left(\frac{p_k - m_k}{p_k}\right)\varepsilon_{jk}}. \tag{21}$$

Note that if the actual price system has a Ramsey structure, $\gamma = 1$.

We can now assess the impact of the marginal price reform on the stochastic dominance curves:

$$\begin{aligned} dD^s(z) &= CD_i^s(z)\,dp_i + CD_j^s(z)\,dp_j \\ &= \left[CD_i^s(z) - \gamma\left(\frac{q_i(p)}{q_j(p)}\right)CD_j^s(z)\right]dp_i \\ &= \left[\overline{CD}_i^s(z) - \gamma\overline{CD}_j^s(z)\right]\frac{q_i(p)}{I}dp_i. \end{aligned} \tag{22}$$

Using (14), (15) and (22), we can now state the following results.

Proposition 1 *A marginal price reform that decreases the price of good i and increases the price of good j in order to keep the firm's profit constant reduces $P(z)$ for any poverty line $z \in [0, z^+]$ and any poverty index $P \in \Pi^s$, iff:*

$$\overline{CD}_i^s(y) - \gamma\overline{CD}_j^s(y) \geq 0 \quad \forall y \leq z^+. \tag{23}$$

Proof. $q_i(p)/I$ is positive and dp_i is negative. If condition (23) holds, it is straightforward to show that $P(z)$ decreases for any poverty line $z \in [0, z^+]$ and any poverty index $P \in \Pi^s$ (and conversely) from (22) and (14). ■

Proposition 1 is similar to the result of Makdissi and Wodon (2002). The main difference is in the structure of γ. In Makdissi and Wodon (2002), this parameter is essentially a function of the demand elasticities because producer prices are assumed to be constant in the context of an indirect tax reform. Here, it is not possible to make this assumption, so that we must take production costs into account. This explains why γ is now a function of both the demand elasticities and the price markups.

Proposition 2 *A marginal price reform that decreases the price of good i and increases the price of good j in order to keep the firm's profit constant increases U for any welfare index $U \in \Omega^s$, if:*

$$\begin{aligned} \overline{CD}_i^s(y) - \gamma \overline{CD}_j^s(y) &\geq 0 \quad \forall y \leq y^{\max} \\ \text{and, if } s &\geq 3, \\ \overline{CD}_i^\sigma(y^{\max}) - \gamma \overline{CD}_j^\sigma(y^{\max}) &\geq 0 \quad \forall \sigma \in \{2, 3, ..., s-1\}. \end{aligned} \tag{24}$$

Proof. Again, $q_i(p)/I$ is positive and dp_i is negative. If condition (24) holds, it is straightforward to show that U increases for any welfare index $U \in \Omega^s$ from (22) and (15). ■

The first part of condition (24) is similar to the result of Duclos, Makdissi and Wodon (2002) except again for the structure of γ. Note also that in Proposition 2, there is an additional test to perform at $y^{\max}$ that was not necessary in Duclos et al. (2002). The reason for this is that Duclos et al. perform their test on $[0, \infty)$. As shown by Duclos and Makdissi (2004), it is not necessary to perform tests at $y^{\max}$ if $y^{\max} \rightarrow \infty$. However, for practical purpose, it is empirically difficult to test on $[0, \infty)$ so we prefer here to give the condition with a finite limit.

Is it always feasible to find a marginal price reform which will increase welfare or reduce poverty at some given order of dominance? Davidson and Duclos (2000) provide a theorem which states that there is indeed a class of welfare functions Ω^s for which $U_b - U_a \geq 0$, for all $U \in \Omega^s$, if $D_a^1(y) - D_b^1(y) \geq 0$ for $y \in [0, \xi]$ for any positive value ξ. This theorem, together with the assumption that $\gamma = 1$ (corresponding to a Ramsey price structure) imply the following result.

