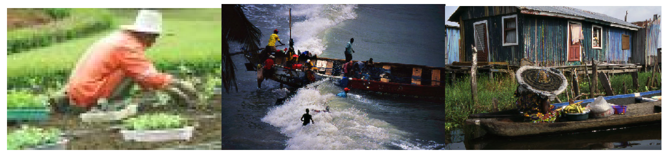

Groupe de Recherche en Économie et Développement International

Cahier de recherche / Working Paper
06-23

# Between-Group Transfers and Poverty-Reducing Tax Reforms


Paul Makdissi

Stéphane Mussard

# Between-Group Transfers and Poverty-Reducing Tax Reforms*

Paul Makdissi†
Université de Sherbrooke

Stéphane Mussard‡
CEPS/INSTEAD, GRÉDI, GEREM

October 2006

**Abstract**

In this paper, we propose the conception of within-group CD-curve, to apprehend the impact of indirect tax reforms on truncated distributions of consumption expenditures. This confers decision makers the ability to perform within-group transfers as well as between-group transfers to reduce poverty in particular groups or to obtain an overall poverty alleviation. Between-group transfers are implemented in order to introduce a fairness element into the indirect tax framework, allowing to test for the robustness of reducing-tax reforms, for any order of stochastic dominance.

**Key-words and phrases**: Between-group redistribution, CD-curves, Stochastic dominance of order $s$, Tax reforms.

**JEL Codes**: D63, H20.

---

*This paper was partly done when Stéphane Mussard was post-doctoral researcher at CEPS/INSTEAD Luxembourg and when he visited the University of Sherbrooke and GRÉDI, which are greatly acknowledged. He is also greatly indebted to the Minsitère de la Recherche du Luxembourg and GRÉDI for financial support.

†Département d'économique and GRÉDI, Université de Sherbrooke, 2500, boulevard de l'Université, Sherbrooke, Québec, Canada, J1K 2R1; Email: paul.makdissi@usherbrooke.ca

‡CEPS/INSTEAD Luxembourg, GRÉDI Université de Sherbrooke, and GEREM Université de Perpignan. Address: GEREM, Département des Sciences Economiques, Université de Perpignan, 52 Avenue Paul Alduy, 66860 Perpignan Cedex, France, E-mail: smussard@adm.usherbrooke.ca

# 1 Introduction

Yitzhaki and Slemrod (1991), subsequently Yitzhaki and Thirsk (1990), Makdissi and Wodon (2002), Duclos, Makdissi and Wodon (2005a), among others, have introduced and analyzed the impact of transfers between individuals according to indirect taxation frameworks in order to yield decision makers the ability to constitute poverty-reducing or welfare-improving fiscal reforms. These standard tax reforms, for couple of goods $\{i, j\}$, consist in financing a decreasing tax on $i$ by increasing the tax on $j$.

Precisely, Makdissi and Wodon (2002) have initiated the use of a new concept, that of Consumption Dominance diagram (CD-curve from now on) in order to apprehend the impact of marginal indirect tax reforms on poverty, for any order of restricted stochastic dominance. In other words, if the CD-curve of good $i$ dominates (lies above) that of good $j$, for any order, and for all incomes below a defined poverty line, and if one increases the tax on the $j$-th commodity and uses the proceeds to subsidize the $i$-th commodity, then overall poverty declines, and conversely.[1]

These taxation procedures are relevant since, as pointed out by Duclos *et al.* (2005a), they are compatible with various poverty indices that belong to an overall class of poverty measures ($\Pi^s$), with heterogeneous agents and with any poverty line corresponding to any group of the population. Furthermore, these dominance tests are appealing since they are less restrictive than parametric tests and can be used for all units of consumption expenditures and for any order of stochastic dominance. Moreover, theses orders of dominance correspond to thorough ethical transfer principles. For instance, dominance of order 2 implicitly assumes that a transfer of amount $\delta > 0$ is made from a higher-income individual to a lower-income one.

In this paper, we analyze the possibility to make transfers within groups or between groups of the population. On the one hand, the within-group CD-curves are introduced to conceive transfers inside a single group. This yields very intuitive applications, e.g., an increasing tax on fuel in an urban area may help to subsidize public transportation in the same area, implying poverty reduction. On the other hand, between-group transfers are characterized by taxing the $j$-th commodity in one group in order to subsidize, in another group, the tax on the same $j$-th good or alternatively on another commodity. One must think about cross-subsidies between different groups

[1] Duclos, Makdissi and Wodon (2005b) have applied this framework to direct transfer reforms and Makdissi and Wodon (2007) to regulatory reforms.

of consumers of a public utility. These theoretical developments are particularly relevant for policy purposes since more freedom is attributed to decision makers, which may easily find fiscal proceeds aiming at subsidizing groups in needs with overall poverty alleviation. Accordingly, we propose many tests based on dominance between within-group CD-curves. This allows us to contemplate doing poverty-reducing tax reforms, provided that within-group CD-curves do not intersect, for any given order of stochastic dominance. This leads to a set of results for which it is not necessary to impose an homogeneous taxation scheme on the whole distribution of goods $i$ and $j$, since it is possible to focus on truncated parts of these distributions.

