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Abstract

A new approach is developed to identify thorough marginal tax reforms for pairs of commodities and to test for the robustness of their impacts on Yaari's dual social welfare functions. *S-concentration curves* are provided for every order of positional dominance and an illustration is performed using Canadian data.

Keywords: Dual Social Welfare Function, Stochastic Dominance, Tax Reform.
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1 Introduction

The non-intersection between concentration curves is usually considered as a necessary condition to gauge the distributive effects of tax reforms (see e.g. Lambert, 1993). The seminal works of Yitzhaki and Thirsk (1990) and Yitzhaki and Slemrod (1991) are devoted to the construction of welfare-improving indirect tax reforms for all social welfare functions respecting the Pigou-Dalton transfer principle. They identify pairs of commodities with non-intersecting concentration curves, for which the analyst can choose the differential in the efficiency cost of raising public funds through the two commodities. Alternatively, Makdissi and Wodon (2002) have introduced the concept of *consumption dominance curve* to determine the impact of marginal indirect tax reforms on poverty for any order of restricted stochastic dominance. This result has been subsequently generalized in Duclos, Makdissi and Wodon (2004), in which consumption dominance curves are used to test for the robustness of welfare-improving indirect tax reforms in a more general framework than the Pigou-Dalton transfer principle.

In this note, following a different path, we aim at using rank dependant social welfare functions (see Yaari, 1987, 1988), instead of the traditional utilitarian criterion, to evaluate the impact of marginal indirect tax reforms on these social welfare functions. We introduce a new approach, *s-concentration curve*, which is a generalization of the usual concentration curve.

The measurement of rank dependant social welfare functions and the associated ethical criteria are discussed in the next section. Section 3 exposes a framework to analyze the impact of a marginal indirect tax reform and introduces the concept of s-concentration curve. Section 4 presents an empirical illustration and a brief section follows to conclude and suggest further researches.

2 Measuring Rank Dependant Social Welfare

Consider the following rank dependant social welfare function:

$$W(F) = \int_0^1 F^{-1}(p)\, v(p)\, dp \tag{A1}$$

where $F\left(y^E\right)$ is the distribution of equivalent income y^E defined over $[0, a]$, which is a subset of non-negative real incomes and where a is the maximum conceivable income. $F^{-1}(p) = \inf\left\{y^E : F\left(y^E\right) \geq p\right\}$ is the left continuous inverse income distribution, and $v(p) \geq 0$ the weight attached to an individual at the pth percentile of the distribution, with $p \in [0, 1]$.

In order to describe the normative implications underlying the functional forms of $v(\cdot)$, we define some positional ethical principles. The first principle stipulates that ordering two distributions of living standards is equivalent to make the living standards "parade" simultaneously alongside each other, and verifying if one parade weakly dominates the other (this exercise was first suggested by Pen, 1971).

Definition 1 *A social welfare function $W(\cdot)$ satisfies the **Pen Parade Principle** if it is increasing in $F^{-1}(p)$ for any p.*

It is straightforward that this principle is satisfied as long as $v(p) \geq 0$ for all $p \in [0, 1]$. The second principle is the well-known Pigou-Dalton Principle of Transfers (Pigou, 1912 and Dalton, 1920).

Definition 2 *A social welfare function $W(\cdot)$ satisfies the **Pigou-Dalton Principle of Transfers** if a transfer from a higher-income individual to a lower-income individual increases social welfare.*

These two first definitions are identical in both utilitarian and rank dependant social welfare approaches. However, it would be appealing to impose more structure on the social welfare function, permitting the decision-maker to choose between a wide range of transfer principles (see e.g. Gajdos, 2002).

In an utilitarian framework, Kolm (1976) has introduced the Principle of Transfer Sensitivity. Mehran (1976) and Kakwani (1980) have adapted this rule to a Positional Principle of Transfer Sensitivity. They stipulate that a small transfer from a higher-income person to a lower-income one, with a given proportion of the population between them, is more valuable if it occurs at lower income levels. In order to propose a formal definition of this principle, let us first define $\Delta_{p,\gamma} W(\delta, F)$ as the variation in $W(F)$ induced by a small transfer δ from the person at rank $p + \gamma$ to the one at rank p, leaving their position unchanged.

