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Public Transfers, Equivalence Scales and Poverty in Canada and the United States

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Abstract

The evaluation of the impact on poverty of social programs depends on how other programs are treated in the analysis and on the assumptions used for estimating poverty measures. This paper applies a simple yet sound method for allocating between various programs the total poverty reduction obtained from several programs. Estimates of program impacts are also provided for a range of equivalence scales, poverty measures, and poverty lines in both Canada and the United States.

Keywords: Poverty, transfers, taxes, Canada, United States
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1 Introduction

The evaluation of the impact of programs and policies on poverty should be an integral part of any strategy for combatting poverty. This was for example recognized on December 13, 2002 in the law to "combat poverty and social exclusion" unanimously adopted by Quebec's National Assembly. The bill suggests the adoption of a National Strategy to Combat Poverty and Social Exclusion, and it commits Quebec's government to prepare Action Plans and annual progress reports. An Advisory Committee on the prevention of poverty and social exclusion will be created, and an "Observatory" will be in charge of enhancing knowledge and develop better indicators on poverty and social exclusion. The Minister of Social Solidarity will need to prepare every three years a report on poverty including an assessment of progress made towards its eradication. Given the fact that the "war on poverty" has long been forgotten in the United States, this law is probably unique today in North America in making poverty reduction an explicit priority, and in requesting a systematic evaluation of the impact of public policies and programs on poverty.

Evaluating such impacts on poverty will not necessarily be easy, however. In this paper, we consider two set of difficulties inherent in any quantitative assessment of the impact of public transfers and taxes on poverty measures. The first set of difficulties is related to the traditional assumptions made in order to assess poverty impacts. Three general conditions must be met in order to compare poverty measures before and after public transfers.

First, there is an identification condition: the analyst must select a threshold (the poverty line) under which basic needs are not met, and the household is declared poor. It may well be, however, that one transfer is better at reducing poverty using one poverty line, while another transfer is better at reducing poverty as defined by another poverty line.

The second condition relates to the choice of the poverty measure for the aggregation procedure whereby the vector of poverty status observed at the household level is translated into a national measure. While most empirical studies rely on the FGT (Foster, Greer, and Thorbecke, 1984) class of poverty measures, other poverty indices could be used as well, and it can happen that the evaluation of the impact of transfers is not robust to the choice of the poverty measure, even within the FGT class (one transfer may have more impact than another with one poverty measure, while the

reverse is observed with another poverty measure).

The third condition relates to how indicators of unmet basic needs are to be compared when households differ in needs. For example, in order to make comparisons of poverty between households of different size, the analyst typically will select an equivalence scale in order to transform household income into equivalent income. The main idea that lies behind the use of an equivalence scale is the fact that part of household total income is used for public goods within the household. Therefore, household needs do not increase in the same proportion than household size. The empirical question is whether the comparisons of unmet basic needs of various groups are robust or not to the assumptions made regarding the extent to which economies of scales exist within households.

In order to deal with these three conditions, one alternative is to test for the robustness of the results to alternative poverty lines, alternative poverty measures, and alternative equivalence scales. This is the approach we follow here (another, more general approach would be to use so-called stochastic dominance criterions, but this is not done here, especially because treating economies of scale in a robust way, for example using sequential stochastic dominance techniques[1], would make the paper overly technical and less appealing to a policy audience).

Consider next the difficulty related to the ordering of multiple programs. As noted by Makdissi and Wodon (2004), the estimates of the impact on poverty of a public transfer program may depend on how other public transfers are treated in the analysis. Consider a country with two identical public transfers. Each tranfer reaches all poor households, and it provides each poor household with exactly enough resources to reach the poverty line, so that no households are poor after the implementation of any one of the two transfers. The impact of each transfer is then measured by comparing the share of the population living in poverty before and after the implementation of the transfer. If the household income used to estimate the impact on poverty of each transfer includes the other transfer, none of the two transfers has any impact on poverty, although each taken individually eradicates poverty. In order to be able to assess the impact of multiple transfers on poverty, and thereby to compare the effectiveness of the various transfers with one another, one needs a method to

[1] On sequential stochastic dominance, see for example Atkinson and Bourguignon (1987), Atkinson (1992), Jenkins and Lambert (1993), Chambaz and Maurin (1998), Duclos and Makdissi (2005), and Duclos, Makdissi and Wodon (2005).

take into account adequately the impact of the other transfers. Such a method was proposed by Makdissi and Wodon (2004). The procedure is based on a game theory result by Shapley (1954). Shapley's rule proposes a unique solution for the allocation of the total surplus obtained in a game from the contribution of each player to the game. In this paper, we apply the same rule for allocating between various programs the total poverty reduction impact obtained from multiple programs.

The structure of the rest of the paper is as follows. In section 2, and in the appendix, we outline our methodology for assessing the impact of various public transfers on poverty in Canada and the United States. We consider five types of transfers: public pensions, unemployment benefits, social assistance transfers, family allowances (this program exists only in Canada), and taxes (an example of negative transfers). Section 3 presents the empirical results. A brief conclusion follows.

