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Inequalities in Poverty : Evidence from Argentina

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INEQUALITIES IN POVERTY: EVIDENCE FROM ARGENTINA

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Abstract: In 1990, Cerioli and Zani introduced an operational multivariate method to analyse and measure poverty, aiming at incorporating several dimensions of poverty. As Dagum and Costa [2004] showed, this study applies the fuzzy set theoretic approach and thus making quantitatively operational the French Social Exclusion Theory and Sen's analysis of functioning and capability.

The literature offers many ways to deal with inequalities in poverty. The most common approach is those of Amartya Sen [1976] with the Gini index of poverty gap ratio, that is, a fundamental component of Sen's poverty index. The problem is that this Gini index does not offer any information on the determinants of the inequalities. For this purpose, Mussard [2004] introduced the bidimensional decomposition of the Gini ratio, where both sub-groups and income sources are jointly analysed in a decomposition context.

These two methodologies are combined in order to evaluate the differences in multidimensional poverty within and between sub-groups of population, and to determine the dimensions that tend to increase inequality in poverty.

The goal of this article is double. Firstly, we present the combined methodology to study the inequality in multidimensional poverty. Secondly, we apply this technique to analyse the differences in poverty within and between the six principal regions of Argentina.

Key words: Fuzzy Set Theory, Gini, Inequality, Multidimensional Poverty, Decomposition.

JEL Classification: I32, D63, D31

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I. Introduction

In the few last decades, the measurement of poverty and inequality has become an important field of public economics, trying to find out the principal determinants that cause poverty and inequality intensities. The domain of inequality measurement has been pioneered by the Atkinson-Kolm-Sen approaches and the poverty one has been initiated by Booth (1892) and Rowntree (1901), followed by Sen (1976), and by Foster, Greer and Thorbecke (1984) with the creation of the well-known FGT measure.

Some researches point out a link between poverty and inequality. For example, on the one hand, Son (2003) provides a new poverty decomposition in which it is possible to explain poverty changes by inequality variations. On the other hand, the famous poverty Sen index can be explained by several components. More specifically, as noted by Xu and Osberg (2001), the Sen index is easily understandable precisely because of its decomposability into three measures: incidence (the poverty rate), depth (poverty gap), and inequality (1 plus the Gini index of poverty gaps). Therefore, as the Sen index depends on the Gini coefficient, it relies on inequality. But, “inequality of what”? The Sen ratio captures the inequalities of poverty gaps, i.e., the inequalities of depth between each poor individual. So, the aim of this study is to explain inequalities of poverty.

This study focuses on the Gini index of poverty gaps using the fuzzy set theory. On the basis of Cerioli and Zani’s approach (1990), Dagum and Costa (2004) introduced a new fuzzy poverty index and its decomposition according to different attributes, that is, dimensions of poverty. Combining this attribute decomposition of poverty with the decomposition properties of the Gini ratio, we provide a multidimensional decomposition of the Gini index, which yields the inequalities of multidimensional poverty following sub-group and attribute decompositions.

The remainder of the paper is organized as follows. *Section 2* addresses a review of the methodology of decomposition by attribute and by sub-group introduced by Dagum and Costa (2004), and Mussard and Pi Alperin (2005), respectively. *Section 3* develops the Gini Index of Poverty (GIP). In *Section 4* this combined methodology is applied to the measurement of the inequalities in multidimensional poverty using the “*Encuesta Permanente de Hogares*” that is a Permanent Survey of Households residence made by the INDEC (Argentina Institute of Statistics and Census). Finally, *Section* 5 is devoted to the conclusions.

II. DAGUM'S APPROACH TO FUZZY SET THEORY AND POVERTY

Recently, Dagum and Costa (2004) introduced, on the basis of the work of Cerioli and Zani (1990), a new approach to study multidimensional poverty. This method permits to obtain: the poverty ratio of each household; the poverty ratio of a population of households; and, the poverty ratio of the population by retained dimension (attribute).

For the application of this method we must define: (*i*) the economic units, the household set in an economic space: $A = \{a_1,..., a_i ,\dots, a_n\}$; and (*ii*) a *m*-dimension vector of socio-economic attributes to study the level of poverty in A: $X = \{X_1,\dots, X_j ,\dots, X_m\}$.

Let B be a fuzzy sub-set of households in A, where $a_i \in B$ stands for the degree of poverty in at least one attribute.

The degree of membership of the *i*-th household ($i = 1,\dots, n$), with respect to the *j*-th attribute ($j = 1,\dots, m$), to the fuzzy sub-set B is defined as:

$$x_{ij} = \mu_B\left(X_j\left(a_i\right)\right),\ 0 \le x_{ij} \le 1. \tag{1}$$

In particular:

- $x_{ij} = 1$, if the *i*-th household does not possess the *j*-th attribute;
- $x_{ij} = 0$, if the *i*-th household possesses the *j*-th attribute;
- $0 < x_{ij} < 1$, if the *i*-th household possesses the *j*-th attribute with an intensity belonging to the open interval (0,1).

