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Redistribution and Micro-simulation

ABSTRACT.

This paper proposes many indicators of vertical and horizontal redistribution throughout a generalized technique of Gini decomposition. Using the TD/BU micro-simulation models with externalities, we simulate income distributions and income source distributions for seven educational groups in Philippines in order to measure the inequality variations between two periods (pre and post policy simulation). We generalize the technique of multi-decomposition, that is, the combination of income source and subgroup decomposition to capture the different indicators of vertical and horizontal redistribution.

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1. INTRODUCTION

In the last few decades, an increasing literature has developed on the Gini decomposition since a debate has opposed approaches. Many of them such as Mookherjee and Shorrocks (1982) refuse to adopt decomposition because a third and curious term appears in the decomposition equation. Usually, decomposable measures yield two terms: a within-group element I_w that explains the inequalities within each group of the population; and a between-group element I_b which gives the inequalities (in mean income) between groups. Mookherjee and Shorrocks name the third term of the Gini decomposition "interaction term" since no interpretation can be attributed to it. However, other researchers such as Pyatt (1976), and Silber (1989) show that this third component is well defined and corresponds to the inequalities of overlap between the groups. Other researchers such as Lerman and Yitzhaki (1991) demonstrate that the interaction term is a good measure of stratification between groups. In a different Gini decomposition, Dagum (1997a, 1997b) explains that the third term corresponds to Gini's transvariation (1916), that is, the between-group inequalities generated from the group with lower mean income. Then, Deutsch and Silber (1999) proposed a new approach to deal with a generalized subgroup Gini decomposition.

In this paper, we propose an alternative approach to generalize the subgroup Gini decomposition which offers many indicators of vertical and horizontal redistribution. For this purpose, we review in Section 2, Dagum's Gini decomposition. In Section 3, we deal with a generalize Gini decomposition with vertical and horizontal indicators of redistribution. Section 4 presents the micro-simulation approach with the introduction of externalities on import tax duties used to generate changes in income distribution for the application of the method. In Section 5, using the simulations of the micro-simulation approach, we apply our decomposition techniques to the Philippines's incomes. Finally, Section 6 is devoted the concluding remarks.

2. DAGUM'S GINI DECOMPOSITION BY SUB-POPULATION (1997A, 1997B)

Consider a population P with n income units: x_i (i, r = 1,…, n), of mean μ, where P is partitioned into k sub-populations (groups) P_j (j=1,…, k) of size n_j and mean income μ_j. Let x_{ij} (i, r =1,…, n_j) be the i-th individual's income that belongs to the j-th group. The well-known Gini index is given by:

$$G = \frac{\sum_{i=1}^{n}\sum_{r=1}^{n}\left|x_i - x_r\right|}{2n^2\mu} . \tag{1}$$

As the Gini index involves $n\times n$ binary income differences, it possible to gather these gaps to bring out the within-group and the gross between-group Gini indices:

$$G = \underbrace{\frac{\sum_{j=1}^{k}\left(\sum_{i=1}^{n_j}\sum_{r=1}^{n_j}\left|x_{ij} - x_{rj}\right|\right)}{2\mu n^2}} + \underbrace{\frac{2\sum_{j=2}^{k}\sum_{h=1}^{j-1}\left(\sum_{i=1}^{n_j}\sum_{r=1}^{n_h}\left|x_{ij} - x_{rh}\right|\right)}{2\mu n^2}} \tag{2}$$

$$= \quad G^{w} \quad + \quad G^{gb} . \tag{3}$$

The expression "gross between-group" singles out G^{gb} from the measures between the groups in mean and signifies that G^{gb} yield a net income inequality between the groups. Let p_j and s_j be the population shares and income shares, respectively:

$$p_j = \frac{n_j}{n}, \quad s_j = \frac{n_j \mu_j}{n\,\mu}. \tag{4}$$

Definition 1.
The Gini index associated with sub-population P_j yields the income inequalities within P_j:

$$G_{jj} = \frac{\sum_{i=1}^{n_j}\sum_{r=1}^{n_j}\left|x_{ij} - x_{rj}\right|}{2n_j^2 \mu_j}. \tag{5}$$

Definition 2.
The Gini index associated with two groups P_j and P_h [*Dagum* (1987)] *gauges the income inequalities between P_j and P_h:*

$$G_{jh} = \frac{\sum_{i=1}^{n_j}\sum_{r=1}^{n_h}\left|x_{ij} - x_{rh}\right|}{(\mu_j + \mu_h)\, n_j\, n_h}, \ \forall\, j, h = 1, \ldots, k. \tag{6}$$

Theorem 1 [Dagum (1997a)].
The Gini coefficient measured on P of size n, partitioned into k sub-populations of size n_j (j = 1,…, k) is given by:

$$\begin{aligned} G &= \sum_{j=1}^{k} G_{jj}\, p_j s_j + \sum_{j=2}^{k}\sum_{h=1}^{j\text{-}1} G_{jh}\left(p_j s_h + p_h s_j\right) \\ &= \qquad G^{w} \qquad + \qquad G^{gb}. \end{aligned} \tag{7}$$

At this phase, we have a two-term Gini decomposition, which differs in the between-group element from the standard decomposed measures of inequality. Indeed, the well-known decomposable measures such as entropy yield a between-group index, which is obtained by replacing each individual's income by its corresponding mean income. Consequently, as Dagum (1997a) states, this between-group statistical measure is questionable since it does not incorporate variance and asymmetry effect. For this reason, the Gini index is a suitable candidate to capture these phenomena and to construct valid between-group measures of inequality.

Lemma 1 [Dagum (1997a)].
The gross economic affluence d_{jh} is a weighted mean of the income differences x_{ij}–x_{rh} for each x_{ij} of P_j which is higher than x_{rh} of P_h, given that P_j is in mean more "affluent" than P_h ($\mu_j > \mu_h$):

$$d_{jh} = \int_0^{\infty} dF_j(x) \int_0^{x} (x - y)\, dF_h(y). \tag{8}$$

Lemma 2 [Dagum (1997a)].
The first-order moment of transvariation p_{jh} between the j-th and the h-th sub-populations (such as $\mu_j > \mu_h$) is:

$$p_{jh} = \int_0^{\infty} dF_h(x) \int_0^{x} (x - y)\, dF_j(y). \tag{9}$$

The transvariation [Gini (1916), Dagum (1959, 1960, 1961)] stands to the fact that income differences are of opposite sign compared with the difference of their corresponding mean incomes. From these two lemma, Dagum (1997a) proposes two between-group indicators.

On the one hand, the net between-group Gini index of inequality,

$$G^{nb} = \sum_{j=2}^{k}\sum_{h=1}^{j-1}\left(d_{jh} - p_{jh}\right)\left(p_j s_h + p_h s_j\right) , \tag{10}$$

gauges the net inequalities between groups. It corresponds to the inequalities of non-overlap between the groups and to the inequalities in mean since: $\mu_j = \mu_h$ ($\forall j$, $h = 1,\ldots, k$) $\Rightarrow G_{nb} = 0$. On the other hand, the intensity of transvariation between-groups,

$$G^{t} = \sum_{j=2}^{k}\sum_{h=1}^{j-1} 2p_{jh}\left(p_j s_h + p_h s_j\right) , \tag{11}$$

measures the inequalities of overlap between groups, that is, the inequalities that correspond to those generated by the high incomes of the poorest groups in mean.

Theorem 2 [Dagum (1997a)].

The Gini coefficient computed on P of size n, partitioned into k sub-populations of size n_j ($j = 1,\ldots, k$) *is given by:*

$$G = G^{w} + G^{nb} + G^{t} . \tag{12}$$

Proof.