2 Methodology

Our use data comes from the Luxembourg Income Study (LIS), specifically surveys for Canada and the United States for 1997. The advantage of the LIS database is that the data are made comparable, at least to some extent, and the information in the data enable the user to identify various income sources. We can thereby estimate the income of households with and without various transfers from the state. For this, we first estimate a "Baseline Income", which corresponds to the following variables in the LIS database: V1, V4, V5, V8, V32, V33, V16, V17, V18, V23, V24S1, V24S2, V24SR, V34, V35, and V36. Details on each variable are provided in the LIS website (at http://lisweb.ceps.lu/). Here, it is enough to mention that roughly speaking, the aggregate baseline incone includes gross labor income, income from assets, basic pensions (both public and private), invalidity and other social security income, income for military personnel, income from social insurance schemes, alimony transfers and other incomes sources that are not state transfers aimed at reducing poverty. Then, we consider in addition the following transfers: a) pensions for the elderly paid by various levels of Government, unemployment assistance (the variable V21 in the database), family-based allowances (the sum of the variables V20, V22 and V24S3), and social assistance (the variables V25 and V26). The sum of our "Baseline Income" and these transfers represents the total gross income of the households. From this gross income, we then deduct taxes (the variables V7, V11 and V13). Details on

the various programs are provided on the web of the Luxembourg Income Study in the institutional documentation for Canada and the United States for the year 1997.

We use Cutler and Katz's (1992) equivalence scale in order to take into account differences in needs between households. The number of adult equivalents in each household is $n_e = n_a + \varphi n_c$, where n_a is the number of adults and n_c is the number of children. The parametre φ takes into account differences in costs of living between children and adults. Cutler et Katz (1992) suggest that a consensus value is 0.40, and this is the value assumed here. The equivalence scale is given by:

$$m\left(n_e, \beta\right) = n_e^{\beta} \tag{1}$$

The parameter β is the elasticity of the equivalence scale, which captures how needs vary with household size. Assuming $\beta = 0$ implies identical needs for all households, while $\beta = 1$ implies no economies of scales (needs are then assumed to be linear in size, after discounting for lower costs for children). While there is some consensus on the value of φ, there is more debate on the value of β. Buhman, Rainwater Schmaus and Smeeding (1984) have analyzed values assumed for β in 30 different equivalence scales used in practice, and found that β varied from 0.12 to 0.84. We will test for the sensitivity of some of our results to alternative values for β, but in general, we will use $\beta = 0.5$, which essentially corresponds to the OECD equivalence scale. If total household income is denoted by X, equivalent income is defined as:

$$Y = \frac{X}{n_e} \tag{2}$$

In order to measure poverty, we use FGT measures (Foster et al., 1984):

$$P_\alpha = \int_0^z \left[1 - \left(\frac{Y}{Z}\right)\right]^\alpha f\left(Y\right) dY \tag{3}$$

where Z is the poverty line. This poverty line is defined relativelly as a specific share λ of the median equivalent adult income:

$$Z_1 = \lambda Y_{médian} \tag{4}$$

We will use two values for λ, namely 0.3 and 0.5.

It was mentioned in the introduction that the estimates of the impact on poverty of a program depend on whether other programs are included in the income aggregate or

not. One theoretically appealing way to avoid such problems is to compute the impact of a program on poverty under all possible combinations of treatment for the other programs, and to take the average impact on poverty under the various combinations as the estimate of the impact of the program. If we have two programs, in order to estimate the impact of the first program, we need to consider only two combinations: an income aggregate which includes the other program, and an aggregate which does not. If we have three programs, we have six combinations. Denoting the programs by #1, #2, and #3, we could consider them in the following orders: #1-#2-#3, #1-#3-#2, #2-#1-#3, #2-#3-#1, #3-#2-#1, and finally #3-#1-#2. Using the first order, namely #1-#2-#3, would mean that when considering program #1, neither program #2, nor program #3 would be included in the income aggregate. Still using the first order, when assessing the impact of program #2, only program #1 would be included in the income aggregate. The use of the first order for assessing the impact of program #3 would imply including both programs #1 and #2 in the income aggregate. Thus, for each program, we compute the impact on poverty uder six different combinations, and then use the average impact on poverty as our best estimate. If we have four different programs, then we need to estimate impacts under a total of 16 combinations. The theoretical rationale for this procedure is discussed in appendix, and more details are available in Makdissi and Wodon (2004).

To illustrate why it is important to proceed in this way, consider table 1. The table provides the minimum and maximum impacts of the four types of transfers in Canada (pensions, unemployment assistance, family-based allowances and social assistance). Remember that the maximum impact is obtained when none of the other programs are included in the income aggregate, while the minimum impact is obtained when all the other programs are included in the aggregate. Consider for example a comparison of the impact on the headcount index of poverty of unemployment benefits and family allowances. The minimum impacts for the two programs are similar, with a reduction in the headcount of 0.66 percentage points for unemployment benefits and 0.60 percentage points for family allowances. The maximum impact is again somewhat higher for unemployment benefits (reduction in headcount of 1.32 percentage point) than for family allowances (reduction in headcount of 1.06 percentage point). However, the maximum impact for family allowances is clearly substantially higher than the minimum impact for unemployment benefits. Clearly, depending on the assumptions used for measuring impacts, it may not always be

straigthforward to rank different programs in terms of which one has the largest impact on the headcount index. This is not an isolated problem since there are several other instances of overlapping in the range of program impacts in table 1.