The degree of membership of the *i*-th household to the fuzzy sub-set B is defined as a weighted average of x_{ij}:

$$\mu_B(a_i) = \sum_{j=1}^{m} x_{ij} w_j \Big/ \sum_{j=1}^{m} w_j \ . \qquad (2)$$

The equation $\mu_B(a_i)$ measures the poverty index of the i-th household, where w_j is the weight attached to the j-th attribute. Following this definition, one obtains:

$$0 \le \mu_B(a_i) \le 1. \qquad (3)$$

In particular:

- $\mu_B(a_i) = 0$, if a_i is completely non-poor in the m attributes;
- $\mu_B(a_i) = 1$, if a_i is totally poor in the m attributes;
- $0 < \mu_B(a_i) < 1$, if a_i is partially or totally deprived in some attributes but not fully deprived in all of them.

As $\mu_B(a_i)$ measures the degree of poverty of the i-th household as a weighted function of the m attributes, it also measures the relative deprivation, the degree of social exclusion, and the insufficient capability of the i-th household to reach the living standard of the society to which it belongs.

The weight w_j attached to the j-th attribute stands for the intensity of deprivation of X_j. It is an inverse function of the degree of deprivation of this attribute by the population of households. The following expression represents this above property. This weight was proposed by Cerioli and Zani (1990):

$$w_j = \log\left[\sum_{i=1}^{n} g(a_i) \Big/ \sum_{i=1}^{n} x_{ij}\, g(a_i)\right], \qquad (4)$$

where $g(a_i) \Big/ \sum_{i=1}^{n} g(a_i)$ is the relative frequency represented by the sample observation a_i in the total population. The denominator of the logarithm in (4) is always positive. Indeed, if $x_{ij} = 0$, $\forall i$, this would be an irrelevant attribute because there is not any deprivation in X_j.

The fuzzy poverty index of the A set is a weighted average of $\mu_B(a_i)$:

$$\mu_B = \sum_{i=1}^{n} \mu_B(a_i) g(a_i) \Big/ \sum_{i=1}^{n} g(a_i) . \qquad (5)$$

Also, the fuzzy set theory allows one to measure an unidimensional poverty index for each one of the *m* attributes:

$$\mu_B(X_j) = \sum_{i=1}^{n} x_{ij} g(a_i) \Big/ \sum_{i=1}^{n} g(a_i) . \qquad (6)$$

$\mu_B(X_j)$ measures the degree of deprivation of the *j*-th attribute for the entire population of *n* households.

We can also write the fuzzy poverty index as a weighted function of the unidimensional poverty indexes:

$$\mu_B = \sum_{j=1}^{m} \mu_B(X_j) w_j \Big/ \sum_{j=1}^{m} w_j \ . \qquad (7)$$

The analysis of the results obtained in (6), for $j = 1,\ldots, m$, enables the policy makers to identify the main causes of poverty and the most urgent areas of structural intervention to raise the poor households to the state of non-poverty.

II.1. HOUSEHOLD POVERTY INDEXES DECOMPOSITION

Furthermore, Dagum and Costa’s (2004) technique enables one to measure the state of poverty inherently to each household by including relative deprivation and social exclusion.

From (2), it is possible to determine the dimensions that tend to increase the level of poverty of each household. Indeed, we can decompose the household poverty index such as:

$$\mu_B(a_i) = \sum_{j=1}^{m} y_{ij} , \qquad (8)$$

where y_{ij} is the contribution of the j-th attribute to the overall amount of the household poverty index $\mu_B(a_i)$:

$$y_{ij} = x_{ij} w_j \Big/ \sum_{j=1}^{m} w_j \ . \tag{9}$$

Then, it is possible to gauge precisely the percentage contribution of each attribute (education, health, etc.) to the index $\mu_B(a_i)$ of each household.

II.2. THE SUB-GROUP DECOMPOSITION

Another way to evaluate the structure of poverty is to provide a decomposition by sub-population [Mussard and Pi Alperin (2005)]. Let us divide the total economic surface into K groups, S_k, of size n_k (k = 1,…, K). The intensity of poverty of the i-th household of S_k is given by:

$$\mu_B(a_i^k) = \sum_{j=1}^{m} x_{ij}^k \ w_j \Big/ \sum_{j=1}^{m} w_j \ , \tag{10}$$

where x_{ij}^k is the degree of membership related to the fuzzy sub-set B of the i-th household (i = 1,…, n) of S_k with respect to the j-th attribute (j = 1,…, m). Then, the fuzzy poverty index associated with group S_k is[1]:

$$\mu_B^k = \sum_{i=1}^{n_k} \mu_B\left(a_i^k\right) g\left(a_i^k\right) \Big/ \sum_{i=1}^{n_k} g\left(a_i^k\right). \tag{11}$$

Following (9), the overall fuzzy poverty index can be computed as a weighted average of the poverty within each group:

$$\mu_B = \sum_{k=1}^{K} \sum_{i=1}^{n_k} \mu_B\left(a_i^k\right) g\left(a_i^k\right) \Big/ \sum_{i=1}^{n} g\left(a_i\right). \tag{12}$$

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[1] $g\left(a_i^k\right)$ is the weight associated with each household observation a_i^k .

Thus, it is possible to measure the contribution of the *k*-th group to the global index of poverty:

$$C_{\mu_B}^{k} = \sum_{i=1}^{n_k} \mu_B\left(a_i^k\right) g\left(a_i^k\right) \Big/ \sum_{i=1}^{n} g\left(a_i\right). \tag{13}$$

This yields to policy makers other thorough possibilities to reduce the overall poverty in focusing on the poorest groups (region, educational group, etc.).