Adding G^{nb} and G^{t} produces:

$$G^{nb} + G^{t} = \sum_{j=2}^{k}\sum_{h=1}^{j-1}\left(p_{jh} + d_{jh}\right)\left(p_j s_h + p_h s_j\right) . \tag{13}$$

Following Dagum (1997), we know that $p_{jh} + d_{jh} = \Delta_{jh}$, where Δ_{jh} is the well-known Gini's mean difference (1912):

$$\Delta_{jh} = \frac{\sum_{i=1}^{n_j}\sum_{r=1}^{n_h}\left|x_{ij} - x_{rh}\right|}{n_j n_h} . \tag{14}$$

Substituting (14) in (13) yields the gross between-group Gini (G^{gb}). Hence, the proof is completed. ■

3. GENERALIZED GINI DECOMPOSITIONS AND REDISTRIBUTION

Dagum's Gini decomposition is interesting since G^{nb} enables to capture both the inequalities in mean between groups as well as inequalities of non-overlap between groups (those generated from the most affluent groups). However, the decomposition is not unique. Indeed, it is possible to compute many coherent decompositions. Dagum (1997a) proposes the net between-group Gini index of inequality on the basis of the following indicator d_{jh}–p_{jh}, that is, the net economic affluence. This entails a transvariation indicator valued to be $2p_{jh}$. Then, it can be seen as an overvaluation of the first-order moment of transvariation, which is valued as p_{jh}. On the one hand, we can propose a between-group measure based only on the gross economic affluence:

$$G^{*b} = \sum_{j=2}^{k}\sum_{h=1}^{j-1} d_{jh}\left(p_j s_h + p_h s_j\right) . \tag{15}$$

This yields a Gini index between groups issued from each x_{ij} of P_j which higher income than x_{rh} of P_h, given that P_j is in mean more "affluent" than P_h ($\mu_j > \mu_h$). On the other hand, the intensity of transvariation can simply be obtained from the first-order moment of transvariation between-groups:

$$G^{*t} = \sum_{j=2}^{k}\sum_{h=1}^{j-1} p_{jh}\left(p_j s_h + p_h s_j\right) . \tag{16}$$

This yields a Gini of transvariation derived from the income differences x_{rh}–x_{ij} for each x_{rh} *of* P_h which is higher than x_{ij} of P_j, given that P_j is in mean more "affluent" than P_h ($\mu_j > \mu_h$).

Proposition 1 [Mussard (2004)].
The Gini coefficient computed on P of size n, partitioned into k sub-groups of size n_j (j = 1,…, k) *can be expressed as:*

$$G = G^w + G^{*b} + G^{*t} . \tag{17}$$

Proof.
Analogous to Theorem 2. ■

This decomposition is interesting since it yields the inequalities issued from the richest groups in mean G^{*b} or from the poorest groups in mean G^{*t}, without any overvaluation of the transvariation. On this basis, this yields an infinite number of decompositions.

Proposition 2.
The Gini coefficient computed on P of size n, partitioned into k sub-populations of size n_j (j = 1,…, k) *can be expressed as:*

$$G = G^w + G^{\alpha nb} + G^{\alpha t} , \tag{18}$$

where,

$$G^{\alpha b} = \sum_{j=2}^{k}\sum_{h=1}^{j-1}\left(\alpha d_{jh} - \beta p_{jh}\right)\left(p_j s_h + p_h s_j\right) . \tag{19}$$

and where,

$$G^{\alpha t} = \sum_{j=2}^{k}\sum_{h=1}^{j-1}\left((1-\alpha) d_{jh} + (1+\beta)\, p_{jh}\right)\left(p_j s_h + p_h s_j\right) . \tag{20}$$

Proof.
It is quite obvious. Let α, β∈$\mathbb{R}_+$. Adding $G^{\alpha b}$ and $G^{\alpha t}$ gives the gross between-group Gini index, and following (3), we have the overall Gini index G. As can be seen, when α = β = 1, we retrieve Dagum's decomposition (see eq. (12)). When α = 1 and β = 0, we retrieve proposition 1. ■

Proposition 2 generalizes the Gini decomposition *via* the between-group elements. However, many combinations of α and β can produce some decomposition without economic interpretation. Let us find another way to construct coherent Gini decompositions, that is, consistent with the between-group components. For this purpose, we formulate again Lemma 1 and 2 in changing the statistics of comparison between the groups. Indeed, following Dagum's approach, d_{jh} and p_{jh} are based on the mean income, since group P_j is more affluent than P_h: $\mu_j > \mu_h$. Consequently, Dagum's approach yields the income differences (inequalities) generated by the most and the least affluent group.

Why not considering other “more affluent than” orderings? For instance, compared with the arithmetic mean, the median is much less sensitive to outliers.[1] This implies the construction of two new indicators between the groups P_j and P_h. Let M_{ej} and M_{eh} be the median of P_j and P_h such as $M_{ej} > M_{eh}$. Then, it would be interesting to capture income inequalities generated by the groups with higher median (P_j), and those by the groups with lower median (P_h). In the same way, it would be attractive to use a poverty index in order to capture the income inequalities caused by the richest groups and by the poorest ones.

To generalize these different between-group inequalities, let I_j and I_h be two given statistics (such as median, poverty index, etc.) related respectively to the groups P_j and P_h. Hence, Lemma 1 and 2 can be rewritten as follows.

Lemma 3.

The generalized gross economic affluence d_{jh} is a weighted mean of the income differences x_{ij}–x_{rh} for each x_{ij} of P_j which is higher than x_{rh} of P_h, given that $I_j > I_h$:

$$\tilde{d}_{jh} = \int_0^\infty dF_j(x) \int_0^x (x-y)\, dF_h(y), \ \forall I_j > I_h. \tag{21}$$

Lemma 4.

The generalized-first-order moment of transvariation p_{jh} between the j-th and the h-th sub-populations with $I_j > I_h$ is:

$$\tilde{p}_{jh} = \int_0^\infty dF_h(x) \int_0^x (x-y)\, dF_j(y), \ \forall I_j > I_h. \tag{22}$$

Proposition 3.

The Gini coefficient computed on P of size n, partitioned into k sub-groups of size n_j (j = 1,…, k) can be expressed as:

$$G = G^w + \tilde{G}^b + \tilde{G}^t, \tag{23}$$

where,

$$\tilde{G}^b = \sum_{j=2}^{k} \sum_{h=1}^{j-1} \left(\alpha \tilde{d}_{jh} - \beta \tilde{p}_{jh}\right)\left(p_j s_h + p_h s_j\right). \tag{24}$$

and where,

$$\tilde{G}^t = \sum_{j=2}^{k} \sum_{h=1}^{j-1} \left((1-\alpha)\tilde{d}_{jh} + (1+\beta)\tilde{p}_{jh}\right)\left(p_j s_h + p_h s_j\right). \tag{25}$$

Proof.

It is straightforward. ■

Example 1.

Let I_j be the well-known FGT (1984) index, which measures poverty level of the *j*-th group. Suppose $\alpha = 1$ and $\beta = 0$. Then, $\tilde{G}^b$ yields the inequalities generated by the groups with higher FGT indices, that is, the inequalities produced by the poorest groups. On the contrary, $\tilde{G}^t$ gives the inequalities between groups issued from the richest ones.

[1] If we compare poorest and richest groups in “mean”, the ordering between rich and poor groups can change immediately if an error occurs in the data. On the contrary, if few bad incomes were included in the date base, the median value would be very near to the true value with a low probability that the ordering changes between poor and rich groups in “median”.

Proposition 4.

The Gini coefficient computed on P of size n, partitioned into k sub-groups of size n_j (j = 1,…, k), with each individual's income separated into q additive factors $x_{ij} = \sum_{m=1}^{q} x_{ij}^{m}$, is multi-decomposable: the subgroup decomposition is factor decomposable.

Proof.