Proposition 3 *If the actual price system has a Ramsey structure, there is always class of welfare functions Ω^s for which a marginal price reform that decreases the price of good i and increases the price of good j in order to keep the firm's profit constant increases U for any welfare index $U \in \Omega^s$, provided we can find $\xi > 0$ such that*

$$\overline{CD}_i^1(y) - \overline{CD}_j^1(y) \geq 0 \quad \forall y \in [0, \xi]. \tag{25}$$

A similar result holds for poverty reduction, whatever the maximum threshold of the poverty line z^+ is chosen to be. That is, we will always find a poverty reducing reform up to z^+ at some order of dominance if the above condition is respected.

4 Impact of Marginal Price Cap Reforms

In the previous two sections, we assumed that prices were directly under the control of the social planner. In many cases however, the regulation of the public firm is implemented through price cap regulations rather than directly through mandated prices. Price cap regulation consists of imposing to the firm a weighted average price ceiling

$$\sum_{i=1}^{n} w_i p_i \leq \overline{p} \tag{26}$$

and letting it free to maximize profit otherwise. If the weights w_i are set equal to future realized quantities, then monopolist profit maximization yields Ramsey prices in structure, as shown by Laffont and Tirole (2000).

As shown by Iozzi, Poritz and Valentini (2002) in a more general framework, it is relatively straightforward to extend this result to Feldstein pricing. Using our own notation, note that when facing price cap regulation, the problem of the monopolist becomes

$$\max_{p,\nu} \sum_{i=1}^{n} p_i q_i(p) - c(q) + \nu \left[\overline{p} - \sum_{i=1}^{n} w_i p_i\right] \tag{27}$$

The first order conditions of this problem are

$$\nu w_i = q_i(p) + \sum_{j=1}^{n} (p_j - m_j) \frac{\partial q_j(p)}{\partial p_i}, \text{ for } i = 1 \text{ to } n. \tag{28}$$

Compare (28) to (9). In order to obtain the same solution as in (9) for utility maximization, the social planner must simply set

$$\nu w_i = -\frac{q_i(p) R_i^U}{\lambda^U}, \text{ for } i = 1 \text{ to } n. \tag{29}$$

For this, the planner may first adjust the budget constraint so as to have $\nu = -1/\lambda^U$, and then choose the weights w_i in order to reflect Feldstein's distributional characteristics for each good times the total quantity sold of the good. A similar equivalence is obtained for the case of poverty minimization. Thus we can state the following result already provided for welfare maximization in Iozzi et al. (2002):

Proposition 4 *Monopolist profit maximization yields Feldstein prices in structure (for the maximization of social welfare or the minimization of poverty) if the firm is regulated under a price cap regime in which the weight are set equal to future realized quantities times Feldstein's distributional characteristic.*

Let us now consider the impact of marginal price cap reforms on social welfare and poverty. As in the case of marginal price reforms, we are motivated to do this because the social planner may not know with certainty the exact social welfare function to be maximized, or the exact poverty measure to be minimized. Here however, we are confronted with an additional difficulty. When assessing the impact on welfare or poverty of a marginal price reform, we had to determine the impact of a price change on consumer welfare. In the case of a price cap reform, we must first determine the impact on the firm's price structure and then only assess the impact of this change in price structure on consumer welfare.

At the consumer level, we have by definition:

$$dD^s(z) = \sum_{k=1}^{n} CD_k^s(z)\, dp_k. \tag{30}$$

Differentiating totally the set of equations given by (28), we get

$$\nu dw_i = \sum_{j=1}^{n} \theta_{ij} dp_j, \text{ for } i = 1 \text{ to } n, \tag{31}$$

where

$$\theta_{ij} = \frac{\partial q_i(p)}{\partial p_j} + \left(1 - \sum_{k=1}^{n} \frac{\partial m_j}{\partial q_k} \frac{\partial q_k(p)}{\partial p_j}\right) \frac{\partial q_j(p)}{\partial p_i} + (p_j - m_j) \frac{\partial^2 q_j(p)}{\partial p_i \partial p_j}, \text{ for } j = 1 \text{ to } n. \tag{32}$$