The remainder of the paper is organized as follows. Section 2 deals with assumptions characterizing the analytical forms of poverty and those of taxation issues. Section 3 is devoted to the test specification of within- and between-group transfers, showing that an equal treatment of the groups in the taxation system provides a poverty-reducing tax reform. Afterwards, introducing an assumption of fair treatment of the groups, we strengthen our test for between-group redistribution and poverty decline, for any order of stochastic dominance. Section 4 draws concluding remarks and advances further researches.

## 2 Assumptions and Definitions

We suggest, on the one hand, a set of assumptions $\mathbb{A}$ in order to formalize the poverty environment on which we intend to derive our results of stochastic dominance. We test for poverty-reducing tax reforms using an additive structure of poverty indices. An additive poverty index is defined as the sum of individuals's poverty $p(\cdot)$:

$$P(F,z) = \int_0^a p\left(y^E(q,y),z\right) dF(y), \tag{1}$$

where $F$ is the cumulative distribution function defined over $[0,a]$, $y^E$ the equivalent incomes, $a$ an integer greater than all $y^E$, and $z$ the poverty line defined in the equivalent income space. The notation $q$ symbolizes a vector of unitary market prices $e$ subject to taxes $t$.

As overall poverty is the sum of individuals's poverty, each agent's equivalent income is compared with a sole common poverty line $z \in \mathbb{R}_{++}$. When the population is partitioned into $k$ groups, $k \in \{1,\dots,K\}$, the advantage of working with such class of additive poverty indices is the possibility to conceptualize the overall poverty as the sum of poverty within each population subgroup. Let $p_k\left(y^E(q,y),z_k\right)$ be the poverty function characterizing group $k$'s poverty, where $z_k \leq z_k^+$, $z_k^+ \in \mathbb{R}_{++}$ being the maximum conceivable poverty line in group $k$.

**Assumption 2.1**: ***Additive Poverty.*** *An additive index is defined as a weighted average of poverty intensity within each group:*

$$P(F,z) = \sum_{k=1}^{K} \theta_k \int_0^a p_k\left(y^E(q,y),z_k\right) dF_k(y), \tag{A1}$$

*where $\theta_k$ is the population share of the $k$-th group and $F_k(y)$ the cumulative distribution function of group $k$ defined over* $[0,a]$.[2]

The poverty level in the $k$-th group and the contribution of the $k$-th group to the overall poverty ratio are respectively given by: $p_k(\cdot)$ and $\theta_k p_k(\cdot)$. If there exists in each group $k$ at least one equivalent income $y^E$ lower than the poverty line $z_k$, then the strict positivity of $p_k(\cdot)$ and $\theta_k p_k(\cdot)$ is guaranteed. On the contrary, these are nil.

In the sequel, focus is principally put on poverty variations and particularly on poverty alleviation. The property of differentiability is then imposed to the class of additive poverty measures.

**Assumption 2.2**: ***Differentiability.*** *The poverty measure is a $s$-time differentiable continuous function almost everywhere over* $[0,a]$ *such as:*

$$(-1)^u p_1^u(\cdot) \geq (-1)^u p_2^u(\cdot) \geq \ldots \geq (-1)^u p_k^u(\cdot) \geq 0, \forall u \in \{1,2,\ldots,s\}, \tag{A2}$$

*where $p_k^u(\cdot)$ is the $u$-th derivative of the $p_k(\cdot)$ function.*

The class of poverty measures satisfying assumptions (A1) and (A2) is denoted by $\Pi^s$. It is a well suited class of indices (see Duclos *et al.* (2005a) and Zheng (1999)) that involves the well-known FGT's measures (Foster, Greer and Thorbecke (1984)). For $s \geq 1$, assumption (A2) implies that an increase in household equivalent income $y^E$ diminishes poverty, for any given household type. Furthermore, it postulates that, for any given household equivalent income $y^E$, the needier the households are, the greater the poverty alleviation may be. Although the difference in household needs

[2] Let $n_k$ be the size of group $k$ and $n$ be the size of the global population. Then, $\theta_k = \frac{n_k}{n}$.