As mentioned earlier, our solution to this issue is to compute impacts under all possible orderings of programs, and take the mean impact as our best estimate of impact. To give a concrete example, table 2 outlines how we proceeded in order to analyze the impact on poverty of pensions still using the Canadian data (the same method is used for assessing the impact of the other transfers). The first column indicates the various income aggregates used to compute poverty measures without pensions. The second column indicates the income sources (the other transfer programs) excluded from the initial income. Columns 3 and 4 provide the poverty measures (in this case, the headcount index of poverty) obtained with and without pensions, for the specific initial income aggregate considered. Column 5 gives the difference in poverty. The weights assigned to each combination give the probability that pensions will be slotted in that specific position when all possible combinations are taken into consideration. The sum of the weights is one. Given these weights, the last column computes the contribution of the specific case to the aggregate Shapley-based estimation of the impact of the policy. The illustration is for $\beta = 0.5$ and $\lambda = 0.3$, and the estimated impact on the headcount index of extreme poverty is -0.1288. Given that the initial headcount was 0.2177, the final headcount is 0.0889.

The same procedure has been applied for values of β ranging from zero to one, and for the poverty gap as well as the squared poverty gap in both countries. The rationale for measuring impacts under various values of β stems from the fact that the impact of an increase (or decrease) in β has an ambiguous effect on the Shapley-based estimation of the impact of transfers on poverty. This is due to the fact that the increase in β has three effects on a poverty measures as argued by Coulter, Cowell and Jenkins (1992) (see appendix for detailed mathematical exposition). The first effect is what Coulter et al. call a pure poverty line effect. A higher β implies a higher poverty line for every household types (except for singles). This in turn implies higher poverty measures for each household types. However this has an ambiguous effect on the impact of a transfer since both the pre and post transfer poverty indices increase and the sign of the measured impact depends on the magnitude of those two changes. The second effect is due to changes in the shape of the income distribution below the poverty line. Finally, the last effect is what Coulter et al. call an indirect poverty

line effect. A higher β implies a lower median equivalent income and hence a lower poverty line. This last effect goes in the opposite direction than the first for a poverty measure. However, as for the first effect, the impact is ambiguous on measures of the poverty reduction obtained with transfers. Thus, because it is unclear how the impact of programs or transfers on poverty changes with a change in β, we estimate the impacts for a range of values of β. The next section provides the empirical results.

3 Empirical results

Tables 3 to 5 give the estimates of the impact of the various categories of programs in both countries on, repectively, the headcount index, the poverty gap, and the squared poverty gap. In each table, the first column gives the value of β, ranging from full economies of scale to no economies of scale at all. The second column gives the value of the poverty measure before the transfers. The next five columns in the case of Canada, and the next 4 columns in the case of the United States gives the estimated impact on poverty of the various transfers and the taxes paid by households (using the Shapley-based procedure). Of course, transfers reduce poverty while taxes increase them, holding everything else constant. The last column gives the poverty measures after transfers and taxes. The procedure is repeated for two relative poverty lines corresponding to 30 percent and 50 percent of the median equivalent adult income.

We will start by using the standard value of 0.5 for β, in which case public transfers in Canada reduce the headcount index of extreme poverty from 0.2178 to 0.0385, for a total reduction of 0.1793. The reduction in the headcount attributed to pensions is -0.1288 (as discussed in some detail in the previous section), while the impact of social assistance is -0.0420. Unemployment benefits (-0.0095) and family allowances (-0.0077) have a smaller impact. The difference between these four contributions and the overall reduction in poverty is the impact of taxes, which increase poverty, but are of course necessary to fund social transfers. When using the headcount of poverty as opposed to the headcount of extreme poverty, we find that poverty is reduced from 0.3010 to 0.1327 (a reduction of 0.1684). Pensions are again by far the biggest contributor to that reduction (-0.1415), but unemployment benefits come now second (-0.0185), followed by social assistance (-0.0181) and family allowances (-0.0126). The fact that after pensions, unemployment benefits have the largest

impact on the headcount of poverty (al least for values of β not too large) while social assistance transfers have the largest impact on the headcount of extreme poverty is not surpsising, given that unemployment benefits tend to be given to previous workers who are often poor, but not extreme poor.

Many of the findings obtained for the poverty gap and the squared poverty gap are similar to those obtained for the headcount. Consider for example the poverty gap, and as before a value of 0.5 for β and the extreme poverty line. The reduction in the (extreme) poverty gap attributed to pensions is 0.0907, versus 0.0418 for social assistance. The two transfers again contribute to most of the reduction in poverty, with the poverty gap decreasing from 0.1526 to 0.0140 (a difference of 0.1386). Note however that with the poverty gap (and even more so with the squared poverty gap), social assistance remains the transfer with the second largest impact on poverty after pensions when we use the moderate poverty line of half the median equivalent income (for the headcount, unemployment benefits were found to have a larger impact for some values of β). This is essentially because the poverty gap takes into account the distance separating the poor from the poverty line (and the squared poverty gap takes into account the square of that distance), which puts more weight on transfers provided to poorer households as opposed to households who are poor, but whose equivalent income is closer to the poverty line.