The multi-decomposition [see Mussard (2004)] consists in a multi-level Gini decomposition. We just need to prove that the three components are factor decomposable, that is, we can measure the contribution of the q factors to G^w, $\tilde{G}^b$ and $\tilde{G}^t$. The discrete expressions of d_{jh} and p_{jh} are respectively:

$$\tilde{d}_{jh} = \frac{\sum_{x_{ij}>x_{rh}} \sum_{i=1}^{n_j} \sum_{r=1}^{n_h} \left(x_{ij} - x_{rh}\right)}{n_j n_h} \; ; \; \tilde{p}_{jh} = \frac{\sum_{x_{rh}>x_{ij}} \sum_{i=1}^{n_j} \sum_{r=1}^{n_h} \left(x_{rh} - x_{ij}\right)}{n_j n_h} \,. \tag{26}$$

As $x_{ij} = \sum_{m=1}^{q} x_{ij}^{m}$, we obtain:

$$\tilde{d}_{jh} = \sum_{m=1}^{q} \frac{\sum_{x_{ij}>x_{rh}} \sum_{i=1}^{n_j} \sum_{r=1}^{n_h} \left(x_{ij}^m - x_{rh}^m\right)}{n_j n_h} \; ; \; \tilde{p}_{jh} = \sum_{m=1}^{q} \frac{\sum_{x_{rh}>x_{ij}} \sum_{i=1}^{n_j} \sum_{r=1}^{n_h} \left(x_{rh}^m - x_{ij}^m\right)}{n_j n_h} \,. \tag{27}$$

Hence, the between-group Gini indices are:

$$\tilde{G}^b = \frac{\alpha \sum_{m=1}^{q} \sum_{x_{ij}>x_{rh}} \sum_{i=1}^{n_j} \sum_{r=1}^{n_h} \left(x_{ij}^m - x_{rh}^m\right) - \beta \sum_{m=1}^{q} \sum_{x_{rh}>x_{ij}} \sum_{i=1}^{n_j} \sum_{r=1}^{n_h} \left(x_{rh}^m - x_{ij}^m\right)}{n^2 \mu} \tag{28}$$

$$\tilde{G}^t = \frac{(1-\alpha) \sum_{m=1}^{q} \sum_{x_{ij}>x_{rh}} \sum_{i=1}^{n_j} \sum_{r=1}^{n_h} \left(x_{ij}^m - x_{rh}^m\right) - (1+\beta) \sum_{m=1}^{q} \sum_{x_{rh}>x_{ij}} \sum_{i=1}^{n_j} \sum_{r=1}^{n_h} \left(x_{rh}^m - x_{ij}^m\right)}{n^2 \mu} \,. \tag{29}$$

For the within-group Gin index, we have:

$$\begin{aligned} G^w &= \frac{\sum_{j=1}^{k} \left(\sum_{i=1}^{n_j} \sum_{r=1}^{n_j} \left| x_{ij} - x_{rj} \right| \right)}{2\mu n^2} = \frac{\sum_{j=1}^{k} \sum_{i=1}^{n_j} \sum_{r=1}^{n_j} \left| \sum_{m=1}^{q} x_{ij}^m - \sum_{m=1}^{q} x_{ir}^m \right|}{2\mu n^2} \\ &= \frac{\sum_{j=1}^{k} \sum_{i=1}^{n_j} \sum_{r=1}^{n_j} \left(\sum_{m=1}^{q} x_{ij}^m + \sum_{m=1}^{q} x_{ir}^m - 2\min\left\{x_{ij}, x_{rj}\right\} \right)}{2\mu n^2} \,. \end{aligned} \tag{30}$$

Let $\sum_{m=1}^{q} x_{irj}^{*m} := \min\{x_{ij}, x_{rj}\}$. Thus:

$$G^w = \sum_{m=1}^{q} \frac{\sum_{j=1}^{k} \sum_{i=1}^{n_j} \sum_{r=1}^{n_j} \left(x_{ij}^m + x_{ir}^m - 2 x_{irj}^{*m} \right)}{2\mu n^2} \,. \tag{31}$$

Finally we obtain a generalized multi-decomposition, that is, an overall breakdown of the Gini index in within-group and between-group Gini indices, which are explained by different additive factors. ■

Example 2.

Let I_j be the median of the j-th group. Suppose α = 1, β = 0 and that each individual's income is disaggregated into consumption expenditures plus savings. Then, $\tilde{G}^b$ characterizes the between-group inequalities generated by the groups with higher medians, and these inequalities are explained by consumption sources and savings. In contrast to this, $\tilde{G}^t$ represents the inequalities between groups obtained from the subpopulations with lower median, and these inequalities are determined by consumption factors and savings.

Proposition 5.

Suppose two periods T and T-1. The Gini coefficient computed on P of size n, partitioned into k sub-populations of size n_j (j = 1,…, k), with each individual's income separated into q additive factor $x_{ij} = \sum_{m=1}^{q} x_{ij}^m$ is time-period multi-decomposable and gives vertical and horizontal measures of redistribution.

Proof.

In a two-period economy, it is possible to compute the Gini index related to each period:

$$G_T = G_T^w + \tilde{G}_T^b + \tilde{G}_T^t \;;\; G_{T-1} = G_{T-1}^w + \tilde{G}_{T-1}^b + \tilde{G}_{T-1}^t . \qquad (32)$$

To get horizontal and vertical indicators of redistribution, we simply perform[2]:

$$G_T - G_{T-1} = G_T^w - G_{T-1}^w + \tilde{G}_T^b - \tilde{G}_{T-1}^b + \tilde{G}_T^t - \tilde{G}_{T-1}^t . \qquad (33)$$

The first element $G_T^w - G_{T-1}^w$ represents changes in within-group inequalities, that is, horizontal redistribution (H). The second one $\tilde{G}_T^b - \tilde{G}_{T-1}^b$, the change in between-group inequalities generated from the groups with $I_j > I_h$, corresponds to vertical redistribution (V_b). In the same manner, $\tilde{G}_T^t - \tilde{G}_{T-1}^t$ symbolizes vertical redistribution (V_t) between groups with $I_j < I_h$. As shown in proposition 4, as the Gini index of inequality is multi-decomposable, this implies that H, V_b, and V_t are explained by the q additive factors. ■

Example 3.

Let I_j be the median income M_{ej} of group P_j. Suppose α = 1, β = 0 and that each individual's income is decomposed by income sources (wages, fringe benefits, capital income, taxes, etc.). Suppose a reduction of H. Then, between T and T-1, we have a redistribution decreasing inequalities within the groups. This drop does not concern all groups, but the time period-multi-decomposition enables one to detect precisely the sub-populations affected by this decline. Moreover, it is possible to know if this decrease comes from changes in wages, capital income or other source of income.

V_b indicates the variation of the inequalities between groups P_j and P_h issued from the groups with higher median incomes. Hence, V_b is a measure of vertical redistribution from the "rich" to the "poor", with the role of each income source.

V_t symbolizes the variation of the inequalities between groups P_j and P_h issued from the groups with lower median incomes. Hence, V_t is a reversed measure of vertical redistribution from the "poor" to the "rich", with the contribution of income components.

[2] Confer Wagstaff (2005) for another approach of vertical and horizontal redistribution indicators.