is usually interpreted as gaps in household sizes, this interpretation is less appropriate in our framework. Here, suitable interpretations may be differences in health (handicapped versus non handicapped individuals), gender (women versus men), ethnic and religious affiliations, as well as differences in regions. For any given example, if divergences in capabilities are recorded at the same income level (see Sen (1992)), such an assumption is relevant. (A2)'s normative implications are more stringent than the usual ones for first-order unidimensional dominance and can be viewed as a weak version of the Pigou-Dalton principle, which is in fact equivalent to Sen's Weak Equity Axiom (see Sen (1997), p. 18). For $s \geq 2$, assumption (A2) postulates that an equalizing transfer of $\delta > 0$ from a richer person to a poorer one decreases poverty, this effect being stronger across needier households. Indeed, for higher $s$, the interpretation of (A2) can be made using Fishburn and Willig's (1984) general transfer principle, for which increasing weights are associated with transfers occurring at the bottom of the distribution as far as $s$ increases. Hence, (A2) makes these properties, viewed as a generalization of Sen's Weak Equity Axiom, normatively more important for needier households. Finally, for any order $s$, we have Fishburn and Willig's normative interpretation of $s$-order unidimensional dominance (that is, the interpretation of $(-1)^s p_k^s(\cdot) \geq 0$), coupled with a weak version of the traditional normative interpretation of $(s{+}1)$-order dominance (the interpretation of $(-1)^s p_k^s(\cdot) \geq (-1)^s p_{k+1}^s(\cdot)$ in a sequential context).

Now, in order to define the indirect taxation environment used to gauge the impact of fiscal reforms on poverty variations, it is possible to follow, on the other hand, a set of assumptions $\mathbb{B}$. We first require a revenue neutrality assumption. This implicitly postulates that an increasing tax on a particular good allows to finance a decreasing tax on another good, the fiscal revenue being constant.

**Assumption 2.3**: ***Revenue Neutrality.*** *If $X_k = \int_0^a x_k(y)dF(y)$ denotes the aggregate average consumption of the k-th good, the per capita government indirect tax revenue is $R = \sum_{k=1}^{K} t_k X_k,\ k \in \{1, \ldots, K\}$, and revenue neutrality is then formalized by:*

$$dR = 0. \tag{B1}$$

Afterwards, in order to capture the efficiency associated with a two-good taxation fashion, the following definitions are required.

**Definition 2.4**: ***Differential Efficiency.*** *Wildasin* (1984) *proposes an efficiency parameter $\gamma_{ij}$ that captures the marginal social cost of raising* \$1 *of* *public funds by taxing the $j$-th commodity and using the proceeds to subsidize the $i$-th commodity. Suppose a $M$-good economy with $i, j, m \in \{1, \ldots, M\}$, then, if producers's prices are constant, equation B1 brings out:*

$$\gamma_{ij} = -\frac{dt_j}{dt_i}\left(\frac{X_j}{X_i}\right) = \frac{1 + \frac{1}{X_i}\sum_{m=1}^{M} t_m \frac{\partial X_m}{\partial t_i}}{1 + \frac{1}{X_j}\sum_{m=1}^{M} t_m \frac{\partial X_m}{\partial t_j}}.$$

Subsequently, Besley and Kanbur (1988) determine the variation of the equivalent income with respect to the tax rate variation of good $i$. Using Roy's identity and assuming that the observed price vector is the vector of reference, they show that the change in equivalent incomes generated by a marginal change of the tax rate of good $i$ is:

$$\frac{\partial y^E}{\partial t_i} = -x_i(q, y), \tag{2}$$

where $x_i(q, y)$ is the Marshallian demand of good $i$. On this basis, Makdissi and Wodon (2002) define CD-curves in order to perform a $s$-order stochastic dominance test.

**Definition 2.5**: ***CD-Curve of order*** $s$. *The CD-Curve of order* 1 *for good $i$ is the ratio between an individual consumption with income $y$ and the aggregate consumption of good $i$: $CD_i^1(y) = x_i(y)/X_i \cdot f(y)$, where $f(y)$ is the density function of per capita income, which is nil outside of the interval* $[0, a]$.[3] *The CD-curve of order $s$ is given by: $CD_i^s(y) = \int_0^y CD_i^{s-1}(u)du$.*

# 3 Stochastic Dominance and Poverty-reducing Taxation

Following the previous assumptions and definitions, Makdissi and Wodon (2002) propose a test that combines dominance between CD-curves, poverty reduction, and indirect tax reforms. In spite of its attractiveness, it is only concerned with overall poverty. To circumvent this issue, Duclos *et al.* (2005a) suggest to deal with heterogenous agents. For this purpose, they use (A1) and define the CD-Curve of order 1 for good $i$ and for group $k$. It represents the ratio between an individual consumption of group $k$ with income $y$ and the aggregate consumption of good $i$: $\widetilde{CD}_{ik}^1(y) = x_{ik}(y)/X_i \cdot f_k(y)$, where $f_k(y)$ is the density function of *per capita* incomes of group $k$, with

[3]Makdissi and Wodon (2002) use the following definition $CD_i^1(y) = x_i(y)/X_i$. However, Duclos, Makdissi and Wodon (2006) show that it is more helpful to use $CD_i^1(y) = x_i(y)/X_i \cdot f(y)$ for estimation purposes.