III. A MULTIDIMENSIONAL GINI INDEX OF POVERTY

The literature offers many ways to deal with inequalities in poverty. The most common approach is those of Amartya Sen (1976) with the Gini index of poverty gap ratio, i.e., a fundamental component of Sen's poverty index. The poverty gap ratio of the *i*-th household is defined as:

$$\beta_i = \begin{cases} \dfrac{z - v_i}{z}, \ \forall z > v_i \\ 0 \ \text{ otherwise} \end{cases} \tag{14}$$

where z is the poverty line and v_i is a variable such as income, consumption, etc.

Then, β_i reflects the difficulty of the *i*-th household to reach a standard level of life. Applying the well-known Gini index on the vector $\beta = \left(\beta_1, ..., \beta_i, ..., \beta_n\right)$ it is possible to implement the Gini index of poverty gaps ratio $G(\beta)$. It allows one to know if poverty gap ratios are equal [$G(\beta)$ tends toward 0] or are unequal in a given population [$G(\beta)$ tends toward 1]. The problem is that $G(\beta)$ does not offer suitable information on the determinants of the inequalities. Are the inequalities in poverty generated by education or health? A simple way to deal with this type of questions is to gauge the poverty gap ratio related to each dimension X_j:

$$\beta_{ij} = \begin{cases} \frac{z_j - v_{ij}}{z_j}, \ \forall z_j > v_{ij} \\ 0 \ \text{otherwise} \end{cases} \quad (15)$$

where β_{ij} is the poverty gap ratio of the *i*-th household associated with the *j*-th attribute X_j.

Afterwards, it is possible to compute the Gini index of these poverty gaps ratios $G(\beta_j)$ where $\beta_j = (\beta_{1j}, ..., \beta_{ij}, ..., \beta_{n_j j})$. But another problem arises. Indeed, $G(\beta_j)$ produces a wide range of poverty inequality indexes, but there is no link between them. Thus, it is not possible to evaluate the contribution of one particular dimension X_j to the global index of inequality.

A solution appears with (8) where the *m* dimensions of poverty are linearly aggregated in the so-called contributions y_{ij}. Recently, Mussard (2004) introduced the bidimensional decomposition of the Gini ratio, where both sub-groups and income sources are jointly analysed in a decomposition context. If *m* factors are linearly aggregated (such as income sources, consumption expenditures, etc.) the Gini ratio makes possible the computation of the couples "factor/within-group" and "factor/between-group" that together account for the whole index of inequality.

Let us explain this decomposition in the fuzzy poverty background[2]. The Gini index that gauges the inequalities in multidimensional poverty can be expressed as:

$$G = \frac{\sum_{i=1}^{n}\sum_{r=1}^{n}\left|\mu_B(a_i) - \mu_B(a_r)\right|}{2\overline{\mu}_B n^2}, \quad (16)$$

where $\mu_B(a_r)$ stands for the poverty index of the *r*-th household, and $\overline{\mu}_B$ stands for the arithmetic mean of the households's poverty index $\mu_B(a_i)$. Then, in order to measure the differences in poverty within each sub-group and the differences in poverty between each and

[2] In Mussard (2004) the dimensions of groups and sources of income are only included in the decomposition. This entails a bidimensional decomposition. In the present paper, we obtain a (*m*+1)-dimensional decomposition since we have *m* dimensions related to the attributes plus one dimension related to the groups.

every pair of sub-groups, we partition the population in K sub-groups (k, h = 1,…, K). The sub-group Gini decomposition in two components is [Dagum (1997)]:

$$G = \frac{\sum_{k=1}^{K}\sum_{i=1}^{n}\sum_{r=1}^{n}\left|\mu_B\left(a_i^k\right)-\mu_B\left(a_r^k\right)\right|}{2\overline{\mu}_B n^2} + \frac{2\sum_{k=2}^{K}\sum_{h=1}^{k-1}\sum_{i=1}^{n}\sum_{r=1}^{n}\left|\mu_B\left(a_i^k\right)-\mu_B\left(a_r^h\right)\right|}{2\overline{\mu}_B n^2} = G_w + G_{gb} \ , \tag{17}$$

where $\mu_B\left(a_r^h\right)$ is the poverty index of the i-th household that belongs to group S_k. The first component of (17) reflects the within-group inequalities of poverty [G_w] and the second one [G_{gb}] the gross between-group inequalities of poverty, i.e., the inequalities between each and every pair of the K sub-populations (that takes into account variance and asymmetric effects contrary to the standard between-group indices that only yield the differences in mean between sub-groups). Hence, given that each household poverty index is linearly desegregated [see (9)], it is possible to compute the following equation:

$$G = \frac{\sum_{k=1}^{K}\sum_{i=1}^{n_k}\sum_{r=1}^{n_k}\left|\sum_{j=1}^{m} y_{ij}^k - \sum_{j=1}^{m} y_{rj}^k\right|}{2\overline{\mu}_B n^2} + \frac{2\sum_{k=2}^{K}\sum_{h=1}^{k-1}\sum_{i=1}^{n_k}\sum_{r=1}^{n_h}\left|\sum_{j=1}^{m} y_{ij}^k - \sum_{j=1}^{m} y_{rj}^h\right|}{2\overline{\mu}_B n^2} \tag{18}$$

