The Welfare Gains of Trade Integration in the European Monetary Union

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Abstract

This paper evaluates the welfare gains arising from a deeper trade integration in the European Monetary Union. To do this, the European Monetary Union is represented in a realistic way by an intertemporal general equilibrium model with incomplete financial markets, sticky prices and home bias in both private consumption and production. The model is estimated and not rejected by the data. Two main results emerge: (i) an increase in vertical (intermediate goods) trade implies welfare gains while (ii) an increase in horizontal (final goods) trade implies welfare losses.

Keywords: trade integration, inflation differentials, welfare analysis, optimal currency areas.

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1 Introduction

This article investigates the welfare effects of a deeper horizontal or vertical trade integration in the European Monetary Union (EMU). Independently of its long-run

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consequences, trade integration usually generates welfare gains through an increased correlation of business cycles and an improved overall adequacy of the common monetary policy to national situations. This paper shows that the picture is more contrasted in a sticky prices environment when financial markets are incomplete and imperfectly integrated.

We lay out an estimated two-country DSGE model of the European Monetary Union (EMU) that accounts for the imperfect integration of both goods and financial markets. As in Ricci (1997), the model encompasses real and monetary arguments for the costs of conducting a single monetary policy in a monetary union characterized by business cycles asymmetries and inflation differentials. Indeed, the model features home bias in private consumption and production technology, incomplete and imperfectly integrated private financial markets, Calvo type sticky prices, and i.i.d. productivity and public spending shocks. These assumptions are also set up to be consistent with the current economic situation of the EMU, characterized by persistent asymmetries in business cycles and significant inflation differentials (see Camacho, Perez-Quiros and Saiz, 2006 and Lane, 2006 for discussions).

In this tractable framework, asymmetric shocks imply asymmetries in business cycles and inflation differentials that can not be addressed by the central bank of the monetary union. These business cycle asymmetries and inflation differentials translate into welfare costs, building on two mains sources: nominal inertia and imperfect risk-sharing combined with a costly access to financial markets. The role of nominal inertia in a monetary union, as well as means to reduce the associated costs, have already been extensively studied in the literature (see among others Bengino, 2004; Beetsma and Jensen, 2005 and Galí and Monacelli, 2008). Less attention has been paid to welfare losses related to imperfectly integrated financial markets in a monetary union. In line with Carré and Collard (2003), we show that imperfect risk-sharing crucially affects the welfare costs of business cycles asymmetries and the size, sign and structure of welfare gains generated by trade integration.

First, we show that horizontal and vertical trade integration both reduce the volatility of national inflation rates. The increase of mutual trade flows improves the correlation of business cycles and the overall adequacy of the common monetary policy to national situations. These results are in the spirit of those of Romer (1993), but the mechanisms that lead to these effects are different.¹

Second, vertical and horizontal trade integration have opposite effects on the pattern of external adjustment to asymmetric shocks. Vertical trade integration reduces the overall need for external adjustment, i.e. the volatility of the current account while horizontal trade increases it. Because financial markets are incomplete and imperfectly integrated, a higher (respectively lower) volatility of the current account increases (re-

¹ According to Romer, loose monetary policies become less appealing when openness increases.
spectively dampens) the welfare costs related to the imperfect integration of financial markets and imperfect risk-sharing.

The result builds on the following mechanism. Using financial markets (the current account) more intensively to smooth their wealth over time, households increase (respectively decrease) their labor supply when contracting debts (respectively accumulating assets) to respect their intertemporal budget constraint. Since we consider a closed monetary union, debts contracted by the representative household of a country are necessarily equal to the cumulated assets of the representative household of the other country. As a result, increased movements of the current account are met by opposite changes in labor supplies. In the equilibrium, the distance of relative hours, relative wages and intermediate terms-of-trade to their natural equilibrium path (characterized by complete financial markets) increases.