$f_k(y) = 0$ outside of the interval $[0, a]$. Subsequently, the $s$-order CD-curve of group $k$ for good $i$ is given by: $\widetilde{CD}^s_{ik}(y) = \int_0^y \widetilde{CD}^{s-1}_{ik}(u)du$.

**Theorem 3.1**. *Under conditions A1, A2 and B1, the two following propositions are equivalent:*
*(ı)* $\sum_{k=1}^{\ell} \theta_k[\widetilde{CD}^s_{ik}(y) - \gamma_{ij}\widetilde{CD}^s_{jk}(y)] \geq 0,\ \forall y \leq z^+,\ \forall \ell \in \{1, 2, \ldots, K\}$
*(ıı)* $dP(F, z) \leq 0,\ \forall z_k \leq z_k^+,\ \forall P(F, z) \in \Pi^s,\ s \in \{1, 2, 3, \ldots\}$.

**Proof.** See the appendix. ■

Theorems 3.1 implies an homogeneous impact of the tax rates $t_i$ and $t_j$ on the distributions of commodity expenditures $i$ and $j$. Indeed, decreasing overall poverty with an increasing tax on the $j$-th commodity may implicitly entail a whole decreasing poverty for which poor can be more solicited than rich, mainly if $j$ is an inferior good. Here after, another issue is analyzed with the assumption of equal treatment of the groups.

**Assumption 3.2**: ***Equal Treatment of the Groups.*** *Let $R^k$ be the fiscal revenue obtained in group $k$. Revenue neutrality is the rule used in each $k$ group, if and only if:*

$$dR^k = 0,\ \forall k \in \{1, \ldots, K\}. \tag{B2}$$

The above assumption may be coherent with a decentralized poverty alleviation program in which each region must have a balanced budget. In such a context, instead of examining $\widetilde{CD}$-curves of any commodity for group $k$ based on the aggregate consumption of the population, we investigate for $CD$-curves concerned with the aggregate consumption of each group $k \in \{1, \ldots, K\}$.

**Definition 3.3**: ***Within-group CD-Curve of Order*** *$s$. The first-order within-group CD-curve of group $k$ for good $i$ is the ratio between an individual consumption with income $y$ and the aggregate consumption of his group $k$ for good $i$: $CD^1_{ik}(y) = x_{ik}(y)/X_{ik} \cdot f_k(y)$. Thus, the $s$-order within-group CD-curve of group $k$ for good $i$ is given by: $CD^s_{ik}(y) = \int_0^y CD^{s-1}_{ik}(u)du$.*

How can we include an assumption improving fairness in an indirect taxation environment? A possibility is to investigate the case of a *per group* taxation design.

**Assumption 3.4**: ***Taxation per Group***. *For any couple of goods* $\{i, j\} \in \{1, \dots, M\}$*, each group of the population imposes his own tax rates. Then, the tax rates* $t_i$ *and* $t_j$ *in group* $k$ *are symbolized as, respectively:*

$$t_i^k,\ t_j^k, \forall k \in \{1, \dots, K\}. \tag{B3}$$

In order to combine the equal treatment of the groups (B2) with a *per group* taxation assumption (B3), another differential efficiency parameter is required.

**Definition 3.5**: ***Within-group Differential Efficiency***. *Suppose that an efficiency parameter* $\widetilde{\gamma}_{ij}^k$ *captures the marginal social cost of raising \$1 of public funds by taxing the* $j$*-th commodity in group* $k$ *and using the proceeds to subsidize the* $i$*-th commodity for the same group. In our* $M$*-good economy, with* $i, j, m \in \{1, 2, ..., M\}$*, B2 and B3 bring out:*

$$\widetilde{\gamma}_{ij}^k = -\frac{dt_j^k}{dt_i^k}\left(\frac{X_{jk}}{X_{ik}}\right) = \frac{1 + \frac{1}{X_{ik}} \sum_{m=1}^{M} t_m^k \frac{\partial X_{mk}}{\partial t_i^k}}{1 + \frac{1}{X_{jk}} \sum_{m=1}^{M} t_m^k \frac{\partial X_{mk}}{\partial t_j^k}}.$$

Now, by invoking the assumptions of additive poverty (A1) and differentiability (A2), it is possible to state a Theorem with equal treatment of the groups.