where y_{ij}^k is the contribution of the j-th dimension to $\mu_B\left(a_i^k\right)$. Subsequently, if we delete the absolute value, we find a multidimensional decomposition of the Gini index of poverty:

$$G = \sum_{j=1}^{m}\frac{\sum_{k=1}^{K}\sum_{i=1}^{n_k}\sum_{r=1}^{n_k}\left(y_{ij}^k + y_{rj}^k - 2y_{k,ir}^{*j}\right)}{2\overline{\mu}_B n^2} + \sum_{j=1}^{m}\frac{2\sum_{k=2}^{K}\sum_{h=1}^{k-1}\sum_{i=1}^{n_k}\sum_{r=1}^{n_h}\left(y_{ij}^k + y_{rj}^h - 2y_{kh,ir}^{*j}\right)}{2\overline{\mu}_B n^2} , \tag{19}$$

where $y_{k,ir}^{*j}$ is an operator that takes the j-th contribution of the minimum between $\mu_B\left(a_i^k\right)$ and $\mu_B\left(a_r^h\right)$; and $y_{kh,ir}^{*j}$ is an operator that takes the j-th contribution of the minimum between the poverty index $\mu_B\left(a_i^k\right)$ and $\mu_B\left(a_r^h\right)$.

Consequently, it is possible to implement the couples "within-group k / attribute X_j" and "between-group k and h / attribute X_j". In other words, the differences in poverty within groups and between groups are determined by the m explanatory dimensions X_j.

IV. A CASE OF STUDY: ARGENTINA

Among the developing countries, Argentina is relatively rich in human and natural resources. In spite of this wealth, the country suffers a significant growth of poverty. That is why we are interested in analyzing, on the one hand, the state of poverty in Argentina (and more precisely in each one of the six principals regions of the country) in May 1998, using the multidimensional approach of fuzzy sets. On the other hand, we study the inequalities in poverty within and between regions in order to identify the main regions and dimensions that explain these inequalities in poverty.

We study 28,511 households for each one of Argentina's provinces. The data base used in this study comes from the "Encuesta Permanente de Hogares (EPH)" Permanent Survey of Households residences. This multidimensional survey has been performed every year since 1974 by the INDEC (Argentina Institute of Statistics and Census). This survey includes information about income, labor market characteristics, demographic characteristics, housing, education and training.

IV.1. THE SOCIO-ECONOMIC ATTRIBUTES SELECTED TO STUDY THE STATE OF POVERTY

The two principal criteria that help the selection of the socio-economic attributes to analyze and to measure the state of poverty in Argentina are: the multidimensional approach of poverty and the information provided by the EPH. This choice is very important because each one of the attributes represents a degree of deprivation and social exclusion of the studied households. The selected attributes are:

- occupancy title and location of the household residence (X_1);
- materials of construction of the household (X_2);
- household size (X_3);
- toilet characteristics (X_4);
- flowing characteristics (X_5);

- household equivalent income[3] (X_6);
- higher level of education completed by the reference person (X_7);
- stability of occupation of the reference person (X_8);
- professional occupation of the reference person (X_9);
- social contributions (X_{10});
- ratio: number of the household members with income and the household size (X_{11}).

IV.2. THE SUB-GROUP DECOMPOSITION

The multidimensional poverty index (MPI) for Argentina in 1998 is μ_B = 0.1638, that means that 16.38% of Argentina's households are structurally poor.

Let us analyze how this percentage can be desegregated using the sub-group decomposition of the MPI. For this, we have chosen to study the six principals regions of Argentina[4]. *Table 1* underlines two kinds of information: (*i*) the multidimensional poverty indexes by region; (*ii*) and their absolute and relative contributions to the MPI.

The results show that the North-east region is the poorest one, followed by the Cuyo, North-west and Pampeana's regions, with 17.92%, 17.90%, 17.34% and 16.97% of poorest households, respectively. But 78.80% of the intensity of poverty of the country's poverty is explained by the GBA and Pampeana regions. This result is plausible since the relative contribution involves the number of persons in each group and these regions have the biggest density of population.

Table 1: Multidimensional poverty index by region, and their absolute and relative contribution to MPI

Decompositions		μ_B^k	Absolute contribution	Relative contribution
Regions	**Cuyo**	0.1790	0.0105	6.4324
	Great Buenos Aires	0.1583	0.0889	**54.2543**
	North-east	**0.1792**	0.0076	4.6173
	North-west	0.1734	0.0132	8.0446
	Pampeana	0.1697	0.0402	**24.5163**
	Patagonia	0.1414	0.0035	2.1350

[3] Divided by the corresponding value of the equivalent scale. See Dagum and Costa (2004) for more details of this method. See Table A.II.1, in Appendix II, for the values of the equivalents scales used in this study.
[4] We can notice that it is possible to chose other kinds of decomposition as gender, civil status, etc..

Table 2 describes the unidimensional poverty indexes (UPI) by attribute and by region. These values, reflecting the degree of deprivation of each attribute for the total population of each sub-group, are different. Then, we can distinguish the truly causes of the state of poverty in the sub-groups. For instance, the level of equivalent income is not necessary the most explicative attribute to understand the problem of poverty in Argentina, because the social contributions, the level of education completed by the reference person, and the professional occupation of the reference person appears, jointly with the income dimension, are the main causes of the state of poverty in each region.