4. THE MICRO-SIMULATION APPROACH

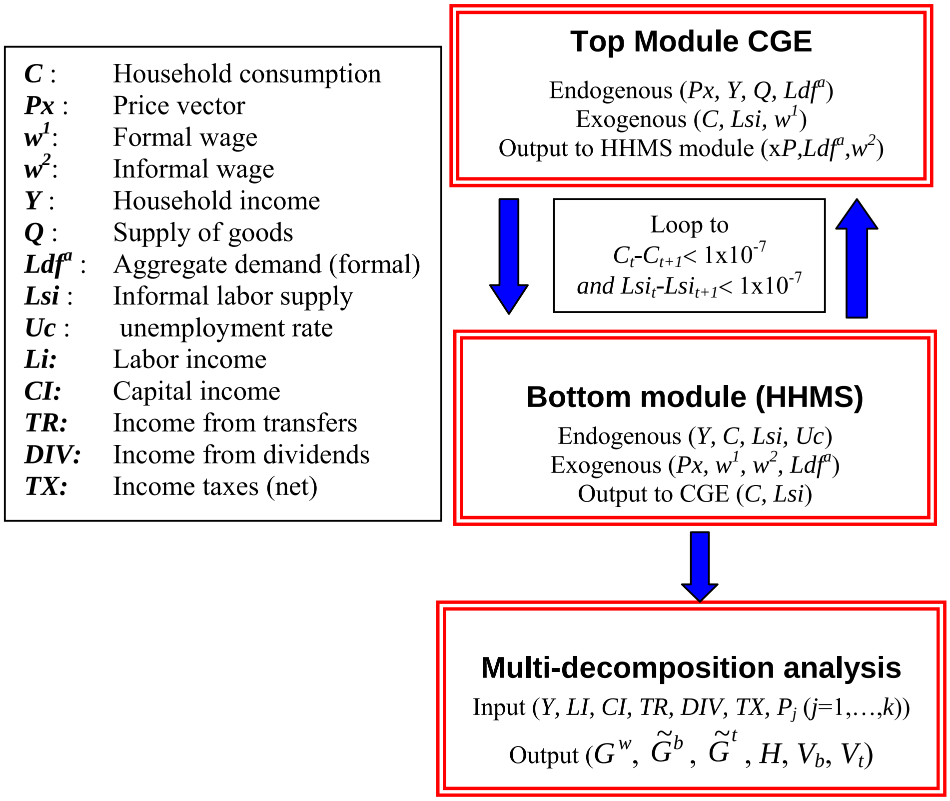
Three main approaches have been developed in the macro-micro modelling context. The main objective of the use of these models is to link macro policy reforms to changes in income distribution and poverty. The first is usually referred to as CGE Integrated Multi-Household (CGE-IMH) approach, the second is generally referred to as the Top-down or micro-simulation sequential approach (CGE-MMS) and the third the Top-Down/Bottom-Up approach (TD/BU).

The CGE-IHM approach first proposed by Decaluwé, Dumont and Savard (1999), relies on inclusion of a large number of households from household survey or all households of the survey into a CGE model. This approach has the advantage of being fully coherent between the micro and macro part of the model albeit data reconciliation can be very problematic [Rutherford *et al*. (2005)] and numerical resolution can also be challenging [Chen and Ravallion (2004)]. The other drawback of the approach is that it can become constraining in terms of the types of behaviours that can be modelled.

The second micro-simulation approach was proposed by Bourguignon, Robilliard and Robinson (2003) to rigorously integrate richer households or individual's behaviour. Their approach is referred to as the CGE Micro-Simulation Sequential approach (CGE-MSS). It consists of constructing a CGE module that feeds price changes into a micro-simulation household module. The main drawback of this process is that it does not always fully take into account the feedback effect of household behaviour being modelled in the micro-simulation module. This critique of the CGE-MMS approach has been raised in two literature reviews of macro-micro modelling for poverty analysis: Hertel and Reimer (2004) as well as Bourguignon and Spadaro (2005).

To circumvent this problem, Savard (2003) proposed the TD/BU approach. The basic idea is to push the CGE-MMS approach further by taking into account the feedback effect of the micro-household module back into the CGE module and imposing convergence. In the paper we apply this third approach to generate changes in income and its subcomponent for each household of a household survey. This information can then be used for our multi-decomposition analysis. Figure 1 presents a synthesis of the methodology.

Figure 1: Resolution procedure of the TD/BU approach



4. 1. Labour market mechanisms

Before presenting the detailed hypothesis of the model it is important to describe the hypothesis of our labour market mechanism. We drew our labour market hypothesis from the Roy (1951) model, revisited by Heckman and Sedlacek (1985) and further enriched by Magnac (1991). The formal market is non competitive with a rigid nominal wage with a cost of entry in the formal sector. The labour supply model is estimated and more details on the results of the estimation can be found in Savard (2005). The wage is flexible on the informal market and allows clearing the endogenous supply with the endogenous demand. In the informal sector, workers will only offer their labour if their estimated reservation wage is inferior to the prevalent nominal wage of the market. The labour market mechanisms described above are found in the CGE modelling context in Fortin, Marceau and Savard (1997), Savard and Adjovi (1998), Devarajan, Ghanem and Thierfelder (1999), Agénor, Izquierdo and Fofack (2003) among others.

4. 2. An application of the CGE-TDBU approach

4. 2. 1. The household micro-simulation module (HHMS)

The household micro-simulation module (HHMS) comprises a representation of the income structure and expenditure behaviour of the household as well as the labour supply decision of households. The household consumption is modelled with a linear expenditure demand system (LES). We use the calibration method proposed by Dervis *et al*. (1982), but with all households having the same income elasticity of each good, and the Frisch parameter. As for the savings rate and income tax rates are calibrated as a fixed share of the household income based on the observed data. All transfers received and paid are exogenous. We consider the capital endowment as fixed to the level observed in the Family Income and Expenditure Survey (FIES-1997). Households are endowed with formal or informal labour based on information found in the FIES and labour force survey (LFS).

4. 2. 2. The CGE model

The CGE model used draws from EXTER model of Decaluwé *et al.* (2001). Without going into great detail to describe the model hypothesis, here are the main features of this CGE-module. Using a Cobb-Douglas production function (for value-added) we assume that producers maximize profits. From this profit maximizing behaviour, we derived the optimal aggregate demand for labour. Value-added is a combination of capital and labour (formal and informal) and is related to intermediate consumption with a fixed share assumption. Capital is assumed fixed, which generates branch-specific returns to capital. This assumption implies a short- to medium-term perspective as it takes time to move capital from one sector to another following a real-life policy shift. The two types of labour are related with a constant elasticity of substitution (CES) function and producers minimize the cost of using both types of labour.

The government draws its receipts through income taxes (imposed on households and firms), goods and services taxes, import duties and transfers from other agents (the rest of the world). It spends by paying subsidies and producing public services. Household income is composed of wage payments (from the two labour categories), capital payments, dividends, and transfers from other agents (households and remittances from abroad). The income tax rate corresponds to the effective rate and not the administrative rate. This holds for all tax rate levels. Modelling taxes in such a way allows us to be coherent with the observed data and implicitly integrates in the model all forms of tax exemptions and evasions. As for closure rules, we also assume that Government expenditure is endogenous, total investment is exogenous and government savings adjust to balance the investment savings constraint (domestic and foreign). The current account balance is endogenous and the numeraire is the GDP deflator. We also assume in a standard fashion that the Philippines is a small open economy with Armington (1969) assumption for the demand of imported goods (imperfect substitution with constant elasticity of substitution function (CES)) and constant elasticity of transformation (CET) functions are used to model for export supply.

We used two versions of our model for our simulation. In the first version described above, public expenditures by government are assumed to provide no benefits to society other then to employ part of the workforce. In our second version, we assume as in Savard and Adjovi (1998), Voyvoda and Yeldan (2000) that public expenditures produce positive externalities to the private sector productivity. These externalities can be of various forms such as improving the efficiency of output with better infrastructure, improved justice system, improved health and training for workers etc. We model these externalities by introducing an externality function with decreasing returns to the public expenditure such as modelled in Savard and Adjovi (1998). We specified sector specific elasticities of public expenditure externalities based on the reliance of the sectors to public services. We selected weak elasticities in our model, to prevent from having externalities play an over weighted role in the results.

5. Simulation and results

For the purpose of illustrating the joint application of horizontal and vertical decomposition with micro-simulation approach we chose to implement commonly analysed policies in the context of macro-micro modelling. We simulate partial trade liberalization, that is, a 40% decrease of import duties on two versions of the TD/BU approach without externalities (WOE) and with externalities (WTE). The objectives of these two simulations are on the one hand to illustrate how we can perform rich horizontal and vertical distributional analysis with a micro-simulation approach and if the introduction of externality of public expenditure hypothesis can modify conclusions or improve the impact of this type of reform.