Taking their effects on national inflation rates and on the pattern of external adjustment into account, we show that vertical and horizontal trade integration have contrasted effects on the aggregate welfare. Quantitatively speaking, vertical trade integration leads to important welfare gains for the whole range of possible parameters of the model. In the baseline estimation, we show that a 10% increase of vertical trade implies an average welfare gain equivalent to a 12.76% rise of permanent consumption for a constant labor effort. On the other hand, horizontal trade generates lower welfare gains or welfare losses, depending on the parametrization. In the baseline estimation, a 10% increase of horizontal trade implies an average welfare loss equivalent to a 2.09% drop of permanent consumption. A sensitivity analysis shows that horizontal trade can lead to welfare gains, especially when the welfare costs of nominal rigidities increase or when the welfare costs of imperfect financial integration decrease.

The remaining of the paper is organized as follows. Section 2 describes a two-country model of an imperfectly integrated monetary union. Based on EMU data, Section 3 provides estimates for the structural parameters of the log-linear approximation of the model. The dynamics properties of the model are analyzed in Section 4. Section 5 provides an extensive welfare analysis of an increase in trade integration and presents some sensitivity analysis. A last section offers some concluding remarks.

2 An imperfectly integrated monetary union

The model describes a two-country world with a common currency. Each nation represents half of this monetary union. It is populated by N infinitely-living households, a government, and firms producing intermediate and final goods. All goods are traded. Monetary policy is delegated to the central bank of the monetary union which controls

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2 This increase fits the actual consensus concerning the effect of the EMU on intrazone trade (see Baldwin, 2006).
the nominal interest rate. The international financial market is incomplete and agents only trade one-period composite bonds.\footnote{Nominal exchange rate issues per se as well as the analysis of the conditions underlying the adoption of a common currency are beyond the scope of the paper.}

### 2.1 Households and national governments

In each country the number of infinitely-living households is normalized to one. The representative household \( j \in [0, 1] \) of nation \( i \in \{h, f\} \) maximizes a welfare index,

\[
\sum_{t=0}^{\infty} \beta^t E_0 \left\{ \frac{C_i^t(j)^{1-\sigma}}{1-\sigma} - \frac{N_i^t(j)^{1+\psi}}{1+\psi} \right\},
\]

subject to,

\[
B_{i+1}^t(j) - R_t B_i^t(j) = W_i^t N_i^t(j) + \Pi_i^t(j) - P_i^t C_i^t(j) - T_i^t(j) - P_{i,t} AC_i^t(j),
\]

and the following transversality condition,

\[
\lim_{T \to \infty} \Pi^T_{t=1} R_s^{-1} E_t \left\{ B_{i+1}^T(j) \right\} = 0.
\]

In Eq. (1), \( \beta \) is the subjective discount factor, \( \sigma \) is the intertemporal elasticity of substitution of private consumption and \( \psi \) is the inverse of the Frisch elasticity. The aggregate consumption bundle of agent \( j \) in country \( i \) is called \( C_i^t(j) \) and the quantity of labor that this agent supplies on the labor market \( N_i^t(j) \). Money holdings are not introduced in the utility function since the money market plays no role for the dynamics when the nominal interest rate is the monetary policy instrument (see Beetsma and Jensen, 2005).

In Eq. (2), \( B_i^t(j) \) is the amount of one-period nominal bonds held by the representative agent of country \( i \) at the end of period \( t-1 \), that pays a gross nominal rate of interest \( R_t \) between periods \( (t-1) \) and \( t \). The consumer price index (CPI) in country \( i \) is called \( P_i^t \) while \( P_{i,t} \) corresponds to the producer price index (PPI) in country \( i \). \( W_i^t \) is the nominal wage in country \( i \) in period \( t \), \( \Pi_i^t(j) = \int_0^1 \Pi_i(k, j) dk \) is the amount of profits paid by monopolistic final goods producers, and \( T_i^t(j) \) is a lump-sum transfer. Finally, in the budget constraint, \( AC_i^t(j) \) is a quadratic portfolio adjustment cost that households have to pay to financial intermediaries to access financial markets. The cost is defined according to,

\[
AC_i^t(j) = \frac{\chi}{2} \left[ B_{i+1}^t(j) - B_i^t(j) \right]^2,
\]
where $B^t(j)$ is the steady state level of net foreign assets. The Euler condition that solves Eqs. (1)-(2) is affected by portfolio adjustment costs since,

$$\frac{\beta R_{t+1}}{1 + \chi P_{t+1}(B^t(j) - B^t(j))} E_t \left\{ \frac{P^t_i C^i_t(j)^\sigma}{P^t_{t+1} C^i_{t+1}(j)^\sigma} \right\} = 1. \quad (3)$$