We now turn to the results obtained for the United States. Using the extreme poverty line and the standard value of 0.5 for β, we find that the various transfers and taxes reduce the headcount of (extreme) poverty by 0.1316, so that the headcount decreases from 0.2001 to 0.0685. As in Canada, pensions account for the bulk of the reduction in the headcount (0.1038), followed by social assistance (0.0321) and unemployment benefits (0.0016). Using the moderate poverty line, the reduction in the headcount is 0.0923, since the headcount decreases from 0.2791 to 0.1868. Using the extreme poverty line, the poverty gap (table 4) drops from 0.1449 to 0.0286, while it drops from 0.1826 to 0.0672 using the moderate poverty line. Table 5 provides the impacts of the transfers and taxes on the squared poverty gap. In all cases, pensions have the largest impact, followed by social assistance, while unemployment benefits have a musch smaller impact.

As expected, the comparison of Canada and the United States suggests that the system of taxes and transfers in place in Canada is better at reducing poverty than the system in the United States. Indeed, in almost all cases, the poverty measures in

Canada are higher than in the United States when all taxes and transfers are excluded from the income aggregate (there is only one exception to this rule: for high values of β and the extreme poverty line, the squared poverty gap in the United States without taxes and transfers is higher than in Canada). However, after taxes and transfers, poverty is systematically lower in Canada than in the United States (there are no exceptions here).

It is also interesting to note that in many cases (although not always), the impact of pensions tends to be higher for lower values of β which correspond to high economies of scale. By contrast, other transfers often (but again not always) have higher impacts when economies of scale are low. This is what we would expect, given that elderly people receiving pension benefits tend to live in smaller households (many leave alone or with their spouse, but without children) than individuals benefitting from family allowances, social assistance, and/or unemployment benefits). Said differently, if we assume in the policy dialogue that there are few economies of scale within households (higher values of β), then it may make more sense to provide more fundings for other transfer programs than for pensions, essentially because assuming few economies of scale is equivalent to saying that the needs of large households are much higher than the needs of small households. This may however be diffiult to achieve in a political economy context where the elderly are active voters, and certainly more so than the very poor who would benefit the most from social assistance.

4 Conclusion

We started this paper by a reference to a law adopted in Quebec to make poverty reduction a priority, and to encourage a systematic evaluation of the impact of programs and policies on poverty. In this paper, we have provided one small and partial contribution to this effort towards better evaluations. In so doing, we have argued that even if the evaluation of the impact of programs and policies on poverty were to focus solely on "recognized methods of international comparison," as Quebec's Law suggests, we would still be confronted to serious difficulties. We considered two such difficulties: the sensitivity of the evaluation results to the traditional assumptions made in order to measure the impact of transfers on poverty (these assumptions include the choice of the poverty line, the choice of the poverty measure, and the assumptions made to compare households who differ in needs), and the sensitivity of

the results to the treatment of other transfer programs (inclusion or not in the income aggregate) when assessing the impact of each specific program. The objective of the paper, then, was to assess the impact of various public transfers on poverty in Canada and the United States, and to test whether the estimated impacts, and the ranking of the various transfers in terms of the magnitude of their impacts, was robust to various assumptions.

Many of the results were as we would have expected. First, the system of transfers and taxes existing in Canada is apparently better (at least in a static way - we did not consider here the dynamic impacts of social policies) at reducing poverty than the system in place in the United States. Indeed, while relative poverty measures without transfers and taxes in Canada are higher than in the United States, the poverty measures in Canada after transfers and taxes are lower than in the United States. Second, we found that pensions were by far the largest contributor to poverty reduction in both countries, especially when large economies of scale are assumed within households. While the ranking of other transfers in terms of impacts depended on the assumptions used for measurement, in many cases social assistance was found to have larger impacts than unemployment benefits or (in Canada only) family allowances.

It is important to emphasize the limits of what we have tried to do in this paper. First, our objective was to evaluate the overall impact of various types of transfers, rather than the impact per dollar spent on each type of transfers. In order to estimate impacts per dollar spent, we would need to consider the total outlays for the various types of programs (and taxes) in both countries. Second, we have focused on income poverty as traditionally defined. But the concept of poverty itself is used in a variety of ways. According to Quebec's law, poverty isn't simply a lack of income. Rather, it is "the condition of a human being who is deprived of the resources, means, choices and power necessary to acquire and maintain economic self-sufficiency and participation in society." As noted by Noel (2003), while the Bill's objective is "to progressively transform Quebec over a ten-year period into one of the industrialized societies with the least poverty, according to recognized methods of international comparison", other goals include improving "the economic and social situation of people living in poverty and marginalized by society", reducing "inequalities that hinder social cohesion", and taking actions to promote "a sense of social solidarity". A large amount of work in each of these areas will be needed to achieve the stated goal of the law to evaluate more systematically the impact of public action on the

poor.