Definition 3 *A social welfare function $W(\cdot)$ satisfies the* ***Positional Principle of Transfer Sensitivity*** *if $\Delta_{p,\gamma} W(\delta, F) \geq \Delta_{p',\gamma} W(\delta, F)$ for all $p' > p$.*

Fishburn and Willig (1984) have suggested a generalized transfer principle for the utilitarian approach. Analogously, Aaberge (2004) has defined a positional version of those generalized transfer principles based on $\Delta^s_{p,\Gamma} W(\delta, F)$, which is recursively deduced. Formally:

$$\Delta^2_{p,\Gamma^2} W(\delta, F) = \Delta_{p,\gamma_1} W(\delta, F) - \Delta_{p+\gamma_2,\gamma_1} W(\delta, F), \tag{1}$$

where $\Gamma^2 = (\gamma_1, \gamma_2)$,

$$\vdots$$

$$\Delta^s_{p,\Gamma^s} W(\delta, F) = \Delta^{s-1}_{p,\Gamma^{s-1}} W(\delta, F) - \Delta^{s-1}_{p+\gamma_s,\Gamma^{s-1}} W(\delta, F), \tag{2}$$

where $\Gamma^s = (\gamma_1, \gamma_2, \ldots, \gamma_s)$.

Definition 4 *A social welfare function* $W(\cdot)$ *satisfies the* ***Positional Generalized Transfer Principle*** *of order* s *if* $\Delta^s_{p,\Gamma^s} W(\delta, F) \geq \Delta^s_{p',\Gamma^s} W(\delta, F)$ *for all* $p' > p$.

The positional transfer principle of order s requires the weight function being continuous and s-time differentiable almost everywhere over $[0, 1]$ with:

$$(-1)^i v^{(i)}(p) \geq 0 \ \forall i = 0, 1, \ldots, s-1, \tag{A2}$$

where $v^{(i)}(\cdot)$ is the ith derivative of the function $v(\cdot)$, $v^{(0)}(\cdot)$ being the function itself. The class of social welfare measures respecting assumptions A1, A2 and the continuity assumption is denoted by Ω^s. Assumption A2 implies that $W(\cdot) \in \Omega^1$ satisfies the Pen Parade Principle, $W(\cdot) \in \Omega^2 \subset \Omega^1$ also satisfies the Pigou-Dalton Principle of Transfer, $W(\cdot) \in \Omega^3 \subset \Omega^2 \subset \Omega^1$ also satisfies the Positional Principle of Transfer Sensitivity and $W(\cdot) \in \Omega^s \subset \cdots \subset \Omega^1$, for $s \in \{4, 5, ...\}$, also satisfies the Positional Generalized Transfer Principle of order s.

3 Analyzing the Impact of a Marginal Tax Reform

Suppose the government is considering an increase in the social welfare by marginally decreasing the indirect tax (or marginally increasing the subsidy) on good i and marginally increasing the indirect tax on good j in order to keep public budget constant. This marginal tax reform entails a variation in equivalent income $F^{-1}(p)$ for an individual at rank p:

$$dF^{-1}(p) = \frac{\partial F^{-1}(p)}{\partial t_i} dt_i + \frac{\partial F^{-1}(p)}{\partial t_j} dt_j. \tag{3}$$

As shown by Besley and Kanbur (1988), if the vector of reference prices used to compute equivalent incomes is the vector of prices before the reform, the change in the equivalent income induced by a marginal change in the tax rate of good i is:

$$\frac{\partial F^{-1}(p)}{\partial t_i} = -x_i(p), \tag{4}$$

where $x_i(p)$ is the Marshallian demand for good i for the individual at rank p in the income distribution.