5.1. TD/BU results

We do not present complete results of the TD/BU micro-simulation approach but focus on the key variables influencing the household income distribution. We present a few key macroeconomic variables as well as the variations of the rental rate of capital as this variable is an important contributor to household income change. In Table 1 we note that the simulation produces strong reduction in government income (-11.60%). This was anticipated as import duties account for approximately 16% of government income. The drop in government income imposes an important reduction in government expenditure (implicitly in government services) of 9.73%.

Table 1: TD/BU micro-simulation macro results

Variables	Definition	Reference	Without externalities (WOE)	With externalities (WE)
Yh	Aggregate household income	86476.9	-0.49%	-0.77%
w^1	Formal wage	1	0.00%	0.00%
w^2	Informal wage	0.5	-1.10%	-1.40%
Yg	Government income	20367	-11.60%	-11.81%
Sg	Government savings	-1163.1	62.38%	57.38%
g	Government expenditure	16818.8	-9.73%	-10.34%
u	Unemployment	16.84	2.31%	2.90%
Ye	Firms income	26172.9	0.41%	0.16%
Se	Firms savings	7810.5	0.70%	0.28%
cab	current account balance	8198.9	4.39%	5.09%
GDP	Gross domestic product	104510.7	-0.60%	-0.91%

This reduction in government services puts strong pressure on the labour market as the reduction in government services is essentially accomplished through the layoff of civil servants. As we have explained, these workers will either choose to become unemployed or to supply their labour on the informal market. This produces an increase in unemployment rate of 2.31% and a reduction of the informal wage of 1.10%. These two variables will play an important role in modifying the household's income concerned by this policy. The other interesting result is that the policy produces a reduction in GDP. In standard CGE models, this effect is generally positive but very small. This stronger reduction is explained by the presence of unemployment and therefore a policy that increases unemployment, reduces the factors used in the economy and therefore the GDP.

When comparing the effect on the income of other agents we observe an increase in the firm's income by 0.41% and a reduction in the aggregate household income. The agent benefiting from this policy is clearly the firms and households and governments are the losers. Let us comment briefly the changes in the rental rate of capital. The factors influencing the rental rate of capital are diverse. First sector most protected are more vulnerable from this policy as they will be challenged by cheaper imported goods, second, the strong reduction in cost in intermediate inputs of certain sectors will also influence the final price of capital and other general equilibrium effects contribute to the final outcome observed.

Table 2: TD/BU micro-simulation sectoral

Variables	branches	Reference	Without externalities (WOE)	With externalities (WE)
r (Return Rate on Capital)	Palay & corn	1	0,73%	-0,34%
	Fruit & vegetable	1	1,27%	0,88%
	Coconut	1	-0,25%	-1,09%
	Livestock	1	1,59%	0,83%
	Fishing	1	1,25%	0,74%
	Other agriculture	1	-1,26%	-1,64%
	Logging % timber	1	1,82%	0,76%
	Mining	1	2,50%	2,02%
	Manufacturing	1	0,29%	0,30%
	Rice manufacturing	1	0,63%	0,21%
	Meat industry	1	1,05%	1,23%
	Food manufacturing	1	-0,44%	-0,03%
	Elec., gas & water	1	-1,63%	-1,56%
	Construction	1	1,98%	1,98%
	Commerce	1	0,37%	0,01%
	Transport & communi.	1	-0,06%	-0,29%
	Finance	1	-2,27%	-2,81%
	Real estate	1	1,07%	1,13%
	Services	1	0,18%	-0,02%

The mining sector (+2.50%), the construction sector (+1.98%) and the logging and timber (+1.82%) are the most favoured sector by this policy. The households endowed with factors in theses sectors will be the biggest gainers. On the other hand, the households endowed with capital in the financial sector (-2.27%), electricity, gas and water (-1.63%) and the other agriculture sector (-1.26% are the biggest losers from this reform.

Moving to the results on the version with externalities we note some changes in results at the macro and sectoral level. The first round effect of the reform on government income is almost the same as in the other version of the model i.e. a reduction of 11.81% and consequently the reduction of public expenditure is very similar (-11.81%) to the WOE version. As the reduction in public expenditure produce negative externalities, the other variables are likely to be more sensitive. In the case of the household aggregate income, the decrease is more the 50% stronger going from -0.49% to -0.77%. The informal wage decreases more at -1.40% and the unemployment is also more sensitive (-2.90%). In this case the positive effect on the firms income is attenuated and increase only by 0.16%. Finally the GDP decreases by 0.91% instead of 0.60% in the WOE version.

The other element important for households is the differences with changes of the rental rate of capital. It is quite interesting that the ranking of the effects have been modified. The two strongest effects are in the same sector i.e. the mining sector (+2.02%), the construction sector (+1.98%) but the third sector changes to the meat industry (+1.23%). On the negative side, the ranking is the same but for some sector the effect is stronger and in other the effect is weaker. We observe qualitative differences in two sectors and differences above 0.5% in seven instances.

5.2. Multi-decomposition results

In the sequel, we illustrate proposition 5, that is, the time-period Gini multi-decomposition *via* seven educational groups: (1) no level of education; (2) primary school level of education without diploma; (3) primary school level of education with diploma; (4) secondary school without diploma (one, two or three years of study); (5) secondary school with diploma; (6) university level without diploma; and (7) university level with diploma. We choose the median statistics to compute between-group inequalities. So that, it is possible to capture inequalities generated from the groups with higher or lower medians (see Table 3). In the application, we note that the higher the educational level, the greater the median.

Table 3: Median per educational groups *

Educational Groups	Reference Period: T-1	Without externalities (WOE): T	With externalities (WE): T
Group 1	14724	14678	14621
Group 2	14951	14861	14804
Group 3	16395	16367	16311
Group 4	17958	17858	17754
Group 5	22625	22394	22320
Group 6	26226	26109	26057
Group 7	41438	41155	41106

* Units: Philippine Pesos

Consequently, we calculate two vertical redistribution indicators. When we measure the variation of the inequalities, say ΔG, between the reference periods T-1 and a given import duties reduction at period T between groups P_j and P_h (generated by the group with higher median $M_{ej} > M_{eh}$), we have: $V_b^{jh} = \tilde{G}_T^{bjh} - \tilde{G}_{T-1}^{bjh}$.

- If $V_b^{jh} < 0$, the group with higher median (P_j) yields less income inequalities between T-1 and T. It is equivalent to a finite number of progressive transfers in incomes from "rich" individuals of group P_j to "poor" individuals of group P_h, that is, a pro-poor vertical redistribution (inequality reducing) by the "rich".

Illustration 1.

In our application, we find no examples in WOE and WE cases where $V_b^{jh} < 0$.[3] Therefore, between the reference period T-1 and those with 40% reduction in import duties T, we find that the between-group inequalities, generated by the groups with higher median (P_j), are increasing. And with the exception of educational groups 2 and 3, V_b^{jh} possesses higher values in the case of changes in import duties without externalities (see the appendix, Table 4). This means that integrating externalities attenuate the increase of between-group inequalities that are inherent to the groups with higher median.

- If $V_b^{jh} > 0$, the group P_j with higher median yields more inequalities between T-1 and T. It corresponds to a finite number of regressive transfers in incomes from

[3] Tables 3 and 4 give V_b^{jh} and V_t^{jh} in percentage, that is, $V_b^{jh}/(G_T\text{-}G_{T\text{-}1})$ and $V_t^{jh}/(G_T\text{-}G_{T\text{-}1})$. Then, as $G_T\text{-}G_{T\text{-}1} > 0$, the percentages in Tables 3 and 4 can be interpreted in the same way than the absolute contributions V_b^{jh} and V_t^{jh}.

Group P_j to Group P_h, or a pro-rich vertical redistribution (inequality increasing) by the "rich".

