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Abstract

In this paper, we show that the rate of unemployment in period t depends on GDP and inflation rate in period $t-1$. We then show that GDP is related to money creation, and subsequently that the rate of unemployment is a decreasing function of this creation.

Key-words and phrases : Creation of Money, Decomposition, GDP, Rate of Unemployment.

Classification JEL : E20, E24.

1 Introduction

Politicians and analysts use to linking the bad results of the unemployment rate in period t, u_t, to the low growth of the real gross domestic product in the same period, GDP_t. However, economists have proved since Okun's law that GDP exerts, with time-lag variations, modifications of the unemployment rate.

In this paper, we formally demonstrate that the rate of unemployment can be decomposed in several economic and demographic variables. Particularly, it brings out the determinants of the real GDP growth : money creation, external balance of good and services, balance of income flows, and inflation rate.

The remainder of the paper is organised as follows. Section 2 is devoted to the unemployment rate decompositions. A brief conclusion follows in Section 3.

2 Decompositions of the rate of unemployment

The hysteresis hypothesis (see e.g. Samuelson (1972)) allows one to conceive u_t as a function of all past GDP's in a given economy. We aim first at deriving such analytical decomposition.

Proposition 1 *The rate of unemployment u_t is a decreasing function of all past GDP's.*

Proof. Let N_t be the total number of persons who are employed in a given economy in period t and $APOP_t$ the active population in t. Formally, we have : $1 - u_t = \frac{N_t}{APOP_t}$. In order to introduce the hysteresis hypothesis, we propose the following trivial expression :

$$1 - u_t = \frac{N_t . \frac{GDP_{t-1}}{GDP_{t-1}} . \frac{GDP_{t-2}}{GDP_{t-2}} \cdots \frac{GDP_{t-n}}{GDP_{t-n}}}{APOP_t}. \tag{1}$$

Subsequently, it is possible to decompose, on the one hand, GDP_{t-u} for all $u \in \{1, 2, \ldots, n\}$, where n is the year number of observed unemployment rates. Let $GDPh_{t-u}$ be the GDP per head and per worked hours, AL_{t-u} the time average of labour per head, and N_{t-u} the total number of employed individuals. We have : $GDP_{t-u} = GDPh_{t-u} . AL_{t-u} . N_{t-u}$, for all $u \in \{1, 2, \ldots, n\}$. On the other hand, we decompose the active population : $APOP_t = RA_t . WPOP_t$, where RA_t is the rate of activity and where $WPOP_t$ is the working-age population (see e.g. IRES (1999)). It then follows that :

$$1 - u_t = \frac{N_t . \prod_{u=1}^{n} \frac{GDP_{t-u}}{GDPh_{t-u} . AL_{t-u} . N_{t-u}}}{RA_t . WPOP_t}. \tag{2}$$

In the same manner, we obtain :

$$1 - u_{t-1} = \frac{N_{t-1}.\prod_{u=2}^{n}\frac{GDP_{t-u}}{GDPh_{t-u}.AL_{t-u}.N_{t-u}}}{RA_{t-1}.WPOP_{t-1}}.\tag{3}$$

Remark that $\log(3) - \log(2)$ yields approximately :

$$u_t - u_{t-1} = \dot{RA}_t + W\dot{POP}_t - \dot{N}_t + \sum_{u=1}^{n}\left(\dot{GDPh}_{t-u} + \dot{AL}_{t-u} + \dot{N}_{t-u} - \dot{GDP}_{t-u}\right),\tag{4}$$

where the notation $\dot{X}_t$ stands for the growth rate of X between $t-1$ and t. $\blacksquare$

This result involves all past GDP's. However, referring to Okun's law, it is sufficient to explain the evolution of the rate of unemployment in period t for $n = 1$:

$$u_t = \dot{RA}_t + W\dot{POP}_t - \dot{N}_t + \dot{GDPh}_{t-1} + \dot{AL}_{t-1} + \dot{N}_{t-1} - \dot{GDP}_{t-1} - u_{t-1}.\tag{5}$$

Expressions (4) or (5) are more realistic than the negative relationship between u_t and GDP_t. Indeed, introducing a time-lag in GDP means that employers analyze the results of their activity at the end of period $t-1$ (throughout $\dot{GDPh}_{t-1}$ and $\dot{GDP}_{t-1}$) and therefore adjust the level of employment at the beginning of period t.

Proposition 2 *The rate of unemployment u_t is an increasing function of the inflation rate in $t-1$, a decreasing function of the creation of money in $t-1$, of the external balance of goods and services in $t-1$ and of the balance of income flows in $t-2$.*

Proof. Let R_{t-1} be the disposal income that agents possess at the end of period $t-1$. Let RR_{t-1} be the sum of all income flows that residents receive from the rest of the world at the end of $t-1$. On the contrary, let RV_{t-1} be the sum of all income flows that residents pay for the rest of the world in the same period. Conventions used in national accountability yield : $R_{t-1} = nGDP_{t-1} + RR_{t-1} - RV_{t-1}$, where $nGDP$ stands for nominal GDP. Let C_t^H be the overall household consumption expenditure at the end of t. Using a time-lag Robertson hypothesis is equivalent to admit that R_{t-1} enables agents to finance their own consumption during period t. Then, saving can be defined as : $S_t = R_{t-1} - C_t^H$. As $R_{t-1} = nGDP_{t-1} + RR_{t-1} - RV_{t-1}$, then $C_t^H = nGDP_{t-1} + RR_{t-1} - RV_{t-1} - S_t$. Let I_t, EXP_t and IMP_t be respectively : investments, exportations and importations in period t. We know that : $nGDP_t = C_t^H + I_t + EXP_t - IMP_t$. The expression of C_t^H yields : $nGDP_t - nGDP_{t-1} = I_t - S_t + EXP_t - IMP_t + RR_{t-1} - RV_{t-1}$. Dividing both sides by $nGDP_{t-1}$ and using the inflation rate Π as the deflator of $n\dot{GDP}$ gives the growth rate of real GDP :

$$\dot{GDP}_t = \frac{I_t - S_t + EXP_t - IMP_t + RR_{t-1} - RV_{t-1}}{GDP_{t-1}} - \Pi_t.\tag{6}$$