Assume that average tax revenue is $R = \sum_{k=1}^{K} t_k X_k$ where X_k is the average consumption of the kth good $X_k = \int_0^1 x_k(p)\,dp$. As shown in Yitzhaki and Slemrod (1991), revenue neutrality $dR = 0$, and constant producer prices imply:

$$dt_j = -\alpha\left(\frac{X_i}{X_j}\right) dt_i \text{ where } \alpha = \frac{1 + \frac{1}{X_i}\sum_{k=1}^{K} t_k \frac{\partial X_k}{\partial t_i}}{1 + \frac{1}{X_j}\sum_{k=1}^{K} t_k \frac{\partial X_k}{\partial t_j}}. \tag{5}$$

Wildasin (1984) interprets α as the differential efficiency cost of raising one dollar of public funds by taxing the jth commodity and using the proceeds to subsidize the ith commodity. Substituting (5) and (4) in (3) yields:

$$dF^{-1}(p) = -\left[\frac{x_i(p)}{X_i} - \alpha\frac{x_j(p)}{X_j}\right] X_i dt_i. \tag{6}$$

Let us now introduce the concept of the s-concentration curve. We start with $s = 1$ and define $C_k^1(p) = x_k(p)/X_k$, which is the consumption of good k for an individual at rank p divided by the average consumption of the good. Next, we define $C_k^s(p) = \int_0^p C_k^{s-1}(u)\,du$ for all integers $s \in \{2, 3, 4, \ldots\}$. For $s = 2$, the curve is the traditional concentration curve which represents the share of total consumption of good k consumed by the individuals whose rank in the income distribution is less than p. Using our notation, equation (6) can be rewritten as:

$$dF^{-1}(p) = -\left[C_i^1(p) - \alpha C_j^1(p)\right] X_i dt_i. \tag{7}$$

The total change in social welfare induced by the reform is then obtained by integrating (7):

$$dW(F) = -X_i dt_i \int_0^1 \left[C_i^1(p) - \alpha C_j^1(p)\right] v(p)\, dp. \tag{8}$$

We can now prove a positional dominance result within our framework for marginal tax reforms.

Proposition 5 *A necessary condition for $dW(F) \geq 0$ for all $W(\cdot) \in \Omega^s$, $s \in \{1, 2, 3, \ldots\}$ is:*

$$\begin{aligned} C_i^s(p) - \alpha C_j^s(p) &\geq 0,\ \forall p \in [0,1], \qquad (9) \\ \textit{and if } s &\geq 4, \textit{ an additional necessary condition is required:} \\ C_i^u(1) - \alpha C_j^u(1) &\geq 0,\ \forall u \in \{3, \ldots, s-1\}. \end{aligned}$$

Proof. If we refer to equation (8), we easily realize that the condition for $s = 1$ is proved by simply noting that $v(p)$ is positive and that dt_i is negative. To prove for $s > 1$, we first need to integrate by parts $\int_0^1 C_k^1(p)\, v(p)\, dp$:

$$\int_0^1 C_k^1(p)\, v(p)\, dp = C_k^2(p)\, v(p)\Big|_0^1 - \int_0^1 C_k^2(p)\, v'(p)\, dp. \tag{10}$$

Now, assume that $s > 2$, and that for some $t \in \{3, 4, \ldots, s-1\}$, we have:

$$\int_0^1 C_k^1(p)\, v(p)\, dp = \sum_{u=2}^{t-1} (-1)^u C_k^u(p)\, v^{(u-2)}(p)\Big|_0^1 + (-1)^{t-2} \int_0^1 C_k^{t-1}(p)\, v^{(t-2)}(p)\, dp. \tag{11}$$

Integrating by parts equation (11), we get:

$$\int_0^1 C_k^1(p)\, v(p)\, dp = \sum_{u=2}^{t} (-1)^u C_k^u(p)\, v^{(u-2)}(p)\Big|_0^1 + (-1)^{t-1} \int_0^1 C_k^{t}(p)\, v^{(t-1)}(p)\, dp. \tag{12}$$

Equation (10) respects the relation depicted in equation (11). We have shown that if equation (11) is true then equation (12) is also true. This implies that equation (12) is true for all integers $t \in \{2, 3, \ldots\}$. We thus have:

$$\int_0^1 C_k^1(p)\, v(p)\, dp = \sum_{u=2}^{s} (-1)^u C_k^u(p)\, v^{(u-2)}(p)\Big|_0^1 + (-1)^{s-1} \int_0^1 C_k^s(p)\, v^{(s-1)}(p)\, dp. \tag{13}$$