Illustration 2.

The most important value V_b^{jh} is concerned with inequalities between groups 5 (secondary school with diploma) and 7 (university level with diploma). These inequalities are created by the group with higher median (group 7), for which the expansion explains 10.68% of the rise of the overall Gini index (ΔG) in the WOE context and only 10.29% in the WE one (see Table 4 and 5 respectively in the column "Total"). The row "Total V_b" indicates that 96.64% of ΔG (in the non-externality context) is determined by the income differences that are generated by the groups with higher median when we compare group pair-wises (93.06% of ΔG in the case of reduction of import duties with externalities).

The second vertical redistribution indicator measures the variation of the between-group inequalities generated by the groups with lower medians (group P_h): $V_t^{jh} = \tilde{G}_T^{tjh} - \tilde{G}_{T-1}^{tjh}$.

- If $V_t^{jh} < 0$, the sub-population with lower median (P_h) creates less income inequalities with group P_j between T-1 and T. It can represent a finite number of progressive transfers between "rich" individuals of group P_h and "poor" individuals of group P_j, that is, a pro-poor vertical redistribution (inequality reducing) by the "poor".

Illustration 3.

In many cases we observe a decrease of V_t^{jh}, which can be considered as a measure of "good" inequalities, since it shows the capacity of the poorest groups in "median" to generate income inequalities with the richest ones. But, as the decrease of import duties shows, we have reductions of these "good" disparities. For instance, the inequalities between groups 3 (primary school level of education with diploma) and 7 (university level with diploma) that are generated by group 3 decline by 2.22% of ΔG and 2.35% of ΔG in externality and non-externality cases respectively. The externality context can be considered as better, since all the decreases are weaker in this case (see Table 5).

- If $V_t^{jh} > 0$, the sub-population with lower median (P_h) generates, between T-1 and T, more inequalities with the sub-population with higher median (P_j). Intuitively, it is equivalent to a finite number of regressive transfers in incomes from group P_h to group P_j, or a pro-rich vertical redistribution (inequality increasing) by the "poor".

Illustration 4.

These "good" inequalities are increasing between T-1 and T for the income differences between groups: 2 and 3 (2.16% of ΔG), 2 and 4 (0.78% of ΔG), 2 and 5 (0.38% of ΔG), 3 and 4 (0.13% of ΔG), 4 and 5 (0.66 of ΔG), 4 and 6 (0.13% of ΔG), 5 and 6 (1.22% of ΔG), and 6 and 7 (0.32% of ΔG) for the externality case. We note that all of these values dominate the ones of the non-externality case.

When there is a pro-poor (pro-rich) redistribution, is there really income inequality reduction (augmentation)? The answer to this question is obviously positive since V_b^{jh} and V_t^{jh} are concerned with total incomes. However, following Proposition 5, V_b^{jh} and V_t^{jh} can be determined by the q income sources (V_b^{m12} and V_t^{m12} from now on), that are: (LI) labour income; (CI) capital income; (TR) net transfers; (DIV) dividends; and (TX) taxes. Then, for instance, there is a global inequality growth between groups 1 and 2 generated by group 2 ($V_b^{12} > 0$), but we have inequality reductions in particular income sources such as transfers (TR: -0.8% of ΔG, see Table 4) and dividends (DIV: -0.01%

of ΔG, see Table 4). On the other hand, we have less inequalities between groups 1 and 2 that are created by group 1 (-0.78% of ΔG), however, labour incomes (0.37% of ΔG), and capital incomes (0.32% of ΔG) increase the inequalities between groups 1 and 2 created by group 1 (see Table 4).

The time-period Gini multi-decomposition provides a third component: the horizontal redistribution effect (*H*). It is captured by the reduction of inequalities within a particular group.

- If $H_{jw} < 0$, the *j*-th group creates less income inequalities between T-1 and T.
- If $H_{jw} > 0$, the *j*-th group generates stronger inequalities between T-1 and T.

Illustration 5.
For both WE and WOE cases, we find only an inequality decline for group 1, that is, $H_{1w} < 0$. This means we obtain a positive horizontal redistribution effect in group 1: we have a reduction of 0.06% of ΔG (see Table 4 and 5). This inequality does not imply a reduction in all incomes sources. As shown previously for the vertical components, the horizontal redistribution within group *j* can be explained by each income source: $H_{jw}^{m} = G_{T}^{mwj} - G_{T-1}^{mwj}$, where G_{T}^{mwj} stands for the contribution of the *m*-th source of the *j*-th group to the overall inequality amount at period *T*. For instance, $H_{1w} = -0.06$, but labour incomes and capital incomes increase ΔG by 0.05% and 0.09% respectively (in each WOE and WE case, see Table 4 and 5).
On the contrary, in the most educated group (group 7), we have in each case, $H_{7w} = 3.71 > 0$. However, taxes reduce ΔG by 0.18%. In WE, we obtain in addition, a decrease of ΔG of 0.01%. The externality case gives greater H_{jw}^{m} values.

Finally, the time-period Gini multi-decomposition with horizontal and vertical effects of redistribution and with the contribution of each factor component is:

$$G_T - G_{T-1} = \sum_{m=1}^{q}\sum_{j=1}^{k} H_{jw}^{m} + \sum_{m=1}^{q}\sum_{j=2}^{k}\sum_{h=1}^{j-1} V_{b}^{mjh} + \sum_{m=1}^{q}\sum_{j=2}^{k}\sum_{h=1}^{j-1} V_{t}^{mjh}. \quad (34)$$

6. Conclusion

The field of subgroup decomposable inequality measures focuses essentially on the indices derived from the second law of entropy (see Theil (1967), Shorrocks (1980), Cowell (1980), and Tsui (1999)) and the subgroup consistency property. Dagum (1997) advanced some critical but justified criticisms about theses measures. We extend Dagum's approach by demonstrating that the Gini index enables one to merge the income source and the subgroup Gini decomposition: the so-called multi-decomposition and its generalization.

In a two-period economy, we show it is possible to use the concept of multi-decomposition to derive some new measures of vertical and horizontal redistribution. To obtain these results, the multi-decomposition is connected with the TD/BU micro-simulation methods, so that, each simulation yields vertical and horizontal indices of redistribution, which are explained by the different sources of income. Our illustration shows that micro-simulations with externalities produce less between-group inequalities issued from the "rich" ("bad" inequalities), and more between-group inequalities issued from the "poor" ("good" inequalities).

REFERENCES

Agénor, P-R., Izquierdo, A., Fofack, H., 2003. The Integrated Macroeconomic Model for Poverty Analysis: A Quantitative Macroeconomic Framework for the Analysis of Poverty Reduction Strategies. *World Bank Policy Research Working Paper* 3092.

Armington, P., 1969. A theory of demand for products distinguished by place of production , *IMF Staff Paper* 16, 159--176.

Bourguignon, F., Robilliard, A-S., Robinson S., 2002. Representative versus real households in the macro-economic modeling of inequality. *Mimeo*, World Bank, Washington.

Bourguignon, F., Spadaro, A., 2005. Microsimulation as a tool for evaluating redistribution policies. *Working paper* 2005-02, Paris-Jourdan sciences économiques, Paris.

Chen, S., Ravallion, M., 2004. Welfare impacts of China's accession to the world trade organization. *The World Bank Economic Review* 18(1), 29--57.

Cowell, F., A., 1980. On the Structure of Additive Inequality Measures. *Review of Economics Studies* 47, 521--531.

Dagum, C., 1959. *Transvariazione fra più di due distribuzioni*, *in: Gini, C.(Eds.) Memorie di metodologia statistica*, Vol II, Libreria Goliardica, Roma.

Dagum, C., 1960. Teoria de la transvariacion, sus aplicaciones a la economia. *Metron* XX, 1--206.

Dagum, C., 1961. Transvariacion en la hipotesis de variables aleatorias normales multidimensionales. *Proceedings of the International Statistical Institute* 38, Book 4, 473--486, Tokyo.