The portfolio adjustment cost parameter ($\chi$) affects the sensitivity of wealth’s accumulation to a variation of the interest rate, as it becomes more or less costly to smooth consumption by accessing financial markets. For instance, when $\chi$ decreases, it is less costly for the households to access the financial markets. The labor supply function is based on the traditional consumption-leisure arbitrage,

$$N^i_t(j)^\sigma C^i_t(j)^\sigma = W^i_t. \quad (4)$$

We assume that preferences in private consumption are affected by a home bias $(1 - \alpha)$ in favor of nationally produced goods. The aggregate consumption of consumer $j$ living in country $i$, $C^i_t(j)$ is defined according to,

$$C^i_t(j) = \left(1 - \alpha \right)^{\frac{1}{\mu}} \left(C^i_{h,t}(j) \right)^{\frac{\mu - 1}{\mu}} + \alpha \left(C^i_{f,t}(j) \right)^{\frac{\mu - 1}{\mu}} \left(1 - \frac{1}{\mu} \right).$$

and the companion consumption price index $P^i_t$ is,

$$P^i_t = \left[(1 - \alpha) \left(P^i_{h,t} \right)^{1-\mu} + \alpha \left(P^i_{f,t} \right)^{1-\mu} \right]^{\frac{1}{1-\mu}}.$$  

In these expressions, the consumption of home and foreign goods $C^i_{h,t}(j)$ and $C^i_{f,t}(j)$ are defined as standard bundles of varieties indexed in $k$ and $\mu$ is the elasticity of substitution between home and foreign goods. It has now become standard to regard $2\alpha$ as a relevant indicator of goods market openness since, in equilibrium, $\alpha$ is the share of imported goods in total private consumption (see Corsetti, 2006 and Galí and Monacelli, 2005). In appendix A, we briefly show that home bias specifications are closely related to iceberg trade costs specifications. The relation between standard iceberg shipping costs $c \in [0,1]$ and home bias $(1 - \alpha)$ is,

$$1 - \alpha = \frac{1}{1 + (1 - c)^{\mu}}. \quad (5)$$

Figure 1 plots the values of $\alpha$ implied by different iceberg shipping costs for different values of the elasticity of substitution between goods.

The relation between iceberg shipping costs and home bias $(1 - \alpha)$ is strictly positive and monotonic, i.e. lower iceberg costs are equivalent to lower home bias. This relation is
clearly consistent with the data. For example, focusing on the US, iceberg costs reaching an average 60% – consistent with the estimation of Anderson and Van Wincoop (2004), yield an average 0.72% home bias with a unitary elasticity of substitution or an average 0.86% home bias when \( \mu = 2 \). The values obtained for homes bias measures \((1 - \alpha)\) are consistent with standard estimates of openness \((2\alpha)\) for the US (see Coeurdacier, 2008) and for the EMU (see Andrè, Ortega and Vallés, 2008). These results are obtained for values of \( \mu \) in the range of estimates based on aggregate time series (see Backus, Kehoe and Kydland, 1993). Figure 1 shows that home bias could be much higher for (higher) micro data-based estimations of \( \mu \).

Finally, we assume that the law of one price holds in the monetary union \((P_{h,t} = P_{h,t} \text{ and } P_{f,t} = P_{f,t})\) and define final terms-of-trade in the monetary union as,

\[
S_t = \frac{P_{f,t}}{P_{h,t}}.
\]

### 2.2 Governments

Governments choose the amount of public spending on the final goods market and balance their budget using lump-sum transfers. The budget constraint of the government
is given by,
\[ \int_0^1 T^i(j) dj + \tau \int_0^1 P_{i,t}(k) Y^i_t(k) dk = P_{i,t} G^i_t, \]
where \( \tau \) is a proportional subsidy to firms. Mixing monopolistic competition and Calvo staggered price contracts on final goods markets introduces several distortions with respect to the Pareto-efficient equilibrium. Nominal rigidities imply inefficient fluctuations of both equilibrium inflation and output while the assumption of monopolistic competition affects the steady state. While monetary and/or fiscal policy may address the first issue, an optimal subsidy \( \tau \) is able to address the second issue and restores the first-best allocation in the steady state (see Benigno and Woodford, 2005).