From equations (8) and (13), we obtain for $s \in \{2, 3, \ldots\}$:

$$\begin{aligned} dW(F) &= (-1)^s X_i dt_i \Bigg\{ \sum_{u=2}^{s} (-1)^u \left[C_i^u(p) - \alpha C_j^u(p)\right] v^{(u-2)}(p)\Big|_0^1 \qquad (14) \\ &\quad + (-1)^{s-1} \int_0^1 \left[C_i^s(p) - \alpha C_j^s(p)\right] v^{(s-1)}(p)\, dp \Bigg\}. \end{aligned}$$

$C_i^u(0) = C_j^u(0) = 0$ for all $u \in \{2, 3, \ldots, s\}$ and $C_i^2(1) = C_j^2(1) = 1$. Thus, we have for $s = 2$:

$$dW(F) = X_i dt_i \int_0^1 \left[C_i^2(p) - \alpha C_j^2(p)\right] v'(p)\, dp. \tag{15}$$

Note that $v'(p)$ is negative and that dt_i is negative. If $C_i^2(p) - \alpha C_j^2(p) \geq 0$ for all $p \in [0, 1]$, then $dW(F) \geq 0$. This proves the proposition for $s = 2$, which is the case studied by Yitzhaki and Slemrod (1991). For $s \in \{3, 4, 5, \ldots\}$ equation (14) becomes:

$$\begin{aligned} dW(F) &= (-1)^s X_i dt_i \Bigg\{ \sum_{u=2}^{s} (-1)^u \left[C_i^u(1) - \alpha C_j^u(1)\right] v^{(u-2)}(1) \qquad (16) \\ &\quad + (-1)^{s-1} \int_0^1 \left[C_i^s(p) - \alpha C_j^s(p)\right] v^{(s-1)}(p)\, dp \Bigg\}. \end{aligned}$$

Note that $v^{(2)}(p)$ is positive and that dt_i is negative. If $C_i^3(p) - \alpha C_j^3(p) \geq 0$ for all $p \in [0, 1]$, then $dW(F) \geq 0$. The same reasoning applies for $s \in \{4, 5, 6, \ldots\}$, except that we must also check if $C_i^u(1) - \alpha C_j^u(1) \geq 0$ for all $u \in \{3, \ldots, s-1\}$ to insure that $dW(F) \geq 0$. ■

Proposition 5 stipulates, for $\alpha = 1$, that the marginal tax reform will increase social welfare at a given order of positional dominance if the s-concentration curve of good i dominates (lies above) the s-concentration curve of good j for every rank in the income distribution.

While the traditional concentration curves can only be used to test for dominance at order $s = 2$, this framework enables the decision-maker to test for any order of positional dominance. For $s \geq 4$ an additional necessary condition is required at $p = 1$. If $\alpha \neq 1$, we can still compare the s-concentration curve for good i with the s-concentration curve of good j provided the latter is multiplied by α.

4 Empirical Illustration

We illustrate our technique on Canadian data using the Survey of Household Spending (2002), which involves 14,655 households. The application is concerned with eight types of expenditures: housing, electricity, transport, gasoline, drugs, scholarship, tobacco, and alcoholic drinks.

The choice of α is important. As Yitzhaki and Slemrod (1991) have pointed out, $\alpha < 1$ $(\alpha > 1)$ indicates, as a consequence of the tax reform, whether a diminution (a rise) of the excess burden occurs. The case for which $\alpha = 1$ is appealing since it yields neither efficiency gain nor efficiency loss for the government but can be welfare-improving. In this section we assume that $\alpha = 1$. Figure 1 exposes the 2-concentration curves and exhibits single crossing curves between housing and electricity and between gasoline and medicaments. In these cases, welfare dominance is not possible. The 2-concentration curves enable us, for instance, to capture any tax reform on education and health, showing that drug expenditures dominate scholarship expenses (see Figure 1).

[Figure 1 about here]

Yitzhaki and Slemrod (1991) introduced the "difference in concentration curve" approach (DCC approach). Our methodology generalizes DCC into "difference in concentration curve of order s" (DCCS). For example, an increase of scholarship taxes entirely financed by a decrease of drug taxes would be welfare improving (see Figure 2).

[Figure 2 about here]

Another atypical tax reform would be welfare-improving by financing a tax reduction on alcoholic drinks by an increasing tax on tobacco.