Dagum, C., 1987. Measuring the Economic Affluence Between Populations of Income Receivers. *Journal of Business and Economic Statistics* 5(1), 5--12.

Dagum, C., 1997a. A New Approach to the Decomposition of the Gini Income Inequality Ratio. *Empirical Economics* 22(4), 515--531.

Dagum, C., 1997b. Decomposition and Interpretation of Gini and the Generalized Entropy Inequality Measures. *Proceedings of the American Statistical Association, Business and Economic Statistics Section, 157th Meeting,* 200--205.

Decaluwé, B., Martens, A., Savard, L., 2001. *La politique économique du développement*, Montréal: Presse de l'Université de Montréal, 509 p.

Decaluwé, B., Dumont, J-C, Savard, L., 1999. How to measure poverty and inequality in general equilibrium framework. *Cahier de recherche CREFA* 99-20, Université Laval, Québec.

Dervis, K., De Melo, J., Robinson, S., 1982. *General equilibrium models for development policy*. London: Cambridge University Press, 526 p.

Deutsch, J., Silber J., 1999. Inequality Decomposition by Population Subgroups and the Analysis of Interdistributional Inequality, in: Silber J. (Eds.), *Handbook of Income Inequality Measurement*, Kluwer Academic Publishers, pp.163--186.

Devarajan, S., Ghanem, H., Thierfelder, K., 1999. Labor market regulations, trade liberalization and the distribution of income in Bangladesh. *Journal of Policy Reform* 3(1), 1--28.

Fortin, B., Marceau, N., Savard, L., 1997. Taxation, wage controls and the informal sector. *Journal of Public Economics* 66(2), 293--312.

FGT - Foster, J. E., Greer J., Thorbecke, E., 1984. Notes and Comments. A Class of Decomposable Poverty Measures. *Econometrica* 52, 761--766.

Gini, C., 1912. Variabilità e mutabilità, *Memori di Metodologia Statistica* 1, *Variabilità e Concentrazione*. Libreria Eredi Virgilio Veschi, Rome, pp. 211--382.

Gini, C., 1916. Il concetto di transvariazione e le sue prime applicazioni. *Giornale degli Economisti e Rivista di Statistica*, in : C. Gini (Eds.) 1959, pp.21--44.

Heckman, J., Sedlacek, G., 1985. Heterogeneity, aggregation and market wage functions: An empirical model of self-selection in the labour market. *Journal of Political Economy* 93, 1077--1125.

Hertel, T., Reimer, J., 2004. Predicting the poverty impacts of trade reform. World Bank Policy Research Working Paper n° 3444, World Bank, Washington.

Lerman, R., Yitzhaki, S., 1991. Income Stratification and Income Inequality. *Review of Income and Wealth* 37(3), 313--329.

Magnac, T., 1991. Segmented of competitive labour markets. *Econometrica* 59, 165--187.

Mookherjee, D., Shorrocks, A., 1982. A Decomposition Analysis of the Trend in UK Income Inequality. *The Economic Journal* 92, 886--902.

Mussard, S., 2004. Décompositions multidimensionnelles du rapport moyen de Gini. Applications aux revenus italiens de 1989 et 2000. *Thesis*, University of Montpellier I.

Pyatt, G. 1976. On the Interpretation and Disaggregation of Gini Coefficients. *Economic Journal* 86, 243-25.

Roy, A.D., 1951. Some thoughts on the distribution of earnings. *Oxford Economic Papers* 3(2), 135--146.

Rutherford, T., Tarr, D., Shepotylo, O., 2005. Poverty effects of Russia's WTO accession: modeling "real" household and endogenous productivity effects. *World Bank policy research working paper* 3473, World Bank, Washington.

Savard, L., Adjovi, E., 1998. Externalités de la santé et de l'éducation et bien-être : Un MEGC appliqué au Bénin. *L'Actualité économique* 74(3), 523--560.

Savard, L., 2003. A Segmented Endogenous Labour Market for Poverty, Income Distribution Analysis in a CGE-Household MS Model: A Top-Down/Bottom-up Approach. *CIRPEE Working Paper* 03-43, Laval, Québec: University of Laval.

Savard, L., 2005. Analyse de pauvreté et distribution des revenus dans la cadre de la modélisation en équilibre général calculable. *Thesis processed*.

Shorrocks, A., F., 1980. The Class of Additively Decomposable Inequality Measures. *Econometrica* 48, 613--625.

Silber, J., 1989. Factor Components, Population Subgroups and the Computation of the Gini Index of Inequality. *Review of Economics and Statistics* 71, 107--115.

Theil, H., 1967. *Economics and Information Theory*, North-Holland Publishing Company, Amsterdam.

Tsui, K., 1999. Multidimensional Inequality and Multidimensional Generalized Entropy Measures : An axiomatic Derivation, *Social Choice and Welfare* 16, 145--157.

Voyvoda, E, Yeldan, E., 2005. IMF Programs, Fiscal Policy and Growth: An Investigation of Macroeconomic Alternatives in an OLG Model of Growth for Turkey. Comparative Economic Studies, 47(1): 41-79

Wagstaff, A., 2005. Decomposing Changes in Income Inequality into Vertical and Horizontal Redistribution and Reranking, with Applications to China and Vietnam. *World Bank Policy Research Working Paper* 3559, April.