**Theorem 3.6**. *Under conditions A1, A2, B2, and B3, the two following propositions are equivalent:*
*(ı)* $\sum_{k=1}^{\ell}[CD_{ik}^s(y) - \widetilde{\gamma}_{ij}^k CD_{jk}^s(y)] \geq 0,\ \forall y \leq z_k^+,\ \forall \ell \in \{1, 2, \dots, K\}$
*(ıı)* $dP(F, z) \leq 0,\ \forall z_k \leq z_k^+,\ \forall P(F, z) \in \Pi^s,\ s \in \{1, 2, 3, \dots\}$.

**Proof.** See the appendix. ■

This result is attractive since it allows one to decrease overall poverty if and only if there is dominance of the sum of the within-group CD-curves for good $i$ (provided that those of good $j$ are multiplied by $\widetilde{\gamma}_{ij}^k$).[4] The condition is that we increase $t_j^k$ for all $k$ and use the proceeds to subsidize $t_i^k$ for all $k$.

It turns out that, this taxation framework could provide consequential freedom to decision makers in respect to the groups they decide to impose

[4]In the same manner as in Theorem 3.1, dominance within each group is not necessary since the *iff* condition guarantees the dominance of the sum. For instance, if the CD-curve of good $i$ in the needier group dominates that of good $j$ (multiplied by $\widetilde{\gamma}_{ij}^k$), and if this dominance is strong enough to compensate for the non dominance within the other groups, then overall poverty decreases and conversely.

the fiscal reform. Indeed, poverty-reducing tax reforms may be performed by taxing one or many groups without affecting the remainder of the population.

**Corollary 3.7**. *Suppose* $dt_i^k \leq 0$ *and* $dt_j^k \geq 0$*, and* $dt_i^\ell = dt_j^\ell = 0$*,* $\forall \ell \neq k \in \{1, \ldots, K\}$*. Then,* $[CD_{ik}^s(y) - \widetilde{\gamma}_{ij}^k CD_{jk}^s(y)] \geq 0 \Longrightarrow dP(F, z) \leq 0$*,* $\forall P(F, z) \in \Pi^s$*,* $s \in \{1, 2, 3, \ldots\}$*.*

**Proof.** It is straightforward. ■

In other words, if decision makers behave in accordance with the result of Theorem 3.6, and if a tax reform is only conducted in group $k$ (tax-rate variations being nil in the other groups), then overall poverty declines. Accordingly, theses *per group* taxation designs exhibit some incentive mechanisms. For instance, in urban areas, it is possible to finance an increasing subsidy on public transport by an increasing tax on fuel. This yields incentives with overall poverty reduction provided the CD-curve of public transport lies above that of fuel in this area (and provided the latter is multiplied by $\widetilde{\gamma}_{ij}^k$), for any chosen order of stochastic dominance.

Now, imagine a tax reform is performed in group $k$ where the number of poor individuals is important. This may entail a weak incentive effect if the proceeds issued from the fiscal revenue are low. Then, instead of increasing the tax on the $j$-th commodity to subsidize the $i$-th commodity in the same group, why not financing a decreasing tax in a poor group with a increasing tax in a rich group? In such a taxation environment, both fiscal revenue and fairness are improved.

**Assumption 3.8**: ***Fair Treatment of the Groups.*** *Let* $R_{ij}^{k\ell}$ *be the per capita indirect tax revenue obtained from group* $k$ *and* $\ell$*. Revenue neutrality is assumed to be the rule between groups* $k$ *and* $\ell$*, if we finance a decreasing tax on good* $i$ *in group* $k$ *by an increasing tax on good* $j$ *in group* $\ell$*:*

$$dR_{ij}^{k\ell} = 0, \ for \ any \ k, \ell \in \{1, \ldots, K\} \ and \ for \ any \ i, j \in \{1, \ldots, M\}. \quad \text{(B4)}$$

The fair treatment of the groups reinforces the *per group* taxation assumption, improving fairness and flexibility in the taxation mechanism. Given this assumption, we may redefine our economic efficiency ratio.

**Definition 3.9**: ***Between-group Differential Efficiency***. *Suppose that an efficiency parameter* $\widehat{\gamma}_{ij}^{k\ell}$ *captures the marginal social cost of raising \$1 of public funds by taxing the* $j$*-th commodity in group* $\ell$ *and using the proceeds* *to subsidize the i-th commodity in group k. In our M-good economy, with $i, j, m \in \{1, 2, ..., M\}$, B3 and B4 bring out:*

$$\widehat{\gamma}_{ij}^{\ell k} = -\frac{dt_j^\ell}{dt_i^k}\left(\frac{X_{j\ell}}{X_{ik}}\right) = \frac{\theta_\ell \left[1 + \frac{1}{X_{j\ell}} \sum_{m=1}^{M} t_m^\ell \frac{\partial X_{m\ell}}{\partial t_j^\ell}\right]}{\theta_k \left[1 + \frac{1}{X_{ik}} \sum_{m=1}^{M} t_m^k \frac{\partial X_{mk}}{\partial t_i^k}\right]}.$$