[Figure 3 about here]

The 3-concentration curves (see Figure 4) can exhibit some dominance since they implicitly involve a progressive positional transfer sensitivity at the lower part of the distribution (from a higher-income person to a lower-income one) coupled with a regressive positional transfer sensitivity at the upper part of the distribution (from a lower-income person to a higher-income one).

[Figure 4 about here]

5 Conclusion

Our methodology – the s-concentration curve – allows decision-makers to test whether the increase in social welfare induced by a marginal tax reform for two commodities is robust over a large set of rank dependant social welfare functions.

Instead of looking for non-intersecting concentration curves at the order 2, the positional dominance approach enables the decision-maker to choose the order of positional dominance of interest. This method can also be used to adapt Duclos, Makdissi and Wodon's (2005) tests for the targeting and allocative efficiency of public transfer programs in a positionalist framework.

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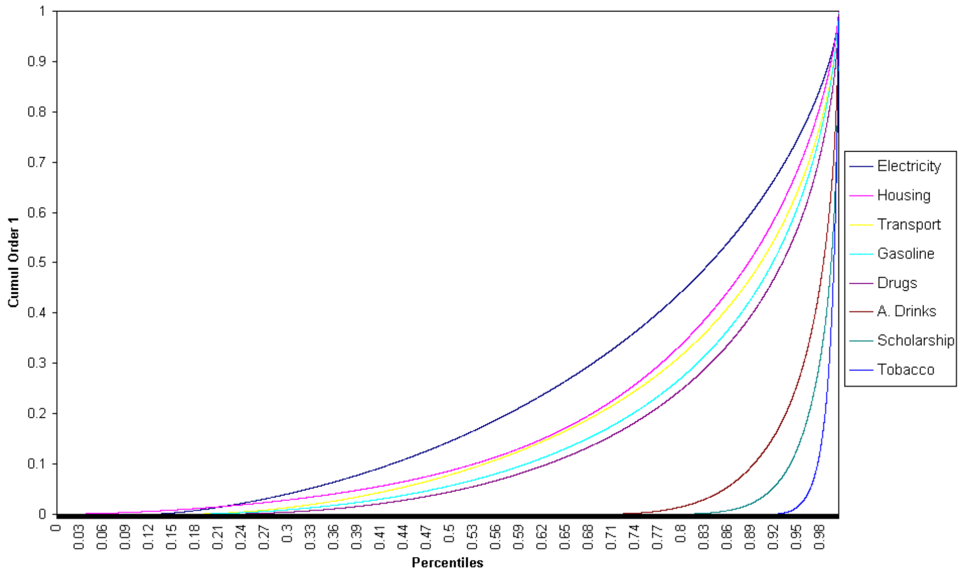
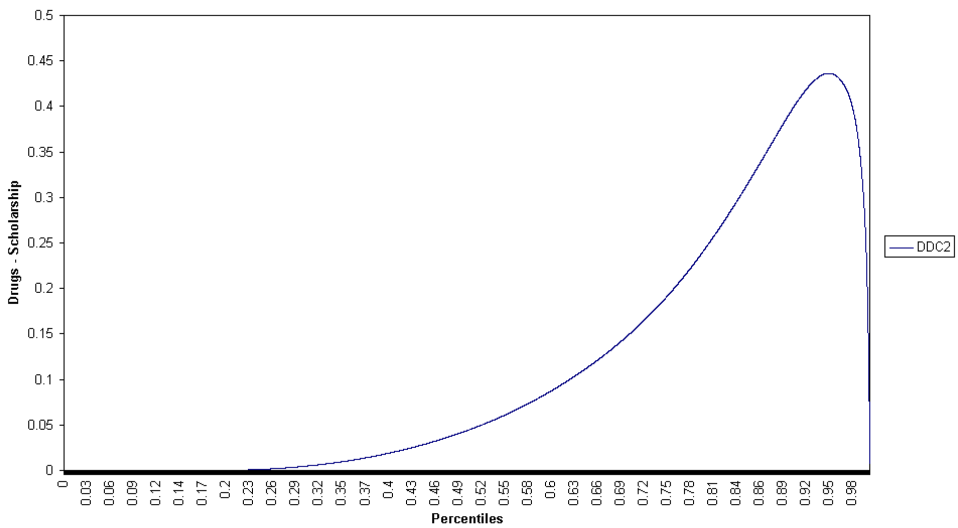
Figure 1**. S-Concentration Curves of Order 2