Using (31) and Cramer's rule, we get

$$\frac{\partial p_j}{\partial w_i} = \frac{\nu \Theta_{ij}}{|\Theta|}, \tag{33}$$

where

$$\Theta = \begin{bmatrix} \theta_{11} & \theta_{12} & \cdots & \theta_{1n} \\ \theta_{21} & \theta_{22} & \cdots & \theta_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ \theta_{n1} & \theta_{n2} & \cdots & \theta_{nn} \end{bmatrix}, \tag{34}$$

and Θ_{ij} is the cofactor of θ_{ij}. At this point, it is useful to define

$$\Phi_{ij}^{k} = \frac{\nu(\Theta_{ik} - \Theta_{jk})}{|\Theta|} \tag{35}$$

Using (35), we can rewrite (30) as

$$\begin{aligned} dD^s(z) &= \sum_{k=1}^{n} \frac{\nu \Theta_{ik}}{|\Theta|} CD_k^s(z)\, dw_i + \sum_{k=1}^{n} \frac{\nu \Theta_{jk}}{|\Theta|} CD_k^s(z)\, dw_j \\ &= \sum_{k=1}^{n} \Phi_{ij}^{k} CD_k^s(z)\, dw_i. \end{aligned} \tag{36}$$

This result is again similar to (23), with one key difference. In the case of direct price reforms, only the prices of two goods change. By contrast, in the case of price cap reforms, all prices may change since the firm will adjust its whole pricing system to take into account the change in the price cap structure. Given (36), we can provide the following result.

Proposition 5 *A marginal price cap regulatory reform that increases w_i and decreases w_j in such a way that $dw_i = -dw_j$ will reduce $P(z)$ for any poverty line $z \in [0, z^+]$ and any poverty index $P \in \Pi^s$, iff:*

$$\sum_{k=1}^{n} \Phi_{ij}^{k} CD_k^s(y) \leq 0 \quad \forall y \leq z^+. \tag{37}$$

Proof. dw_i is positive. If condition (37) holds, it is straightforward to show that $P(z)$ decreases for any poverty line $z \in [0, z^+]$ and any poverty index $P \in \Pi^s$ (and conversely) from (36) and (14). ■

Proposition 6 *A marginal price cap regulatory reform that increases w_i and decreases w_j such that $dw_i = -dw_j$ will increase U for any welfare index $U \in \Omega^s$, if:*

$$\begin{aligned} \sum_{k=1}^{n} \Phi_{ij}^k CD_k^s(y) &\leq 0 \quad \forall y \leq y^{\max}, \\ \textit{and, if } s &\geq 3, \\ \sum_{k=1}^{n} \Phi_{ij}^k CD_k^\sigma(y^{\max}) &\leq 0 \quad \forall \sigma \in \{2, 3, ..., s-1\}. \end{aligned} \tag{38}$$

Proof. dw_i is positive. If condition (38) holds, it is again straightforward to show that U increases for any welfare index $U \in \Omega^s$ from (36) and (15). ■

As already mentioned, in order to assess the impact of marginal price cap reforms, we need to take into consideration the CD curves for all goods since a change in the weight of any one good may have an impact on the price of all goods. There is however an exception if cross-price elasticities of demand are zero (i.e. when $\varepsilon_{ij} = 0$ for all i and j). In this case, we have

$$\frac{\partial p_i}{\partial w_i} = \nu \left[2\frac{\partial q_i(p)}{\partial p_i} - \frac{\partial m_i}{\partial q_i} \left(\frac{\partial q_i(p)}{\partial p_i} \right)^2 + (p_i - m_i) \frac{\partial^2 q_i(p)}{\partial p_i^2} \right] = \theta_i \tag{39}$$

and

$$\begin{aligned} dD^s(z) &= CD_i^s(z) \frac{\partial p_i}{\partial w_i} dw_i + CD_j^s(z) \frac{\partial p_j}{\partial w_j} dw_j \\ &= \left[\theta_i CD_i^s(z) - \theta_j CD_j^s(z) \right] dw_i. \end{aligned} \tag{40}$$

Then, if we define $\phi = \theta_j / \theta_i$, we can state the following results.