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A Derivation of the Shapley Measure

This appendix explains the logic behind our method for estimating program impacts on poverty under multiple government transfers and taxes. We follow closely Makdissi and Wodon (2004), although with a slightly different and improved notation. Denote by W the set of programs implemented by the Government with $|W| = w$. The poverty measure used for evaluating the impact of these programs is P and the reduction in poverty obtained from a given program is $\Delta_i(W, P)$ where i denotes a specific program from W. For the evaluation of the impact of the various programs on poverty, we require that the evaluation method respect three axioms.

Axiom 1 ***Focus:*** *if* $P(F \cup i) = P(F), \forall F \subset W$ *then :* $\Delta_i(W, P) = 0$.

The focus axiom states that if a program does not benefit at least one poor person, its impact on poverty is zero.

Axiom 2 ***Additivity:*** $\Delta_i(W, \alpha P' + \beta P'') = \alpha \Delta_i(W, P') + \beta \Delta_i(W, P'') \forall\ i \in W$.

In the additivity axiom, $\Delta^{\alpha}(S)$ denotes the reduction of poverty obtained with a set of programs S. We are interested in assessing the poverty reduction obtained from two additional programs not included in S, namely g_i and g_j. When both programs are added to the programs already included in S, the set of programs under implementation is $S \cup \{g_i, g_j\}$. The axiom states that the additional poverty

reduction from the implementation of the two new programs is the sum of the poverty reduction obtained from each. This property corresponds to the additive nature of the FGT poverty measures.

Axiom 3 ***Equal treatment of equivalent policies (P):*** *If* $P(P \cup i) = P(P \cup j)$ $\forall\ F \subseteq W \setminus \{i, j\}$ *then* $\Delta_i(W, P) = \Delta_j(W, P)$.

The equal treatment axiom states that if the impact of two programs is always the same independently of which other programs have already been taken into account in the income aggregate, then the two programs have the same poverty reduction impact.

In the context of cooperative game theory, Shapley (1953) demontrated that one and only one rule for allocating the total surplus obtained from the contribution of each player respected these three axioms[2]. Shapley's rule consists in computing the contribution of each player for every potential ordering of the other players, and in taking as the allocation rule for each player the average of all his contributions under all possible scenarios. Makdissi and Wodon (2003) used Shapley's theorem to show that in the context of poverty measurement, one and only one measure of the impact on poverty of the various programs satisfies the above axioms.

Proposition 4 *The only measure of poverty reduction which satisfies Axioms 1, 2, and 3, which we will refer to as the Shapley poverty reduction impact of program* g_i*, is:*

$$\Delta_i^S(W, P) = \sum_{k=0}^{N-1} \frac{k!\ (W-k-1)!}{W!} \sum_{\beta_{-i}(k)} \{P(K) - P(K \cup i)\}, \tag{5}$$

where $\beta_{-i}(k) = \{K \mid i \notin K \ \ et\ \ |K| = k\}$.

The proposition 4 states that in order to respect our three axioms, the impact of program g_i on poverty, $\Delta^\alpha(\{g_i\})$, must be the average of the poverty reduction obtained with g_i for all possible permutations of the various programs. Using equation (5) eases a bit the computation of the average by allowing the analyst to compute only one time the poverty reduction for all permutation in which a given policy g_i has the same rank and follows the same other policies, whatever the order of those other policies.

[2]Mathematicaly, our focus axiom is equivalent to Shapley's dummy player axiom. Since our two other axioms are the same as those stated by Shapley, we can use his theorem and apply it directly to our own context.

B The Impact of a Change in β

Coulter et al. (1992) have derived the theoretical impact of a change in β on the poverty index P_α. It is useful to exploit the additive structure of this index and rewrite it as

$$P_\alpha = \sum_{j=1}^{J} \theta_i \int_0^{Z_j} \left[1 - \left(\frac{X}{Z_j}\right)\right]^\alpha f_j(X)\, dX, \tag{6}$$

where θ_j is the population share of households of structure j, $f_j(\cdot)$ is their density of total household income and Z_j is their poverty line (in term of total income). J is the number of different household structures in the population. The poverty line is given by

$$Z_j = m(n_e(j), \beta) Z_1, \tag{7}$$

where $n_e(j)$ is the number of equivalent adults in a household of structure j and Z_1 is the poverty line for a single person. In this framework, Coulter et al. (1992) show that if $\alpha = 0$, the impact of a change in β on the poverty measure is given by

$$\frac{\partial P_\alpha}{\partial \beta} = \sum_{j=1}^{J} \theta_j \log n_e(j) D_\alpha(j) [1 - T(j)], \tag{8}$$

where

$$D_\alpha(j) = \begin{cases} f_j(Z_j) Z_j & \text{if } \alpha = 0 \\ \alpha \int_0^{Z_j} \left[1 - \left(\frac{X}{Z_j}\right)\right]^{\alpha-1} \left(\frac{X}{Z_j}\right) f_j(X)\, dX & \text{if } \alpha \in \{1, 2, 3, ...\} \end{cases} \tag{9}$$

and

$$T(j) = \left(\frac{1}{\log n_e(j)}\right) \left[\frac{COV\left(\overline{Y}_j, \log n_e(j)\right)}{\overline{Y}_j} + \log n_e(j)\right]. \tag{10}$$

The terms θ_j, $\log n_e(j)$ et $D_\alpha(j)$ are necessarily positive. However, $[1 - T(j)]$ may be positive or negative. This implies that expression (8) may be positive or negative.