National public spending are entirely home biased, i.e.,
\[ G^i_t = \left[ \int_0^1 G^i_t(k) \frac{q-1}{\sigma} dk \right]^{\frac{\sigma}{q-1}}, \]
where the level of aggregate public spending in country \( i \) evolves according to,
\[ G^i_{t+1} = (1 - \rho_g) G^i_t + \rho_g G^i_t + \zeta^i_{g,t+1}, \]
where \( \zeta^i_{g,t} \) is an i.i.d. innovation.

2.3 Firms

The production of final goods consists in a two-step process. First, in each country \( i \), a continuum of identical firms (normalized to one) produce the same intermediate good and sell it on a competitive market. The production function of these firms is given by,
\[ X^i_t = A^i_t L^i_t. \]

\( L^i_t \) is the labor demand and \( A^i_t \) is the level of labor productivity evolving according to,
\[ A^i_{t+1} = (1 - \rho_a) A^i_t + \rho_a A^i_t + \zeta^i_{a,t+1}, \]
where \( \zeta^i_{a,t} \) is an i.i.d. innovation.

Second, intermediate goods are traded within the monetary union and combined by monopolistic final goods producers \( k \in [0,1] \). Intermediate goods are sold at their marginal cost \( W^i_t / A^i_t \) and intermediate terms-of-trade are,
\[ \Sigma_t = \frac{W^f_t / A^f_t}{W^h_t / A^h_t}. \]
The production function of final goods producer \( k \) located in country \( i \) is given by,

\[
Y^i_t(k) = \left[ (1 - \gamma_i)^{\frac{1}{\phi}} X^i_{h,t}(k)^{\frac{\phi - 1}{\phi}} + (\gamma_i)^{\frac{1}{\phi}} X^i_{f,t}(k)^{\frac{\phi - 1}{\phi}} \right]^{\frac{1}{\phi - 1}}. \tag{6}
\]

In this expression, \( X^i_{h,t}(k) \) is the consumption of intermediate goods produced in country \( h \) of firm \( k \) located in country \( i \). The parameter \( \phi \) is the elasticity of substitution between home and foreign intermediate goods. The parameter \((1 - \gamma_i) \in [0, \frac{1}{2}]\) is the home bias in production. This parameter also introduces a link between iceberg shipping costs and the trade of intermediate goods. Regarding the equivalence relation (5), notice that \( 2\gamma_i \) is also a measure of openness of trade of intermediate goods, i.e. vertical trade. The companion nominal marginal cost of firm \( k \) in country \( i \), \( MC^i_t(k) \), is given by,

\[
MC^i_t(k) = \left[ (1 - \gamma_i) \left( W^i_h / A^i_h \right)^{1 - \phi} + (\gamma_i)^{\frac{1}{\phi}} \left( W^i_f / A^i_f \right)^{1 - \phi} \right]^{\frac{1}{1 - \phi}}. 
\]

Final goods prices are governed by standard Calvo contracts. Each period, only a fraction \((1 - \eta^i)\) of randomly selected firms located in country \( i \in \{ h, f \} \) are allowed to set new prices. These firms choose the following optimal price \( P^i_{t,t}(k) \),

\[
P^i_{t,t}(k) = \theta \frac{\sum_{v=0}^{\infty} \left( \eta^i \beta \right)^v E_t \left\{ Y^i_{t+u}(k) MC^i_{t+u} \right\}}{(\theta - 1)(1 - \tau)} \frac{\sum_{v=0}^{\infty} \left( \eta^i \beta \right)^v E_t \left\{ Y^i_{t+u}(k) P^i_{t+u} C^i_{t+u} \right\}}{P^i_{t+u} C^i_{t+u} \left( \eta^i P^i_{t+u} \right)^\phi}. 
\]