Note that in the above definition, in contrast to the within-group efficiency ratio, the between-group differential efficiency ratio includes population shares of the groups concerned with the tax reform. Remember that a (within-group) differential efficiency ratio gauges, under budget neutrality condition, the *per capita* budgetary impact of the reform (in each group). Consequently, in a two-group taxation framework, using the weights of population shares enables us to assess the total impact on the public budget. Indeed, imagine we finance a decreasing tax on the $i$-th commodity in one group (say $k$) with an increasing tax on the $j$-th commodity in another group (say $\ell$). Then, if $\theta_\ell \gg \theta_k$, the decreasing tax on good $i$ in group $k$ can be performed with a very marginal growth of $t_j^\ell$.

**Theorem 3.10**. *Under conditions A1, A2, B3, and B4, the two following propositions are equivalent:*
*(ı)* $CD_{ik}^s(y) - \widehat{\gamma}_{ij}^{\ell k} CD_{j\ell}^s(y) \geq 0$, $\forall y \leq z_k^+$, $\forall \ell \in \{k+1, k+2, \ldots, K\}$
*(ıı)* $dP(F, z) \leq 0$, $\forall P(F, z) \in \Pi^s$, $s \in \{1, 2, 3, \ldots\}$.

**Proof.** See the appendix. ■

This result is very helpful to get poverty-reducing tax reforms with a two-good taxation scheme. For instance, assume $j$ represents all goods, whereas $i$ stands for housing expenditures. In Canada, as natives are exempted of VAT, the project aiming at increasing subsidizes on $i$ for natives may be achieved with a slight increasing VAT on $j$ for non natives.

Alternatively, this technique may be applied in the one-good case, which is useful when we consider cross-price subsidies for public utilities between different consumer groups.

**Corollary 3.11**. *Under conditions A1, A2, B3, and B4, the two following propositions are equivalent:*
*(ı)* $CD_{ik}^s(y) - \widehat{\gamma}_{ii}^{\ell k} CD_{i\ell}^s(y) \geq 0$, $\forall y \leq z_k^+$, $\forall \ell \in \{k+1, k+2, \ldots, K\}$
*(ıı)* $dP(F, z) \leq 0$, $\forall P(F, z) \in \Pi^s$, $s \in \{1, 2, 3, \ldots\}$.

**Proof.** See the appendix. ■

Finally, contrary to the previous results, Theorem 3.10 and Corollary 3.11 allow to test for dominance with curves belonging to two different groups. Therefore, a *per group* taxation model is relevant with between-group transfers, provided that commodity consumptions in a given group are more concentrated among the poor than in the other group.

## 4 Conclusion

Indirect tax reforms exert, throughout the use of within-group CD-curves, a non-homogenous impact on the distributions of commodity expenditures since the underlying *per group* taxation assumption yields modifications on truncated parts of theses distributions. This confers decision makers the possibility to perform within-group transfers as well as between-group transfers to reduce poverty in particular groups or to obtain an overall poverty alleviation. Between-group transfers are implemented in order to introduce a fairness element into the indirect taxation system which strengthens the *per group* taxation fashion.

This methodology can contribute to open the way on new topics. Indeed, between-group indirect tax reforms may be studied to capture the impact on the diminution of overall inequalities, of between-group inequalities using the Gini index between populations of income receivers (see Dagum (1987)), or to analyze mobility (see e.g. Van Kerm (2004)). Moreover, it would be interesting to adapt these stochastic dominance tests in order to apprehend the dynamics of reducing-poverty tax reforms as well as the efficiency of the redistribution mechanism in measuring their significance over time with Davidson and Duclos's (2000) test.

Finally, the fact that poverty-reducing indirect tax reforms might be analyzed with the nature of the goods (luxury goods or inferior goods) is left for future researches.

## Appendix

**Proof. Theorem 3.1.**

$(\imath) \implies (\imath\imath)$: See Duclos, Makdissi and Wodon (2005a).