Here, we are interested to know the marginal impact of a change in β on ΔP_α which is the difference in the poverty index when a transfer policy is added. This impact is given by

$$\frac{\partial \Delta P_\alpha}{\partial \beta} = \sum_{j=1}^{J} \theta_j \log n_e(j) \Delta D_\alpha(j) [1 - T(j)] \tag{11}$$

where

$$\Delta D_{\alpha}(j) = \begin{cases} \left[f_j(Z_j) - \widetilde{f}_j(Z_i)\right] Z_j & \text{if } \alpha = 0 \\ \alpha \int_0^{Z_j} \left[1 - \left(\frac{X}{Z_j}\right)\right]^{\alpha-1} \left(\frac{X}{Z_j}\right) \left[f_j(X) - \widetilde{f}_j(X)\right] dX & \text{if } \alpha \in \{1, 2, 3, ...\} \end{cases}. \tag{12}$$

This last term may be positive or negative. This implies that (11) may again be positive or negative. This will be true for each term of the Shapley value so that the sign of the total effect of a marginal change of β on the impact of a single policy is uncertain.

Table 1: Differences in impacts due to definition of baseline income (Canada, 1997)

		Pensions	Unemployment benefits	Family allowances	Social Assistance
Extreme poverty H	Minimum impact	12.21	0.66	0.60	4.02
	Maximum impact	13.59	1.32	1.06	4.41
Moderate poverty H	Minimum impact	12.98	1.42	0.88	1.26
	Maximum impact	15.34	2.29	1.64	2.35
Extreme poverty PG	Minimum impact	8.38	0.27	0.26	3.51
	Maximum impact	9.80	0.71	1.02	4.88
Moderate poverty PG	Minimum impact	10.39	0.57	0.66	3.39
	Maximum impact	11.77	1.16	1.01	3.94
Extreme poverty SPG	Minimum impact	6.74	0.16	0.17	2.90
	Maximum impact	8.21	0.50	1.15	4.69
Moderate poverty SPG	Minimum impact	8.55	0.33	0.37	3.30
	Maximum impact	9.97	0.79	1.05	4.51

Source: Authors' estimation using LIS database.

Table 2: Estimating program impact: The Shapley methodology (Canada 1997, pensions)

Included transfers (apart from pensions)	Excluded transfers	Poverty without pensions	Poverty with pensions	Change in poverty	Weight	Contribution
None	(1) + (2) + (3) + (4)	21.77	9.41	12.36	1/5	2.47
(1) Unemployment benefits	(2) + (3) + (4)	20.82	8.61	12.21	1/20	0.61
(2) Family allowances	(1) + (3) + (4)	21.08	8.71	12.37	1/20	0.62
(3) Social assistance	(1) + (2) + (4)	17.75	5.33	12.42	1/20	0.62
(4) Taxes	(1) + (2) + (3)	23.45	9.90	13.55	1/20	0.68
(1) + (2)	(3) + (4)	20.23	8.01	12.22	1/30	0.41
(1) + (3)	(2) + (4)	16.79	4.52	12.27	1/30	0.41
(1) + (4)	(2) + (3)	22.13	8.78	13.35	1/30	0.45
(2) + (3)	(1) + (4)	16.77	4.40	12.37	1/30	0.41
(2) + (4)	(1) + (3)	22.68	9.11	13.57	1/30	0.45
(3) + (4)	(1) + (2)	19.33	5.74	13.59	1/30	0.45
(1) + (2) + (3)	(4)	15.97	3.75	12.22	1/20	0.61
(1) + (2) + (4)	(3)	21.51	8.15	13.36	1/20	0.67
(2) + (3) + (4)	(1)	18.26	4.72	13.54	1/20	0.68
(1) + (3) + (4)	(2)	18.03	4.63	13.40	1/20	0.67
(1) + (2) + (3) + (4)	None	17.20	3.84	13.36	1/5	2.67
Shapley value (average)						12.88

Source: Authors' estimation using LIS database.

Table 3: Impact of public transfers on the headcount index of poverty, Canada and United States, 1997