Proposition 7 *If cross-price elasticities of demand are all zero, a marginal price cap regulatory reform that increases w_i and decreases w_j in such a way that $dw_i = -dw_j$ will reduce $P(z)$ for any poverty line $z \in [0, z^+]$ and any poverty index $P \in \Pi^s$, iff:*

$$CD_i^s(y) - \phi CD_j^s(y) \leq 0 \quad \forall y \leq z^+. \tag{41}$$

Proposition 8 *If cross-price elasticities of demand are all zero, a marginal price cap regulatory reform that increases* w_i *and decreases* w_j *in such a way that* $dw_i = -dw_j$ *will increase* U *for any welfare index* $U \in \Omega^s$*, if:*

$$\begin{aligned} CD_i^s(y) - \phi CD_j^s(y) &\leq 0 \quad \forall y \leq y^{\max}, \\ \text{and, if } s &\geq 3, \\ CD_i^\sigma(y) - \phi CD_j^\sigma(y) &\leq 0 \quad \forall \sigma \in \{2, 3, ..., s-1\}. \end{aligned} \tag{42}$$

In other words, in the particular case where cross-price elasticities of demand are zero, robust estimates of the impact on poverty or social welfare of marginal price cap reforms can be assessed by simply checking for non-intersection between the CD curves of the two goods for which the weighting system is modified. Note here that, unlike γ which is necessarily positive, the parameter ϕ can theoretically be positive or negative.

5 Conclusion

As noted by Feldstein (1975), many reforms are intrinsically "piecemeal". In this paper, our main objective has been to show how small marginal changes in the price of publicly provided goods, or in price cap regulations, can be deemed to improve welfare or reduce poverty for wide classes of welfare and poverty measures. Our results have been obtained using stochastic dominance techniques, which have the advantage to be valid not only for a discussion of the impact of policy reforms on social welfare as a whole, but also on poverty or "censored" welfare functions. One key difference versus previous work devoted to assessing the impact of indirect tax reforms on poverty and welfare is that the cost structure of the firm(s) producing the goods must be taken into account. Another differences is that it can be shown that regulation through price caps leads to the same results as direct regulation through mandated price changes.

References

[1] Davidson, R. and J.Y. Duclos (2000), Statistical Inference for Stochastic Dominance and the for the Measurement of Poverty and Inequality, *Econometrica*, 68, 1435-1465.

[2] Davies, J. and M. Hoy (1994), The Normative Significance of Using Third-Degree Stochastic Dominance in Comparing Income Distributions, *Journal of Economic Theory*, 64, 520-530.

[3] Duclos, J.-Y. and P. Makdissi (2004), Restricted and Unrestricted Dominance for Welfare, Inequality and Poverty Orderings, *Journal of Public Economic Theory*, 6, 145-164.

[4] Duclos, J.-Y., P. Makdissi and Q. Wodon (2002), Socially-Efficient Tax Reforms, Cahier de recherche 02-01, Département d'économique, Université de Sherbrooke.

[5] Duclos, J.-Y., P. Makdissi and Q. Wodon (2004), Poverty-Reducing Tax Reforms with Heterogeneous Agents, forthcoming in *Journal of Public Economic Theory.*

[6] Estache, A., V. Foster and Q. Wodon (2002), *Accounting for Poverty in Infrastructure Reform. Learning from Latin America's Experience*, WBI Development Studies, Washington, D.C.

[7] Feldstein, M.S. (1972), Distributional Equity and the Optimal Structure of Public Prices, *American Economic Review*, 62, 32-36.

[8] Feldstein, M.S. (1975), The Income Tax and Charitable Contributions: Part I-Aggregate and Distributional Effects, *National Tax Journal*, 28, 81-100.

[9] Fishburn, P.C. and R.D. Willig (1984), Transfer Principles in Income Redistribution, *Journal of Public Economics*, 25, 323-328.

[10] Iozzi, A., J. A. Poritz, and E. Valentini (2002), Social Preferences and Price Cap Regulation, *Journal of Public Economic Theory*, 4, 95-114.

[11] King, M.A. (1983), Welfare Analysis of Tax Reforms Using Household Data, *Journal of Public Economics*, 21, 183-214.

[12] Kolm, S.C. (1976), Unequal Inequlity: I, *Journal of Economic Theory*, 12, 416-442.

[13] Laffont, J.-J. and J. Tirole (2000), *Competition in Telecommunications*, MIT Press, Cambridge, MA.