Table 2: UPI by attribute and by region

	Attributes										
	X_1	**X_2**	**X_3**	**X_4**	**X_5**	**X_6**	**X_7**	**X_8**	**X_9**	**X_{10}**	**X_{11}**
Cuyo	0.2221	0.1280	0.0938	0.0279	0.1463	0.4576	0.6227	0.2037	0.4488	0.7191	0.1229
GBA	0.1569	0.0097	0.0673	0.0888	0.1990	0.3828	0.6174	0.2243	0.4775	0.6925	0.1475
N - east	0.2082	0.0285	0.1345	0.1086	0.1268	0.5685	0.6353	0.2075	0.4618	0.6898	0.1441
N - west	0.1861	0.0174	0.1415	0.1015	0.1177	0.4849	0.6254	0.2428	0.4899	0.7355	0.1287
Pamp.	0.1906	0.0126	0.0921	0.0654	0.1821	0.4747	0.6068	0.2332	0.5081	0.7275	0.1695
Patag.	0.1997	0.0581	0.0927	0.0500	0.0597	0.2983	0.6158	0.1894	0.4141	0.5955	0.0996

IV.3. INEQUALITIES IN MULTIDIMENSIONAL POVERTY

In this section we study the inequalities in multidimensional poverty, using the multidimensional Gini index of poverty presented in *Section 3*. Each one of the groups represents one region as follow: Group 1, Cuyo; Group 2, Great Buenos Aires; Group 3, North-east; Group 4, North-west; Group 5 Pampeana; and Group 6, Patagonia.

Table 3: Overall Decomposition of Poverty Inequalities***

Indices	**Attributes**											**Total**
	X_1	**X_2**	**X_3**	**X_4**	**X_5**	**X_6**	**X_7**	**X_8**	**X_9**	**X_{10}**	**X_{11}**	
Within-group Inequalities G_w	0.0077	0.0047	0.0099	0.0107	0.0077	0.0076	0.0027	0.0083	0.0038	0.0024	0.007	**0.0725**
	[2.09]	[1.28]	[2.69]	[2.91]	[2.09]	[2.07]	[0.73]	[2.26]	[1.03]	[0.65]	[1.90]	**[19.71]**
Between-group Inequalities G_{gb}	0.0301	0.0197	0.0388	0.0408	0.0333	0.0327	0.012	0.0344	0.0156	0.009	0.029	**0.2954**
	[8.18]	[5.35]	[10.55]	[11.09]	[9.05]	[8.89]	[3.26]	[9.35]	[4.24]	[2.45]	[7.88]	**[80.29]**
Total	**0.0378**	**0.0244**	**0.0487**	**0.0515**	**0.041**	**0.0403**	**0.0147**	**0.0427**	**0.0194**	**0.0114**	**0.036**	***G*=0.3679**
	[10.27]	**[6.63]**	**[13.24]**	**[14.00]**	**[11.14]**	**[10.96]**	**[3.99]**	**[11.61]**	**[5.27]**	**[3.10]**	**[9.78]**	**[100]**

*** [.] *: % contribution to G*

Table 3 exposes the overall decomposition of poverty inequalities. This decomposition offers two kinds of information. On the one hand, the Gini index of poverty is G = 0.3679. This Gini can be explained by the poverty differences within groups and between groups. Indeed, the between-group inequalities represent 80.29% of the overall inequality, and the within-group inequalities 19.71%. On the other hand, it is also possible to distinguish the main dimensions that have the biggest contribution to the global poverty inequalities. Precisely, toilet characteristics (14%), household size (13.24%), stability of occupation (11.61%) and flowing characteristics (11.14%).

Tables 4 and *5* present more details about the characteristics of inequalities within-groups and between-groups, respectively.

Table 4: Within-group Inequalities of Poverty*

Indices	Attributes											
	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}	X_{11}	Total
Group 1: G_{w1}	0.0007 [0.19]	0.0007 [0.19]	0.0007 [0.19]	0.0005 [0.14]	0.0008 [0.22]	0.0006 [0.16]	0.0002 [0.05]	0.0007 [0.19]	0.0003 [0.08]	0.0002 [0.05]	0.0004 [0.11]	**0.0058** **[1.58]**
Group 2: G_{w2}	0.0005 [0.14]	0.0002 [0.05]	0.0007 [0.19]	0.0009 [0.24]	0.0007 [0.19]	0.0008 [0.22]	0.0002 [0.05]	0.0007 [0.19]	0.0003 [0.08]	0.0002 [0.05]	0.0006 [0.16]	**0.0058** **[1.58]**
Group 3: G_{w3}	0.0042 [1.14]	0.0023 [0.63]	0.0055 [1.49]	**0.0063** **[1.71]**	0.0038 [1.03]	0.0035 [0.95]	0.0014 [0.38]	0.0041 [1.11]	0.0018 [0.49]	0.0012 [0.33]	0.0036 [0.98]	**0.0377** **[10.25]**
Group 4: G_{w4}	0.0005 [0.14]	0.0002 [0.05]	0.0008 [0.22]	0.0007 [0.19]	0.0005 [0.14]	0.0006 [0.16]	0.0003 [0.08]	0.0006 [0.16]	0.0003 [0.08]	0.0002 [0.05]	0.0006 [0.16]	**0.0053** **[1.44]**
Group 5: G_{w5}	0.0006 [0.16]	0.0002 [0.05]	0.0007 [0.19]	0.0008 [0.22]	0.0007 [0.19]	0.0007 [0.19]	0.0002 [0.05]	0.0006 [0.16]	0.0003 [0.08]	0.0001 [0.03]	0.0006 [0.16]	**0.0055** **[1.49]**
Group 6: G_{w6}	0.0012 [0.33]	0.0011 [0.30]	0.0015 [0.41]	0.0015 [0.41]	0.0012 [0.33]	0.0014 [0.38]	0.0004 [0.11]	0.0016 [0.43]	0.0008 [0.22]	0.0005 [0.14]	0.0012 [0.33]	**0.0124** **[3.37]**
Total: G_w	**0.0077** **[2.09]**	**0.0047** **[1.28]**	**0.0099** **[2.69]**	**0.0107** **[2.91]**	**0.0077** **[2.09]**	**0.0076** **[2.07]**	**0.0027** **[0.73]**	**0.0083** **[2.26]**	**0.0038** **[1.03]**	**0.0024** **[0.65]**	**0.007** **[1.90]**	**G_w=0.0725** **[19.71]**