$(\imath\imath) \implies (\imath)$: Let us take a set of functions $p_k\left(y^E(q,y), z_k\right)$, for which the

$(s-1)$-th derivative is:

$$p_k^{(s-1)}\left(y^E(q,y),z_k\right)=\left\{\begin{array}{cc}(-1)^{s-1}\,\epsilon & \text{if } y\le\overline{y}\\ (-1)^{s-1}\left(\overline{y}+\epsilon-y\right) & \text{if } \overline{y}<y\le\overline{y}+\epsilon\\ 0 & y>\overline{y}+\epsilon\end{array}\right.,\forall k\in\{1,2,\ldots,K\}.$$

Poverty indices whose functions $p_k\left(y^E(q,y),z_k\right)$ have the above form for $p_k^{(s-1)}\left(y^E(q,y),z_k\right)$ belong to the class $\Pi^s$. This yields:

$$p_k^{(s)}\left(y^E(q,y),z_k\right)=\left\{\begin{array}{cc}0 & \text{if } y\le\overline{y}\\ (-1)^{s} & \text{if } \overline{y}<y\le\overline{y}+\epsilon\\ 0 & y>\overline{y}+\epsilon\end{array}\right.,\forall k\in\{1,2,\ldots,K\}. \quad (3)$$

Imagine now that $\sum_{k=1}^{\ell}\theta_k[\widetilde{CD}_{ik}^{s}(y)-\gamma\widetilde{CD}_{jk}^{s}(y)]<0$ on an interval $[\overline{y},\overline{y}+\epsilon]$ for some $\ell$, for $\overline{y}<z_\ell^+$, and for $\epsilon$ that can be arbitrarily close to 0. For $p_k\left(y^E(q,y),z_k\right)$ indices with $s$-order derivatives defined as in (3), the marginal tax reform induces an increase of poverty. Hence it cannot be that $\sum_{k=1}^{\ell}\theta_k[\widetilde{CD}_{ik}^{s}(y)-\gamma\widetilde{CD}_{jk}^{s}(y)]<0$ for some $\ell$, $y\in[\overline{y},\overline{y}+\epsilon]$ when $\overline{y}<z_\ell^+$. This proves the necessity of the condition. ■

For the following demonstrations, one needs Abel's lemma.

**Lemma 4.1**: ***Abel's lemma*** (*see Jenkins and Lambert* (1993) *and Duclos et al.* (2005)). *Let $x_j$ and $y_i$ be two real variables. If $x_n\ge x_{n-1}\ge\ldots\ge x_1\ge 0$, then $\sum_{i=j}^{n}y_i\ge 0\ \forall j$ is a sufficient condition for $\sum_{i=1}^{n}x_iy_i\ge 0$. Contrary to this, if $x_n\le x_{n-1}\le\ldots\le x_1\le 0$, then $\sum_{i=j}^{n}y_i\ge 0\ \forall j$ is also a sufficient condition for $\sum_{i=1}^{n}x_iy_i\le 0$.*

**Proof. Theorem 3.6.**
$(\imath)\implies(\imath\imath)$: In this context:

$$dp_k(\cdot)=-p_k^{(1)}(\cdot)\left[\frac{x_{ik}(y)}{X_{ik}}-\widetilde{\gamma}_{ij}^{k}\frac{x_{jk}(y)}{X_{jk}}\right]X_{ik}dt_i^k.$$

Now, remember that $\frac{x_{ik}(y)}{X_{ik}}\cdot f(y)=CD_{ik}^1(y)$, then:

$$dP(F,z)=-\sum_{k=1}^{K}\underbrace{(dt_i^k\theta_kX_{ik})}_{\lambda_k}\int_0^a p_k^{(1)}(\cdot)\left[CD_{ik}^1(y)-\widetilde{\gamma}_{ij}^{k}\ CD_{jk}^1(y)\right]dy.$$

Along the line of Duclos, Makdissi and Wodon (2005a), integrating by parts $\int_0^a p_k^{(1)}(\cdot) CD_{ik}^1$ $s$ times and using an induction reasoning implies that:

$$dP(F,z) = (-1)^s \sum_{k=1}^{K} \lambda_k \int_0^a p_k^{(s)}(\cdot) \left[CD_{ik}^s(y) - \widetilde{\gamma}_{ij}^k \ CD_{jk}^s(y)\right] dy$$
$$= \int_0^a \sum_{k=1}^{K} \lambda_k (-1)^s p_k^{(s)}(\cdot) \left[CD_{ik}^s(y) - \widetilde{\gamma}_{ij}^k \ CD_{jk}^s(y)\right] dy.$$

As $dt_i^k < 0$ for all $k \in \{1, \ldots, K\}$, it can be noticed that $\lambda_k \leq 0$, for all $k$. Using Abel's lemma, in order to get $\sum_{k=1}^{K} (-1)^s p_k^{(s)} \lambda_k [CD_{ik}^s(y) - \widetilde{\gamma}_{ij}^k \ CD_{jk}^s(y)] \leq 0$, it is then sufficient to have $\sum_{k=1}^{\ell} [CD_{ik}^s(y) - \widetilde{\gamma}_{ij}^k \ CD_{jk}^s(y)] \geq 0$, $\forall \ell \in \{1, 2, \ldots, K\}$. Thus, $\sum_{k=1}^{\ell} [CD_{ik}^s(y) - \widetilde{\gamma}_{ij}^k \ CD_{jk}^s(y)] \geq 0$, $\forall \ell \in \{1, 2, \ldots, K\}$, $\forall y \in \left[0, z_k^+\right]$, implies $dP(F,z) \leq 0$.
$(\imath\imath) \implies (\imath)$: The proof is similar to the one of Theorem 3.1. ■