[14] Makdissi, P. and Q. Wodon (2002), Consumption Dominance Curves: Testing for the Impact of Indirect Tax Reforms on Poverty, *Economics Letters*, 75,227-235.

[15] Mayshar, J. and S. Yitzhaki (1995), Dalton Improving Tax Reform, *American Economic Review*, 85, 793-807.

[16] Mayshar, J. and S. Yitzhaki (1996), Dalton-Improving Indirect Tax Reforms When Households Differ in Ability and Needs, *Journal of Public Economics*, 62, 399-412.

[17] Shorrocks, A.F. and J.E. Foster (1987), Transfer Sensitive Inequality Measures, *Review of Economic Studies*, 54, 485-497.

[18] Yang, C. C. (1993), Distributional Equity and the Pricing of Public Final and Intermediate Goods, *Economics Letters*, 41, 429-34

[19] Yitzhaki, S. and J.D. Lewis (1996), Guidelines on Searching for a Dalton-Improving Tax Reform: An Illustration with Data from Indonesia, *World Bank Economic Review*, 10, 541-562.

[20] Yitzhaki, S. and J. Slemrod (1991), Welfare Dominance: An Application to Commodity Taxation, *American Economic Review*, 81, 480-496.

[21] Yitzhaki, S. and W, Thirsk (1990), Welfare Dominance and the Design of Excise Taxation in the Côte d'Ivoire, *Journal of Development Economics*, 33, 1-18.

94-01 BILODEAU, Marc et Al SLIVINSKI, *Toilet Cleaning and Department Chairing: Volunteering a Public Service.*

94-02 ASCAH, Louis, *Recent Retirement Income System Reform: Employer Plans, Public Plans and Tax Assisted Savings.*

94-03 BILODEAU, M. et Al SLIVINSKI, *Volunteering Nonprofit Entrepreneurial Services.*

94-04 HANEL, Petr, *R&D, Inter-Industry and International Spillovers of Technology and the Total Factor Productivity Growth of Manufacturing Industries in Canada, 1974-1989*

94-05 KALULUMIA, Pene et Denis BOLDUC, *Generalized Mixed Estimator for Nonlinear Models: A Maximum Likelihood Approach.*

95-01 FORTIN, Mario et Patrice Langevin, *L'efficacité du marché boursier face à la politique monétaire*

95-02 HANEL, Petr et Patrice Kayembe YATSHIBI, *Analyse de la performance à exporter des industries manufacturières du Québec 1988.*

95-03 HANEL, Petr, *The Czech Republic: Evolution and Structure of Foreign Trade in Industrial Goods in the Transition Period, 1989-1994.*

95-04 KALULUMIA, Pene et Bernard DÉCALUWÉ, *Surévaluation, ajustement et compétitivité externe : le cas des pays membres de la zone franc CFA.*

95-05 LATULIPPE, Jean-Guy, *Accès aux marchés des pays en développement*

96-01 ST-PIERRE, Alain et Petr HANEL, *Les effets directs et indirects de l'activité de R&D sur la profitabilité de la firme.*

96-02 KALULUMIA, Pene et Alain MBAYA LUKUSA, *Impact of budget deficits and international capital flows on money demand: Evidence From Cointegration and Error-Correction Model.*

96-03 KALULUMIA, Pene et Pierre YOUROUGOU, *Money and Income Causality In Developing Economies: A Case Study Of Selected Countries In Sub-Saharan Africa*

96-04 PARENT, Daniel, *Survol des contributions théoriques et empiriques liées au capital humain (A Survey of Theoretical and Empirical Contributions to Human Capital)*

96-05 PARENT, Daniel, *Matching Human Capital and the Covariance Structure of Earnings*

96-06 PARENT, Daniel, *Wages and Mobility : The Impact of Employer-Provided Training*

97-01 PARENT, Daniel, *Industry-Specific Capital and the Wage Profile : Evidence From the NLSY and the PSID.*