*[.] *: % contribution to G*

Table 4 shows that inequalities in the North-east region, the poorest one, explains 10.25% of the global inequalities in poverty; and 52% of the total within-group inequalities[5], and the main dimensions that generate these inequalities are the toilet characteristics (which explains 1.71% of total inequalities) and household size (1.49%) in this region. The second major contribution is done by the richest region, Patagonia, which explains 3.36% of the global inequalities. This is a very important result because even if Patagonia is the region with less

[5] With this method we can easily calculate the relative contribution of each one of the dimensions and each region to the global within-group and between-group inequalities.

important poor households, we can notice a significant difference in intensity of poverty between them.

Another important result is that even that the region Great Buenos Aires has the biggest contribution to understand the state of poverty in Argentina (it explains 54.25% of the total poverty) the intensity of poverty of the poor households that belong to this region is not so different.

Table 5: Between-group Inequalities of Poverty**

Indices	**Attributes**											**Total**
	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}	X_{11}	
G_{gb12}	0.0012 [0.33]	0.001 [0.27]	0.0014 [0.38]	0.0013 [0.35]	0.0016 [0.43]	0.0014 [0.38]	0.0005 [0.14]	0.0014 [0.38]	0.0006 [0.16]	0.0003 [0.08]	0.0011 [0.30]	**0.0118** **[3.21]**
G_{gb13}	0.0033 [0.90]	0.0029 [0.79]	0.0039 [1.06]	0.0035 [0.95]	0.0037 [1.01]	0.0029 [0.79]	0.0012 [0.33]	0.0033 [0.90]	0.0016 [0.43]	0.0008 [0.22]	0.0026 [0.71]	**0.0297** **[8.07]**
G_{gb14}	0.0012 [0.33]	0.0011 [0.30]	0.0015 [0.41]	0.0012 [0.33]	0.0014 [0.38]	0.0012 [0.33]	0.0005 [0.14]	0.0013 [0.35]	0.0006 [0.16]	0.0003 [0.08]	0.001 [0.27]	**0.0113** **[3.07]**
G_{gb15}	0.0012 [0.33]	0.001 [0.27]	0.0014 [0.38]	0.0013 [0.35]	0.0015 [0.41]	0.0013 [0.35]	0.0005 [0.14]	0.0013 [0.35]	0.0006 [0.16]	0.0003 [0.08]	0.001 [0.27]	**0.0114** **[3.10]**
G_{gb16}	0.0017 [0.46]	0.0018 [0.49]	0.0021 [0.57]	0.0017 [0.46]	0.0022 [0.60]	0.0019 [0.52]	0.0007 [0.19]	0.0021 [0.57]	0.001 [0.27]	0.0006 [0.16]	0.0015 [0.41]	**0.0173** **[4.70]**
G_{gb23}	0.003 [0.82]	0.0014 [0.38]	0.0039 [1.06]	0.0047 [1.28]	0.0033 [0.90]	0.0034 [0.92]	0.0012 [0.33]	0.0034 [0.92]	0.0015 [0.41]	0.0009 [0.24]	0.0031 [0.84]	**0.0298** **[8.10]**
G_{gb24}	0.0011 [0.30]	0.0004 [0.11]	0.0014 [0.38]	0.0016 [0.43]	0.0013 [0.35]	0.0014 [0.38]	0.0005 [0.14]	0.0013 [0.35]	0.0006 [0.16]	0.0003 [0.08]	0.0012 [0.33]	**0.0111** **[3.02]**
G_{gb25}	0.0011 [0.30]	0.0004 [0.11]	0.0013 [0.35]	0.0017 [0.46]	0.0014 [0.38]	0.0015 [0.41]	0.0005 [0.14]	0.0013 [0.35]	0.0006 [0.16]	0.0003 [0.08]	0.0012 [0.33]	**0.0113** **[3.07]**
G_{gb26}	0.0016 [0.43]	0.001 [0.27]	0.002 [0.54]	0.0023 [0.63]	0.0019 [0.52]	0.0021 [0.57]	0.0007 [0.19]	0.0021 [0.57]	0.0009 [0.24]	0.0006 [0.16]	0.0018 [0.49]	**0.017** **[4.62]**
G_{gb34}	0.003 [0.82]	0.0015 [0.41]	0.0043 [1.17]	0.0045 [1.22]	0.0028 [0.76]	0.0028 [0.76]	0.0012 [0.33]	0.0032 [0.87]	0.0015 [0.41]	0.0009 [0.24]	0.0028 [0.76]	**0.0285** **[7.75]**
G_{gb35}	0.0031 [0.84]	0.0015 [0.41]	0.004 [1.09]	0.0046 [1.25]	0.0032 [0.87]	0.0031 [0.84]	0.0011 [0.30]	0.0032 [0.87]	0.0014 [0.38]	0.0008 [0.22]	0.0029 [0.79]	**0.0289** **[7.86]**
G_{gb36}	0.0043 [1.17]	0.003 [0.82]	0.0059 [1.60]	**0.0064** **[1.74]**	0.0044 [1.20]	0.0047 [1.28]	0.0016 [0.43]	0.0053 [1.44]	0.0024 [0.65]	0.0015 [0.41]	0.0042 [1.14]	**0.0437** **[11.88]**
G_{gb45}	0.0011 [0.30]	0.0005 [0.14]	0.0015 [0.41]	0.0016 [0.43]	0.0012 [0.33]	0.0012 [0.33]	0.0005 [0.14]	0.0012 [0.33]	0.0005 [0.14]	0.0003 [0.08]	0.0012 [0.33]	**0.0108** **[2.94]**
G_{gb46}	0.0016 [0.43]	0.0011 [0.30]	0.0022 [0.60]	0.0021 [0.57]	0.0015 [0.41]	0.0018 [0.49]	0.0007 [0.19]	0.002 [0.54]	0.0009 [0.24]	0.0006 [0.16]	0.0017 [0.46]	**0.0162** **[4.40]**
G_{gb56}	0.0016 [0.43]	0.0011 [0.30]	0.002 [0.54]	0.0023 [0.63]	0.0019 [0.52]	0.002 [0.54]	0.0006 [0.16]	0.002 [0.54]	0.0009 [0.24]	0.0005 [0.14]	0.0017 [0.46]	**0.0166** **[4.51]**
Total: G_{gb}	**0.0301** **[8.18]**	**0.0197** **[5.35]**	**0.0388** **[10.55]**	**0.0408** **[11.09]**	**0.0333** **[9.05]**	**0.0327** **[8.89]**	**0.012** **[3.26]**	**0.0344** **[9.35]**	**0.0156** **[4.24]**	**0.009** **[2.45]**	**0.029** **[7.88]**	**G_{gb}=0.2954** **[80.29]**