APPENDIX

TABLE 4: 40% INCREASE OF TAX DUTIES WITHOUT EXTERNALITIES

Sources → Indices ↓	LI	CI	TR	DIV	TX	Total
V_b^{m12}	0.08	0.52	-0.80	-0.01	1.12	**0.91**
V_t^{m12}	0.37	0.32	-0.22	-0.05	-1.20	**-0.78**
V_b^{m13}	0.20	0.38	-0.37	0.05	0.45	**0.71**
V_t^{m13}	0.44	0.27	-0.39	-0.06	-1.13	**-0.86**
V_b^{m14}	0.26	0.18	-0.21	0.01	0.42	**0.66**
V_t^{m14}	0.23	0.11	-0.23	-0.02	-0.61	**-0.52**
V_b^{m15}	1.01	0.40	-0.31	0.01	0.58	**1.69**
V_t^{m15}	0.40	0.05	-0.55	-0.05	-0.99	**-1.14**
V_b^{m16}	1.09	0.26	-0.32	0.00	0.48	**1.50**
V_t^{m16}	0.25	-0.02	-0.40	-0.03	-0.64	**-0.84**
V_b^{m17}	2.68	0.46	0.39	0.00	-0.21	**3.32**
V_t^{m17}	0.26	-0.34	-1.04	-0.02	0.10	**-1.04**
V_b^{m23}	0.77	1.24	-0.31	0.23	-0.87	**1.06**
V_t^{m23}	0.47	1.71	-3.06	-0.06	2.86	**1.92**
V_b^{m24}	1.04	0.61	-0.27	0.06	0.48	**1.91**
V_t^{m24}	0.24	0.72	-1.69	-0.02	1.35	**0.59**
V_b^{m25}	4.06	1.49	-0.02	0.02	0.17	**5.72**
V_t^{m25}	0.29	0.80	-3.34	-0.05	2.33	**0.04**
V_b^{m26}	4.51	1.02	-0.53	0.00	0.64	**5.64**
V_t^{m26}	0.15	0.28	-2.27	-0.03	1.40	**-0.46**
V_b^{m27}	11.34	2.21	3.18	-0.02	-2.83	**13.89**
V_t^{m27}	0.13	-0.89	-4.27	-0.02	3.39	**-1.66**
V_b^{m34}	1.07	0.53	-0.54	0.05	0.70	**1.81**
V_t^{m34}	0.40	0.40	-0.57	0.12	-0.36	**-0.01**
V_b^{m35}	4.13	1.42	-0.43	0.01	0.51	**5.64**
V_t^{m35}	0.46	0.27	-1.42	0.19	-0.57	**-1.08**
V_b^{m36}	4.58	0.99	-0.79	0.00	0.87	**5.66**
V_t^{m36}	0.20	-0.04	-1.07	0.13	-0.43	**-1.20**
V_b^{m37}	11.63	2.31	3.15	-0.02	-2.74	**14.33**
V_t^{m37}	0.06	-1.17	-3.32	0.10	1.96	**-2.35**
V_b^{m45}	1.97	0.61	-0.32	0.01	0.17	**2.44**
V_t^{m45}	0.69	0.13	-0.80	0.05	0.53	**0.59**
V_b^{m46}	2.21	0.44	-0.46	0.00	0.39	**2.58**
V_t^{m46}	0.35	-0.02	-0.58	0.02	0.31	**0.08**
V_b^{m47}	5.72	1.13	1.58	-0.01	-1.47	**6.95**
V_t^{m47}	0.17	-0.57	-1.68	0.02	1.40	**-0.65**
V_b^{m56}	3.21	0.57	-0.99	0.00	0.68	**3.46**
V_t^{m56}	1.43	0.30	-0.70	0.01	0.15	**1.18**
V_b^{m57}	8.87	1.77	2.46	-0.02	-2.41	**10.68**
V_t^{m57}	0.65	-0.68	-2.61	0.00	2.11	**-0.53**
V_b^{m67}	5.09	1.04	1.48	-0.01	-1.51	**6.09**
V_t^{m67}	0.90	-0.35	-1.86	-0.01	1.61	**0.30**
Total: V_b	75.53	19.57	5.58	0.35	-4.38	**96.64**
Total: V_t	8.55	1.29	-32.07	0.22	13.57	**-8.43**
Total: $V_b + V_t$	84.08	20.86	-26.49	0.57	9.19	**88.21**
H_{1w}^m	0.05	0.09	-0.17	-0.01	-0.02	**-0.06**
H_{2w}^m	0.37	1.85	-2.44	-0.05	2.37	**2.10**
H_{3w}^m	0.86	1.06	-0.83	0.22	-0.53	**0.79**
H_{4w}^m	0.51	0.20	-0.35	0.03	0.31	**0.70**
H_{5w}^m	2.71	0.72	-0.90	0.01	0.32	**2.87**
H_{6w}^m	1.74	0.25	-0.69	0.00	0.38	**1.68**
H_{7w}^m	3.24	0.44	0.24	0.00	-0.22	**3.71**
Total: H_w^m	**9.49**	**4.62**	**-5.13**	**0.20**	**2.61**	**11.79**
Total: G_T-G_{T-1}	**93.57**	**25.48**	**-31.62**	**0.77**	**11.80**	**100.00**

TABLE 5: 40% INCREASE OF TAX DUTIES WITH EXTERNALITIES

Sources → Indices ↓	LI	CI	TR	DIV	TX	Total
V_b^{m12}	0.07	0.49	-0.75	-0.01	1.03	**0.84**
V_t^{m12}	0.35	0.31	-0.20	-0.04	-0.95	**-0.54**
V_b^{m13}	0.19	0.35	-0.35	0.05	0.35	**0.59**
V_t^{m13}	0.42	0.26	-0.37	-0.05	-0.87	**-0.61**
V_b^{m14}	0.25	0.17	-0.20	0.01	0.36	**0.58**
V_t^{m14}	0.22	0.10	-0.21	-0.02	-0.47	**-0.39**
V_b^{m15}	0.95	0.38	-0.29	0.01	0.48	**1.52**
V_t^{m15}	0.38	0.05	-0.52	-0.04	-0.78	**-0.90**
V_b^{m16}	1.03	0.25	-0.31	0.00	0.43	**1.40**
V_t^{m16}	0.24	-0.02	-0.38	-0.03	-0.50	**-0.69**
V_b^{m17}	2.54	0.44	0.37	0.00	-0.23	**3.12**
V_t^{m17}	0.25	-0.33	-0.99	-0.02	0.16	**-0.93**
V_b^{m23}	0.72	1.16	-0.27	0.22	-0.65	**1.19**
V_t^{m23}	0.45	1.61	-2.87	-0.05	3.02	**2.16**
V_b^{m24}	0.98	0.57	-0.25	0.05	0.53	**1.88**
V_t^{m24}	0.22	0.68	-1.59	-0.02	1.49	**0.78**
V_b^{m25}	3.83	1.41	-0.01	0.01	0.27	**5.52**
V_t^{m25}	0.28	0.75	-3.14	-0.04	2.54	**0.38**
V_b^{m26}	4.28	0.96	-0.49	0.00	0.72	**5.48**
V_t^{m26}	0.14	0.26	-2.14	-0.03	1.54	**-0.23**
V_b^{m27}	10.76	2.10	3.02	-0.01	-2.58	**13.29**
V_t^{m27}	0.13	-0.85	-4.03	-0.02	3.28	**-1.49**
V_b^{m34}	1.02	0.49	-0.49	0.04	0.72	**1.78**
V_t^{m34}	0.37	0.38	-0.52	0.11	-0.21	**0.13**
V_b^{m35}	3.89	1.34	-0.38	0.01	0.57	**5.43**
V_t^{m35}	0.43	0.25	-1.32	0.19	-0.35	**-0.81**
V_b^{m36}	4.34	0.93	-0.73	0.00	0.95	**5.49**
V_t^{m36}	0.19	-0.04	-1.00	0.12	-0.27	**-0.99**
V_b^{m37}	11.04	2.18	2.99	-0.01	-2.49	**13.70**
V_t^{m37}	0.06	-1.10	-3.14	0.10	1.86	**-2.22**
V_b^{m45}	1.85	0.58	-0.30	0.01	0.27	**2.41**
V_t^{m45}	0.64	0.13	-0.75	0.04	0.60	**0.66**
V_b^{m46}	2.10	0.41	-0.43	0.00	0.47	**2.55**
V_t^{m46}	0.34	-0.02	-0.55	0.02	0.35	**0.13**
V_b^{m47}	5.43	1.07	1.50	-0.01	-1.31	**6.69**
V_t^{m47}	0.16	-0.55	-1.59	0.02	1.33	**-0.63**
V_b^{m56}	3.05	0.54	-0.93	0.00	0.79	**3.45**
V_t^{m56}	1.34	0.28	-0.66	0.01	0.25	**1.22**
V_b^{m57}	8.43	1.68	2.34	-0.01	-2.15	**10.29**
V_t^{m57}	0.61	-0.64	-2.48	0.00	2.00	**-0.51**
V_b^{m67}	4.83	0.98	1.41	-0.01	-1.34	**5.87**
V_t^{m67}	0.86	-0.33	-1.75	-0.01	1.55	**0.32**
Total: V_b	71.58	18.48	5.46	0.34	-2.81	**93.06**
Total: V_t	8.08	1.17	-30.19	0.22	15.57	**-5.16**
Total: V_b+V_t	79.65	19.65	-24.73	0.56	12.76	**87.90**
H_{1w}^m	0.05	0.09	-0.16	-0.01	-0.01	**-0.06**
H_{2w}^m	0.35	1.75	-2.29	-0.04	2.57	**2.10**
H_{3w}^m	0.81	1.00	-0.76	0.21	-0.34	**0.79**
H_{4w}^m	0.49	0.19	-0.32	0.03	0.35	**0.73**
H_{5w}^m	2.56	0.69	-0.84	0.01	0.46	**2.87**
H_{6w}^m	1.66	0.25	-0.65	0.00	0.47	**1.72**
H_{7w}^m	3.09	0.42	0.23	-0.01	-0.18	**3.71**
Total: H_w^m	**9.01**	**4.37**	**-4.79**	**0.19**	**3.32**	**11.87**
Total: G_T-G_{T-1}	**88.66**	**24.03**	**-29.52**	**0.75**	**16.09**	**100.00**