97-02 PARENT, Daniel, *Methods of Pay and Earnings: A Longitudinal Analysis*

97-03 PARENT, Daniel, *Job Characteristics and the Form of Compensation*

97-04 FORTIN, Mario et Michel BERGERON, Jocelyn DUFORT et Pene KALULUMIA, *Measuring The Impact of Swaps on the Interest Rate Risk of Financial Intermediaries Using Accounting Data*

97-05 FORTIN, Mario, André LECLERC et Claude THIVIERGE, *Testing For Scale and Scope Effects in Cooperative Banks: The Case of Les Caisses populaires et d'économie Desjardins.*

97-06 HANEL, Petr, *The Pros and Cons of Central and Eastern Europe Joining the EU*

00-01 MAKDISSI, Paul et Jean-Yves DUCLOS, *Restricted and Unrestricted Dominance Welfare, Inequality* *and Poverty Orderings*

00-02 HANEL, Petr, John BALDWIN et David SABOURIN, *Les déterminants des activités d'innovation dans les entreprises de fabrication canadiennes : le rôle des droits de propriété intellectuelle*

00-03 KALULUMIA, Pene, *Government Debt, Interest Rates and International Capital Flows: Evidence From Cointegration*

00-04 MAKDISSI, Paul et Cyril TÉJÉDO, *Problèmes d'appariement et politique de l'emploi*

00-05 MAKDISSI, Paul et Quentin WODON, *Consumption Dominance Curves: Testing for the Impact of Tax Reforms on Poverty.*

00-06 FORTIN, Mario et André LECLERC, *Demographic Changes and Real Housing Prices in Canada.*

00-07 HANEL, Petr et Sofiene ZORGATI, *Technology Spillovers and Trade: Empirical Evidence for the G7 Industrial Countries.*

01-01 MAKDISSI, Paul et Quentin WODON, *Migration, poverty, and housing: welfare comparisons using sequential stochastic dominance*. Avril 2001, 23 p.

01-02 HUNG Nguyen Manh et Paul MAKDISSI, *Infantile mortality and fertility decisions in a stochastic environment*. Mars 2001, 12 p.

01-03 MAKDISSI, Paul et Quentin WODON, *Fuel poverty and access to electricity: comparing households when they differ in needs*. Juin 2001, 19 p.

01-04 MAKDISSI, Paul et Yves GROLEAU, *Que pouvons-nous apprendre des profils de pauvreté canadiens ?* Juillet 2001, 47 p.

01-05 MAKDISSI, Paul et Quentin WODON, *Measuring poverty reduction and targeting performance under multiple government programs* Août 2001, 16 p.

01-06 DUCLOS, Jean-Yves et Paul MAKDISSI, *Restricted inequality and relative poverty* Août 2001, 31 p.

01-07 TÉJÉDO, Cyril et Michel TRUCHON, *Serial cost sharing in multidimensional contexts* Septembre 2001, 37 p.

01-08 TÉJÉDO, Cyril, *Strategic analysis of the serial cost sharing rule with symmetric cost function*. Février 2001, 25 p.

01-09 HANEL, Petr, *Current intellectual protection practices by manufacturing firms in Canada*. Septembre 2001, 57 p.

02-01 DUCLOS, Jean-Yves, Paul MAKDISSI et Quentin WODON, *Socially-efficient tax reforms*, Janvier 2002, 47 p.

02-02 MAKDISSI, Paul, *La décroissance démographique : Pourquoi pas?*, Février 2002, 20 p.

02-03 LECLERC, André et Mario FORTIN, *Production et rationalisation des intermédiaires financiers : leçons à tirer de l'expérience des caisses populaires acadiennes* Février 2002, 24 p.

02-04 HANEL, Petr et Snezana VUCIC, *L'impact économique des activités de recherche de l'Université de Sherbrooke,* Février 2002, 44 p.

02-05 TÉJÉDO, Cyril et Michel TRUCHON, *Monotonicity and bounds for cost shares under the path serial rule*, Mars 2002, 18 p.

02-06 PORET, Sylvaine et Cyril TÉJÉDO, *Analyse horizontale du marché des biens illicites*, Mai 2002, 15 p.

02-07 KALULUMIA, Pene, *Effects of government debt on interest rates : evidence from causality tests in Johansen-type models*, Juillet 2002, 21 p.