** [.] *: % contribution to G*

Finally, analyzing the between-region inequalities of poverty (see *Table 5*), we can notice that the major contribution to the inequality (11.88%) is concerned with differences in intensity of poverty between the North-east region (the poorest one) and the Patagonia region (the richest one); the main dimension that explains this inequality is the household size that contributes with a 1.74% to the overall inequality. Other important between-region inequalities are those generated by differences between the Cuyo and North-east regions, GBA and North-east regions, North-east and North-west regions, and North-east and Pampeana regions, which explain 8.07%, 8.10%, 7.75%, and 7.86% of the overall inequalities in multidimensional poverty respectively.

It is important to notice that North-east is the region generating the most significant differences in intensity of poverty between regions.

V. CONCLUSION

This article is three-fold. First, we make a synthesis of Dagum's work between the Gini index of income inequality measure [Dagum (1997)] and the measurement of poverty in the background of fuzzy set theory [Dagum and Costa (2004)]. This synthetic approach shows that is possible to use the property of sub-group decomposition [Dagum (1997)], source decomposition, and the multidimensional decomposition to compute inequalities in poverty.

Second, we contemplate a multidimensional inequality measure of poverty, whereas most of the researches are connected with the causal effect of inequality on the level of poverty as Son (2003). In the same way, the Sen index includes a component that represents the dimension of inequality. These components also represent inequalities in poverty, but they fail to explain inequalities in a multidimensional context, that is, involving the dimensions of socio-economic status such as education or professional occupation. The Gini index of poverty discussed in this article enables one to measure inequality following these different dimensions.

Third, using a multidimensional decomposition we show that inequalities in poverty are due to poverty differences within-group, poverty differences between each pair of sub-populations, poverty differences in each attribute, and poverty differences following the couples "within-group/attribute" and "between-group/attribute".

The illustration on the Argentinean's households shows that the North-east area possesses the most important inequalities of poverty level among its households. Indeed, this region has the biggest ratio of poverty, $\mu_B^k = 0.1792$. This means that 17.92% of the households are structurally poor. On the other hand, as this region has a low population share, its contribution to the overall poverty is low (only 4.6%). Nevertheless, the Gini index of poverty verifies that this region creates important differences of poverty. Furthermore, this region also generates significant differences with the other area since the inequalities between the North-east and the other region are the principal ones. The attribute inequalities are mainly explained by toilet characteristics and household sizes. Finally, combining the attributes and the group decomposition, we find that toilet characteristics of North-east is the major combination "within-group/attribute" of the overall inequalities of poverty (1.71%), and that the differences in toilet characteristics between North-east and Patagonia is the principal couple "between-group/attribute", which yields the Gini index of poverty (1.74%).