**Proof. Theorem 3.10 and Corollary 3.11.**
$(\imath) \implies (\imath\imath)$: We only present the sufficiency of Theorem 3.10, that of Corollary 3.11 being a particular case for $j = i$. In this context, we have:

$$dp_k(\cdot) = -p_k^{(1)}(\cdot) \left[\frac{x_{ik}(y)}{X_{ik}}\right] X_{ik} dt_i^k$$

and,

$$dp_j(\cdot) = -p_j^{(1)}(\cdot) \left[-\widehat{\gamma}_{ij}^{\ell k} \frac{x_{jk}(y)}{X_{jk}}\right] X_{ik} dt_i^k.$$

This entails:

$$dP(F,z) = -\left[\lambda_k \int_0^a p_k^{(1)}(\cdot) CD_{ik}^1(y) dy - \lambda_k \widehat{\gamma}_{ij}^{\ell k} \int_0^a p_k^{(1)}(\cdot) CD_{jk}^1(y) dy\right].$$

The remaining of the proof is straightforward if we apply the result of Theorem 3.6.
$(\imath\imath) \implies (\imath)$: The proof is similar to the one of Theorem 3.1. ■

# References


[1] Besley, T. and R. Kanbur (1988), Food Subsidies and Poverty Alleviation, *Economic Journal*, 98, 701-719.

[2] Dagum, C. (1987), Measuring the Economic Affluence Between Populations of Income Receivers, *Journal of Business and Economic Statistics*, 5(1), 5-12.

[3] Davidson, R., J.-Y. Duclos (2000), Statistical Inference for Stochastic Dominance and for the Measurement of Poverty and Inequality, *Econometrica*, 68(6), 1435-1464.

[4] Duclos, J.-Y., P. Makdissi and Q. Wodon (2005a), Poverty-Reducing Tax Reforms with Heterogeneous Agents, *Journal of Public Economic Theory*, 7(1), 107-116.

[5] Duclos, J.-Y., P. Makdissi and Q. Wodon (2005b), Poverty-Dominant Transfer Programs: The Role of Targeting and Allocation Rules, *Journal of Development Economics*, 77(1), 53-73.

[6] Duclos, J.-Y., P. Makdissi and Q. Wodon (2006), Socially-Improving Tax Reforms, mimeo.

[7] Fishburn, P. C. and R. D. Willig (1984), Transfer Principles in Income Redistribution, *Journal of Public Economics*, 25(3), 323-328.

[8] Foster, J. E., J. Greer and E. Thorbecke (1984), Notes and Comments. A Class of Decomposable Poverty Measures, *Econometrica*, 52, 761-766.

[9] Jenkins, S.P. and P.J. Lambert (1993), Ranking Income Distributions when Needs Differ, *Review of Income and Wealth*, 39, 337-356.

[10] Makdissi, P. and Q. Wodon (2002), Consumption Dominance Curves: Testing for the Impact of Indirect Tax Reforms on Poverty, *Economics Letters*, 75, 227-235.

[11] Makdissi, P. and Q. Wodon (2007), Poverty-Reducing and Welfare-Improving Marginal Public Price and Price Cap Reforms, forthcoming in *Journal of Public Economic Theory.*

[12] Sen, A. (1992), *Inequality Reexamined*, Cambridge: Harvard University Press.

[13] Sen, A. (1997), *On Economic Inequality*, Oxford: Clarendon Press.

[14] Van Kerm, P. (2004), What Lies Behind Income Mobility? Reranking and Distributional Change in Belgium, Western Germany and the USA, *Economica*, 71, 223-239.

[15] Wildasin, D.E. (1984), On Public Good Provision With Distortionary Taxation, *Economic Inquiry*, 22, 227-243.

[16] Yitzhaki, S. and W. Thirsk (1990), Welfare Dominance and the Design of Excise Taxation in the Côte d'Ivoire, *Journal of Development Economics*, 33, 1-18.

[17] Yitzhaki, S. and J. Slemrod (1991), Welfare Dominance: An Application to Commodity Taxation, *American Economic Review*, 81, 480-496.

[18] Zheng, B. (1999), On the Power of Poverty Orderings, *Social Choice and Welfare*, 3, 349-371.