02-08 MAKDISSI, Paul et Quentin WODON, *Can safety nets offset the impact of risk on wage inequality and* *social welfare?* Août 2002, 12 p.

02-09 DUCLOS, Jean-Yves, Paul MAKDISSI et Quentin WODON, ***Poverty-reducing tax reforms with heterogeneous agents***, Février 2002, 10 p.

02-10 MAKDISSI, Paul et Quentin WODON, ***Fuzzy targeting indices and orderings***, Mai 2002, 11 p.

02-11 DUCLOS, Jean-Yves, Paul MAKDISSI et Quentin WODON, ***Poverty-efficient transfer programs : the role of targeting and allocation rules***, Mai 2002, 25 p.

02-12 MAKDISSI, Paul et Quentin WODON, ***Environmental regulation and economic growth under education externalities***, Août 2002, 8 p.

02-13 CHARTRAND, Frédéric et Mario FORTIN, ***L'impact du régime d'accession à la propriété sur la demande de logement***, Novembre 2002, 46 p.

03-01 MAKDISSI, Paul et Quentin WODON, ***Gini decomposition and gini income elasticity under income variability***, Avril 2003, 11p.

03-02 MAKDISSI, Paul et Quentin WODON, ***Robust comparisons of natural resources depletion indices***, Avril 2003, 11 p.

03-03 MAKDISSI, Paul, Yannick THERRIEN et Quentin WODON, ***L'impact des transferts publics et des taxes sur la pauvreté au Canada et aux Etats-Unis*** 28 p.

03-04 MAKDISSI, Paul et Quentin WODON, ***Corruption, inequality, and environmental regulation*** Mai 2003, 16 p.

03-05 MAKDISSI, Paul et Quentin WODON, ***Robust poverty comparisons and marginal policy reform orderings under income variability***, Juillet 2003, 13 p.

03-06 CRUCES, Guillermo, Paul MAKDISSI et Quentin WODON, ***Poverty Measurement Under Risk Aversion Using Panel Data,*** Août 2003, 20 p.

03-07 LECLERC, André et Mario FORTIN, **Mesure de la production vancaire, rationalisation et efficacité des Caisses Populaires Desjardins**, Novembre 2003, 46 p.

03-08 BERNARD, André et Gérald ROY, **Étude des distorsions de nievau des tests de Johansen pour la cointégration**, Novembre 2003, 47 p.

03-09 HANEL, Petr, **Impact of innovation motivated by environmental concerns and government regulations on firm performance: a study of survey data**, Décembre 2003, 31 p.

04-01 KALULUMIA, Pene et Denis BOLDUC, **Generalized mixed estimation of a multinomial discrete-continuous choice model for electricity demand**, Janvier 2004, 20 p.

04-02 HANEL, Petr, **Impact of government support programs on innovation by Canadian manufacturing firms** Janvier 2004, 33 p.

04-03 HANEL, Petr, **Innovation in the Canadian Service Sector,** Janvier 2004, 52 p.

04-04 MAKDISSI, Paul et Quentin WODON, **Poverty-Reducing and Welfare-Improving Marginal Public Price and Price Cap Reforms**, mai 2004, 17 p.

* *Tous ces cahiers de recherche sont **disponibles** sur notre **site WEB** (www.usherbrooke.ca/economique) ou au Centre de documentation de la FLSH A3-330 (UdeS).*

Prière d'adresser vos commentaires ou demandes d'exemplaires d'un cahier de recherche antérieur (1976 à 1990) à monsieur Paul MAKDISSI, responsable des Cahiers de recherche du Département d'économique, Tél : 819) 821-8000, poste 2269 Télécopieur : 819) 821-7237 Courriel : paul.makdissi@usherbrooke.ca.

Comments or requests for copies of previous Working Papers (1976 to 1990) should be made to the Research Papers Supervisor at the "Département d'économique ", Mr. Paul MAKDISSI. Tel: (819) 821-8000, extension 2269 FAX:819) 821-7237 E-mail: paul.makdissi@usherbrooke.ca.

Révisé le 14 mai 2004.