Consequently, the growth of GDP is a decreasing function of the inflation rate, an increasing function of the Investment/Save difference $(I - S)$, an increasing function of the external balance of goods and services $(EXP - IMP)$ and an increasing function of the balance of income flows $(RR - RV)$. Substituting back this expression into (5), with a $t - 1$ lag, entails :

$$
\begin{aligned}
u_t =& \dot{RA}_t + W\dot{POP}_t - \dot{N}_t + G\dot{DPh}_{t-1} + \dot{AL}_{t-1} + \dot{N}_{t-1} \\
& - \frac{I_{t-1} - S_{t-1}}{GDP_{t-2}} - \frac{EXP_{t-1} - IMP_{t-1}}{GDP_{t-2}} - \frac{RR_{t-2} - RV_{t-2}}{GDP_{t-2}} \\
& + \Pi_{t-1} - u_{t-1}.
\end{aligned}
\tag{7}
$$

The positivity of $I_{t-1} - S_{t-1}$ is equivalent to $I_{t-1} + C_{t-1}^H > S_{t-1} + C_{t-1}^H$. This means that agents have the possibility to engage more expenses than the amount of their disposal incomes – since $I_{t-1} + C_{t-1}^H$ represents the spending flow of agents during $t - 1$ whereas $S_{t-1} + C_{t-1}^H$ represents their disposal incomes at the end of period $t - 2$. Consequently, for all t, I_{t-1} is higher than S_{t-1} if and only if there is creation of money. ∎

3 Conclusion

The rate of unemployment is a function of economic and demographic variables. We demonstrate that the equality (or identity) $I = S$ is debatable since the creation of money is a relevant determinant of the unemployment rate. The best empirical examples are those of the United-States's and United-Kingdom's economies, for which the ratio $\frac{I_t - S_t}{GDP_{t-1}}$ has been still important with a decreasing u_t since the 90's. Contrary to this, the ratio $\frac{I_t - S_t}{GDP_{t-1}}$ decreases for continental economies such as France, Italy and Germany while u_t increases (see Table 1).

Our approach can also be useful to test wether u_t is more sensitive to demographic variables such as $\dot{RA}_t$, $W\dot{POP}_t$, $\dot{N}_t$ than economic variables. For instance, it may be of interest to test, for continental economies, the sensibility of the unemployment rate when the average time of labour decreases. These analyzes can be performed using non-parametric issues such as bootstrap inference to measure the significance of the variables over time.

Finally, the model presented in (7) exhibits other decomposition possibilities. Indeed, the literature on production theory indicates that productivity indices such as Malmquist measures (see e.g. see Mussard and Peypoch (2006)) are decomposable in two components : efficiency change and technical change. Consequently, *via GDPh*, one may associate the rate of unemployment with the efficiency with which the employers use their inputs (labour, capital, etc.) and the possibility they have to increase technical progress.

4 References

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Table 1 : Ratio of Money Creation (%) : $\frac{I_t - S_t}{GDP_{t-1}}$

Years	USA	UK	Italy	France	Germany
1971	8.566	10.482	8.238	9.688	16.125
1972	10.365	11.475	8.591	9.982	14.720
1973	11.449	17.958	23.080	12.074	15.597
1974	8.578	19.733	31.788	16.048	10.153
1975	8.181	29.496	14.435	11.607	8.653
1976	11.539	21.856	27.235	16.121	13.902
1977	12.576	16.420	20.279	12.438	11.615
1978	14.293	14.547	15.047	12.645	11.938
1979	12.745	17.336	20.948	13.566	14.783
1980	9.330	14.612	28.312	15.183	13.954
1981	12.604	7.306	22.191	15.204	10.773
1982	4.637	7.710	19.555	18.759	8.432
1983	10.249	8.502	15.836	12.917	10.000
1984	14.153	7.268	14.813	9.366	9.378
1985	10.255	8.500	12.682	8.314	7.521
1986	8.905	8.333	9.105	7.633	7.236
1987	9.477	11.329	8.907	6.269	4.942
1988	10.117	15.694	11.048	8.705	6.935
1989	9.246	14.275	9.793	8.425	7.993
1990	7.271	11.184	10.404	6.331	9.667
1991	3.886	6.248	9.083	4.627	9.232
1992	6.566	5.483	5.316	3.725	7.787
1993	6.508	6.161	-0.200	-0.165	2.928
1994	8.025	6.778	2.188	2.698	4.947
1995	6.297	6.097	3.833	2.225	3.313
1996	7.087	6.755	1.011	1.214	0.588
1997	7.759	6.049	0.453	0.321	0.826
1998	7.244	7.138	0.860	1.756	1.200
1999	8.995	7.011	1.354	1.000	1.485
2000	10.070	7.443	4.687	4.529	2.150
2001	6.945	7.354	3.396	2.680	0.395
2002	7.603	8.352	2.734	1.659	-3.088
2003	9.616	8.436	2.527	1.945	-3.225
2004	12.681	8.837	3.287	3.768	-2.644

*Source : OECD