Aggregating among final firms and assuming behavioral symmetry of monopolistic producers, the average producer price level of final goods in country \( i \in \{ h, f \} \) is,

\[
P^i_{t,t} = \left[ (1 - \eta^i) P^i_{t,t}(k) \right]^{1 - \phi} + \eta^i P^i_{t,t-1} \right]^{\frac{1}{1 - \phi}}. 
\]

2.4 Monetary policy

The central bank of the monetary union sets the nominal interest rate according to the following rule,

\[
R_{t+1} = (1 - \rho_c) R + \rho_c R_t + \varphi \left( \pi^u_t - \pi^w \right),
\]

where \( \pi^u_t = \frac{1}{2} \pi^h_t + \frac{1}{2} \pi^i_t \) and \( \pi^i_t = P^i_t / P^i_{t-1} \). This rule is commonly used in the literature (see among others Taylor, 1993; Clarida, Galí and Gertler, 1998 and Rudebusch and Svensson, 1999). Furthermore, it is a fair approximation of the monetary policy of the European Central Bank with respect to its mission, i.e. the stabilization of aggregate inflation in the EMU. Finally, the rule features a smoothing parameter \( \rho_c \) since a large empirical literature underlines the smoothness of the nominal interest rate variations in the Euro area (see among others Peersman and Smets, 1999 and Gerlach and Schnabel, 2000).
2.5 Markets equilibrium

We solve the model assuming that each country is the mirror image of the other on the goods market. Posing $\alpha_h = \alpha$ and $\gamma_h = \gamma$ we simply get $\alpha_f = (1 - \alpha)$ and $\gamma_f = (1 - \gamma)$.

A competitive equilibrium is defined as a sequence of quantities,

$$\{Q_t\}_{t=0}^{\infty} = \{C^h_t, C^f_t, N^h_t, N^f_t, Y^h_t, Y^f_t, L^h_t, L^f_t, B^h_{t+1}, B^f_{t+1}, AC^h_t, AC^f_t\},$$

and a sequence of prices,

$$\{P_t\}_{t=0}^{\infty} = \{\bar{P}_{h,t}(k), \bar{P}_{f,t}(k), P_{h,t}, P_{f,t}, P^h_t, P^f_t, W^h_t, W^f_t, R_{t+1}\},$$

such that:

(i) For a given sequence of exogenous shocks $\{S_t\}_{t=0}^{\infty} = \{A^h_t, A^f_t, G^h_t, G^f_t\}$ and prices $\{P_t\}_{t=0}^{\infty}, \{Q_t\}_{t=0}^{\infty}$ respects households first order conditions and maximizes the profits of final and intermediate goods producers.

(ii) For a given sequence of shocks $\{S_t\}_{t=0}^{\infty}$ and quantities $\{Q_t\}_{t=0}^{\infty}, \{P_t\}_{t=0}^{\infty}$ clears final and intermediate goods markets, labor markets,

$$N^i_t = \int_0^1 N^i_t(j) dj = L^i_t,$$

and financial markets,

$$\int_0^1 B^h_t(j) dj + \int_0^1 B^f_t(j) dj = 0.$$

In the equilibrium, net foreign assets evolve as follows,

$$B^h_{t+1} - B^h_t = (R_t - 1) B^h_t + \alpha \left( P^f_t C^f_t - P^h_t C^h_t \right) + \gamma \left( MC^f_t Y^f_t DP^f_t - MC^h_t Y^h_t DP^h_t \right),$$

where $DP^i_t$ is the dispersion of final goods production prices in country $i$.