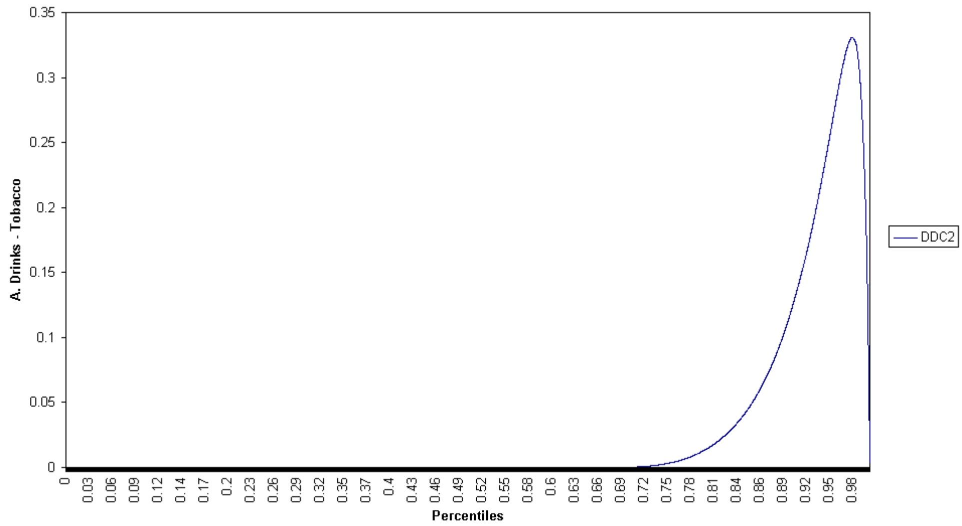
*Source: Canadian Survey of Household Spending (2002)
**The curves are ranked by dominance: Electricity (the highest), Housing, Transport, and so on.

Figure 2*. Difference in Concentration Curves of Order 2: Drugs - Scholarship



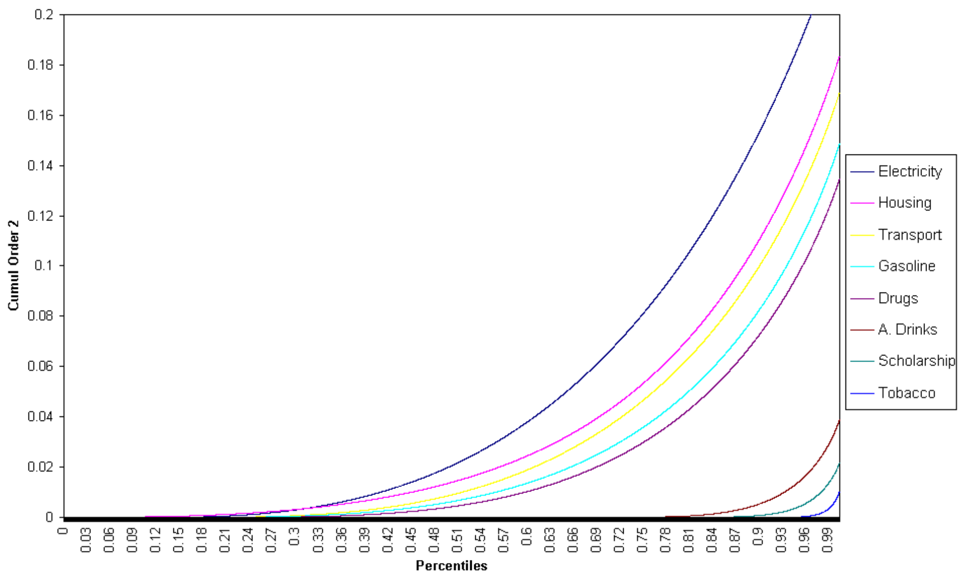
*Source: Canadian Survey of Household Spending (2002)

Figure 3*. Difference in Concentration Curves of Order 2: Alcoholic Drinks - Tobacco



*Source: Canadian Survey of Household Spending (2002)

Figure 4**. S-Concentration Curves of Order 3



*Source: Canadian Survey of Household Spending (2002)
**The curves are ranked by dominance: Electricity (the highest), Housing, Transport, and so on.