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APPENDIX I: Degree of membership of the socio-economic attributes studies

A.I.1. Occupancy title and location of the household residence

Occupancy title and location of the household residence	Owner of the house and terrain	Owner of the house only	Rented	Occupied under reddumption agreement	Occupied free of charges
House	0	0.3	0.4	0.5	1
Apartment	0	0.3	0.4	0.5	1
House residence at work	0	0.4	0.5	0.6	1
Rooms for rent	0	0.6	0.6	0.7	1
Hotel	0	0.6	0.75	0.8	1
Not ability household	0.5	0.8	0.9	0.9	1
Run-down neighborhood	0.7	1	1	1	1

A.I.2. Toilets characteristics

Characteristics		Degree of membership	
		There is	There isn't
The toilets has	WC with water flow	0	1
	WC without water flow	0.75	1
	Latrines	1	1

A.I.3. Flow characteristics

Characteristics		Degree of membership	
		There is	There isn't
The flow goes to	Waste water disposal or sewer	0	1
	Antiseptic room	0.25	1
	Water sump	1	1

A.I.4. Materials of construction of the household (of the principals' walls)

Materials	Degree of membership
Masonry (brick, concrete, and others)	0
Wood	0.25
Metal or fibrocement	0.50
Adobe	0.75
Carton or waste	1
Others	1

A.I.5. Household size: σ = number of households members/number of rooms of the household[6]

Ratio (σ)	Degree of membership
$\sigma \leq 1$	0
$1 < \sigma \leq 2$	0
$2 < \sigma \leq 3$	0.5
$\sigma > 3$	1

[6]We have not considered the bathrooms or the kitchen.

A.I.6. Household equivalent income[7]

Income level (y_i^e)	Degree of membership
If $y_i^e \le y_{0,15}^e$	1
If $y_{0,15}^e < y_i^e \le y_{0,60}^e$	$(y_{0,60}^e - y_i^e)/(y_{0,60}^e - y_{0,15}^e)$
If $y_i^e > y_{0,60}^e$	0

A.I.7. Stability of occupation of the reference person[8]

		Degree of membership		
		< 25 years old	**25-65 years old**	**> 65 years old**
Male employed head of household	**Permanent**	0	0	0
	Temporary	0.1	0.1	0
	Unknown	0.2	0.3	0,1
	Little job	0.4	0.5	0,1
Male unemployed head of household		1	1	1
Male inactive		0.5	0.6	0.2
Female employed head of household	**Permanent**	0	0	0
	Temporary	0.1	0.2	0
	Unknown	0.2	0.4	0,1
	Little job	0.4	0.6	0,1
Female unemployed head of household		1	1	1
Female inactive		0.5	0.8	0.2

A.I.8. Higher level of education completed by the reference person

Level of education	Degree of membership
None	1
Primary school	1
National School	0.5
Commercial school	0.5
Normal school	0.5
Technical school	0.25
Others	0.25
Associate' s university degree (3 years of study)	0.1
University studies	0

A.I.9. Professional occupation of the reference person

Occupation	Degree of membership
Manager or employer	0
Self employed	0
Office worker	0.3
Non salary worker	1

[7] Where $y_{0.15}^e$ and $y_{0.60}^e$ are the equivalent income for the 15th and 60th percentile.

[8] We made an adaptation of the degree of membership proposed by Dagum and Costa (2004) for this attribute.

A.I.10 Pension and others benefits for the employed person[9]

Pensions and others	Degree of membership
Pension only	0.5
Combinations with pension	0.25
Combinations without pension	0.9
All the benefits	0
Without any benefit	1
Employed without salary	1
Unemployed	1

A.I.11 : Ratio: the number of the household members with income and the household size [10]

Number of rooms of the household	Value of the ratio	Degree of membership
1	**0**	1
1	**1**	0
2	**0**	1
2	**$\geq$ 0.5**	0
3	**0**	1
3	**$\geq$ 0.33**	0
4	**0**	1
4	**0.25**	0.4
4	**$\geq$ 0.5**	0
5	**0**	1
5	**0.2**	0.5
5	**$\geq$ 0.4**	0
6	**0**	1
6	**0.16**	0.75
6	**0.33**	0.25
6	**$\geq$ 0.5**	0
$\geq$ 7	**0**	1
$\geq$ 7	**0.14-0.29**	0.75
$\geq$ 7	**0.3-0.58**	0.25
$\geq$ 7	**>0.58**	0

APPENDIX II : Equivalent scales[11]

Table A.II.1.: Values of the equivalent scale used in the present article

Household Size	Equivalent scale
1 person	73
2 persons	82
3 persons	91
4 persons	**100**
5 persons	109
6 persons	118
7 persons or more	127

[9] The benefits are: holiday period, worker compensation, pension, social security and dismissal's indemnity.
[10] Degree of membership proposed by Dagum and Costa (2004) for this attribute.
[11] To transform the level of income of an *N*-size household into its equivalent income as it will be an *N**-size household, we had used the approach to build an equivalence scale proposed by Dagum and Costa (2004). For the application of this method is necessary to calculate a crossed elasticity between the level of income and the size of the household. The data base used for this estimation comes from the expenditure of household survey proposed by the World Bank in 2002.