	Canada							United States					
Beta	Poverty Before transfers	Pensions	Unemployment benefits	Family allowances	Social assistance	Taxes	Poverty After transfers	Poverty Before transfers	Pensions	Unemployment benefits	Social assistance	Taxes	Poverty After transfers
Extreme Poverty (Poverty line = 0.30*Median equivalent income)													
0.0	22.78	13.22	0.90	0.67	3.28	-1.15	5.86	20.80	9.08	0.16	2.56	-0.90	9.90
0.1	22.52	13.21	0.92	0.68	3.43	-0.98	5.28	20.59	9.38	0.17	2.60	-0.84	9.29
0.2	22.26	13.10	0.92	0.69	3.55	-0.94	4.93	20.49	9.66	0.17	2.72	-0.67	8.60
0.3	22.10	13.03	0.93	0.69	3.86	-0.89	4.48	20.28	9.94	0.16	2.88	-0.66	7.95
0.4	21.92	12.94	0.97	0.76	4.00	-0.89	4.14	20.17	10.18	0.16	3.07	-0.62	7.37
0.5	21.78	12.88	0.95	0.77	4.20	-0.87	3.85	20.01	10.38	0.16	3.21	-0.60	6.85
0.6	21.58	12.75	0.97	0.79	4.40	-0.85	3.52	19.89	10.59	0.16	3.36	-0.63	6.41
0.7	21.37	12.60	0.97	0.81	4.56	-0.81	3.24	19.73	10.62	0.16	3.45	-0.64	6.15
0.8	21.20	12.45	0.99	0.84	4.79	-0.80	2.94	19.70	10.69	0.16	3.50	-0.64	5.98
0.9	21.09	12.33	1.01	0.85	4.79	-0.79	2.90	19.59	10.67	0.16	3.55	-0.66	5.87
1.0	20.91	12.12	1.04	0.88	4.82	-0.79	2.85	19.59	10.67	0.17	3.54	-0.65	5.87
Poverty (Poverty line = 0.50*Median equivalent income)													
0.0	31.26	9.72	1.58	1.05	1.52	-2.57	19.95	28.87	7.15	0.22	2.36	-2.33	21.47
0.1	31.01	10.25	1.65	1.09	1.58	-2.41	18.85	28.58	7.39	0.23	2.36	-2.32	20.92
0.2	30.62	11.17	1.68	1.13	1.68	-2.42	17.38	28.36	7.71	0.24	2.35	-2.27	20.33
0.3	30.50	12.18	1.75	1.15	1.62	-2.34	16.14	28.19	8.02	0.24	2.36	-2.17	19.73
0.4	30.34	13.18	1.81	1.19	1.65	-2.27	14.79	27.89	8.37	0.25	2.35	-2.20	19.11
0.5	30.10	14.15	1.85	1.26	1.81	-2.23	13.27	27.91	8.74	0.26	2.37	-2.14	18.68
0.6	29.97	14.66	1.89	1.32	1.92	-2.16	12.34	27.86	9.05	0.27	2.31	-2.11	18.34
0.7	29.81	15.09	1.92	1.37	1.91	-2.13	11.65	27.70	9.37	0.26	2.32	-2.15	17.90
0.8	29.58	15.05	1.94	1.42	2.03	-2.10	11.25	27.73	9.69	0.27	2.30	-2.26	17.74
0.9	29.54	14.91	1.96	1.45	2.20	-2.14	11.15	27.77	9.96	0.27	2.28	-2.28	17.53
1.0	29.44	14.82	1.96	1.51	2.47	-2.29	10.98	27.87	10.24	0.27	2.33	-2.34	17.36

Source: Authors' estimation using LIS database.

Table 4: Impact of public transfers on the poverty gap, Canada and United States, 1997

	Canada							United States					
Beta	Poverty Before transfers	Pensions	Unemployment benefits	Family allowances	Social assistance	Taxes	Poverty After transfers	Poverty Before transfers	Pensions	Unemployment benefits	Social assistance	Taxes	Poverty After transfers
Extreme Poverty (Poverty line = 0.30*Median equivalent income)													
0.0	16.07	9.61	0.47	0.60	3.85	-0.52	2.05	15.10	8.59	0.09	2.86	-0.31	3.86
0.1	15.89	9.50	0.47	0.61	3.93	-0.50	1.88	14.97	8.64	0.09	2.92	-0.29	3.60
0.2	15.70	9.38	0.46	0.61	4.01	-0.48	1.72	14.83	8.67	0.09	2.97	-0.28	3.36
0.3	15.55	9.28	0.46	0.61	4.07	-0.48	1.60	14.70	8.69	0.09	3.03	-0.27	3.15
0.4	15.42	9.19	0.46	0.62	4.13	-0.47	1.50	14.59	8.68	0.09	3.08	-0.26	2.99
0.5	15.26	9.07	0.46	0.62	4.18	-0.47	1.40	14.49	8.67	0.09	3.12	-0.25	2.86
0.6	15.12	8.95	0.46	0.63	4.21	-0.46	1.33	14.40	8.63	0.10	3.16	-0.25	2.77
0.7	14.95	8.81	0.45	0.64	4.24	-0.46	1.26	14.32	8.58	0.10	3.19	-0.25	2.71
0.8	14.80	8.68	0.45	0.65	4.25	-0.46	1.23	14.25	8.52	0.10	3.21	-0.25	2.67
0.9	14.66	8.55	0.45	0.66	4.24	-0.46	1.21	14.18	8.46	0.10	3.23	-0.26	2.65
1.0	14.52	8.42	0.46	0.67	4.24	-0.47	1.19	14.13	8.39	0.10	3.26	-0.26	2.64
Poverty (Poverty line = 0.50*Median equivalent income)													
0.0	20.50	10.41	0.78	0.74	3.26	-1.01	6.32	18.99	8.42	0.14	2.71	-0.80	8.52
0.1	20.27	10.64	0.79	0.76	3.34	-0.96	5.70	18.83	8.57	0.14	2.77	-0.76	8.12
0.2	20.03	10.84	0.80	0.77	3.42	-0.92	5.12	18.65	8.71	0.14	2.82	-0.72	7.70
0.3	19.90	10.98	0.82	0.79	3.49	-0.89	4.71	18.49	8.85	0.14	2.87	-0.69	7.31
0.4	19.73	11.06	0.83	0.81	3.57	-0.87	4.33	18.36	8.97	0.14	2.93	-0.67	6.99
0.5	19.54	11.06	0.84	0.83	3.66	-0.85	4.00	18.26	9.08	0.14	2.98	-0.66	6.72
0.6	19.38	11.01	0.85	0.84	3.73	-0.83	3.78	18.16	9.17	0.14	3.03	-0.65	6.47
0.7	19.20	10.90	0.86	0.86	3.82	-0.82	3.58	18.09	9.24	0.14	3.08	-0.65	6.27
0.8	19.04	10.78	0.87	0.87	3.91	-0.82	3.43	18.04	9.30	0.15	3.12	-0.65	6.13
0.9	18.90	10.65	0.88	0.89	3.98	-0.82	3.32	17.99	9.33	0.15	3.17	-0.67	6.01
1.0	18.78	10.52	0.89	0.90	4.05	-0.84	3.26	17.96	9.35	0.15	3.20	-0.69	5.95

Source: Authors' estimation using LIS database.

Table 5: Impact of public transfers on the squared poverty gap, Canada and United States, 1997

	Canada							United States					
Beta	Poverty Before transfers	Pensions	Unemployment benefits	Family allowances	Social assistance	Taxes	Poverty After transfers	Poverty Before transfers	Pensions	Unemployment benefits	Social assistance	Taxes	Poverty After transfers
Extreme Poverty (Poverty line = 0.30*Median equivalent income)													
0.0	13.41	8.01	0.32	0.63	3.67	-0.50	1.27	13.06	7.91	0.07	2.88	-0.48	2.69
0.1	13.25	7.90	0.32	0.63	3.71	-0.49	1.19	12.96	7.89	0.07	2.91	-0.45	2.55
0.2	13.09	7.78	0.31	0.63	3.73	-0.49	1.13	12.85	7.85	0.07	2.94	-0.43	2.42
0.3	12.95	7.67	0.31	0.63	3.75	-0.49	1.08	12.75	7.80	0.07	2.96	-0.40	2.31
0.4	12.83	7.57	0.31	0.64	3.77	-0.50	1.04	12.66	7.76	0.07	2.98	-0.38	2.23
0.5	12.67	7.45	0.30	0.64	3.77	-0.50	1.01	12.58	7.70	0.07	3.01	-0.37	2.17
0.6	12.53	7.33	0.30	0.65	3.77	-0.51	0.99	12.51	7.64	0.07	3.03	-0.35	2.12
0.7	12.37	7.20	0.30	0.65	3.77	-0.52	0.97	12.43	7.58	0.07	3.05	-0.34	2.07
0.8	12.21	7.07	0.30	0.66	3.75	-0.53	0.96	12.37	7.52	0.07	3.07	-0.33	2.04
0.9	12.06	6.95	0.29	0.66	3.74	-0.55	0.96	12.31	7.45	0.07	3.09	-0.32	2.02
1.0	11.91	6.82	0.29	0.67	3.73	-0.56	0.97	12.25	7.38	0.07	3.11	-0.32	2.00
Poverty (Poverty line = 0.50*Median equivalent income)													
0.0	16.66	9.47	0.52	0.66	3.60	-0.67	3.08	15.71	8.33	0.10	2.81	-0.53	5.00
0.1	16.46	9.48	0.52	0.66	3.66	-0.64	2.78	15.58	8.39	0.10	2.86	-0.50	4.72
0.2	16.27	9.45	0.52	0.67	3.73	-0.62	2.52	15.43	8.44	0.10	2.91	-0.47	4.45
0.3	16.14	9.41	0.53	0.68	3.78	-0.61	2.35	15.30	8.48	0.10	2.95	-0.44	4.20
0.4	15.98	9.34	0.53	0.69	3.84	-0.60	2.19	15.19	8.51	0.10	3.00	-0.43	4.01
0.5	15.82	9.24	0.53	0.70	3.89	-0.59	2.06	15.10	8.53	0.10	3.04	-0.41	3.84
0.6	15.67	9.13	0.53	0.71	3.93	-0.59	1.95	15.01	8.53	0.10	3.09	-0.41	3.70
0.7	15.49	9.00	0.53	0.71	3.98	-0.59	1.86	14.93	8.52	0.10	3.13	-0.40	3.59
0.8	15.34	8.87	0.54	0.72	4.01	-0.59	1.79	14.88	8.50	0.10	3.16	-0.40	3.51
0.9	15.19	8.74	0.54	0.73	4.03	-0.60	1.75	14.81	8.46	0.11	3.19	-0.40	3.45
1.0	15.06	8.61	0.55	0.75	4.05	-0.61	1.72	14.76	8.42	0.11	3.22	-0.41	3.43

Source: Authors' estimation using LIS database.