3 Estimation

We estimate the log–linear version of the model using the Simulated Method of Moments (SMM) of Hansen (1982). In the symmetric competitive flexible price steady state, we assume that $A^i = A = 1$ and that $\tau = (1 - \theta)^{-1}$. Other steady state relations are given by,

$$Y = (1 - \kappa)^{-\frac{\tau}{\tau + \sigma}}, \ C = (1 - \kappa)^{\frac{\phi}{\tau + \sigma}}, \ G = \kappa (1 - \kappa)^{-\frac{\tau}{\tau + \sigma}},$$

$$N = (1 - \kappa)^{-\frac{\phi}{\tau + \sigma}}, \ W/P = 1 \ and \ R = \beta^{-1}.$$

The log–linear approximation of the model is presented in Table B.1 in the appendix.
We use quarterly data from EMU countries (OECD Economic Outlook quarterly database) posterior to the German reunification, i.e. ranging from 1992 to 2006. Aggregates are converted in the same currency and we focus on the following seasonally adjusted series: GDP (without investment), private consumption, employment, GDP deflator, trade balance and current account balance (as a percentage of GDP). We also take into account the evolution of the average nominal short-term interest rate in the EMU.

We build two regions based on the levels of nominal rigidities of EMU countries (see Benigno, 2004). Table 1 indicates the percentage of goods prices in the consumer price index changing every month in EMU countries (data are borrowed from Alvarez et al., 2006). We consider that countries in which less than 15% of CPI goods prices change every month belong to the group of high nominal rigidities and countries in which more than 15% of CPI goods prices change every month belong to the group of low nominal rigidities. Consequently, in the first group (region $h$ in the model), we have Germany, Spain and Italy and in the second group (region $f$ in the model), we have all remaining countries.\(^5\)

<table>
<thead>
<tr>
<th>Region</th>
<th>% goods in the CPI</th>
<th>% of country’s GDP in the EMU GDP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Germany</td>
<td>$h$</td>
<td>13.5</td>
</tr>
<tr>
<td>France</td>
<td>$f$</td>
<td>23.9</td>
</tr>
<tr>
<td>Italy</td>
<td>$h$</td>
<td>10.0</td>
</tr>
<tr>
<td>Spain</td>
<td>$h$</td>
<td>13.3</td>
</tr>
<tr>
<td>Netherlands</td>
<td>$f$</td>
<td>16.2</td>
</tr>
<tr>
<td>Belgium</td>
<td>$f$</td>
<td>17.6</td>
</tr>
<tr>
<td>Luxembourg</td>
<td>$f$</td>
<td>23.0</td>
</tr>
<tr>
<td>Finland</td>
<td>$f$</td>
<td>20.3</td>
</tr>
<tr>
<td>Portugal</td>
<td>$f$</td>
<td>21.1</td>
</tr>
</tbody>
</table>

Once both regions of the monetary union are defined, we aggregate series given the relative time-varying weights of countries in terms of GDP in the region. Inflation rates are computed using GDP deflators. Finally, we take the log of GDP, consumption and employment and detrend all series using the HP-filter. We estimate the model using a large sample of second order moments. We focus on three types of moment: standard

\(^5\) Austria, Greece and Ireland are not taken into account because data are unavailable or not reliable.
deviations (absolute or relative to standard deviation of output), first-order autocorrelations and cross-correlations. Standard deviations and autocorrelations concern all variables and cross-correlations are those of output with private consumption, output with hours and private consumption with hours.

Some parameters of the model are calibrated. We set $\beta = 0.988$, which corresponds to an annual steady state real interest rate of 4.7%, consistent with the average real interest rate over the corresponding period in the EMU. Following Rotemberg and Woodford (1997), the elasticity of substitution between varieties is $\theta = 7$, implying an average 16-17% steady state mark-up (compensated at the equilibrium by the optimal subsidy). The average share of public spending in the GDP is set to $\kappa = 0.25$ (see Galí and Monacelli, 2008). The elasticity of substitution between intermediate goods is $\phi = 1.5$ (see Hairault, 2002). Finally, we calibrate parameters of the nominal interest rate rule using standard values for the smoothing parameter $\rho_r = 0.7$ and for the feedback coefficient on aggregate inflation $\varphi = 1.5$ (see Gerlach and Schnabel, 2000).

Other parameters are estimated. The results of the estimation are reported in Table 2. The test allowed by over-identifying conditions implies a 0.8035% p-value, which indicates that the model is not rejected by the data.

<table>
<thead>
<tr>
<th>Estimated parameters</th>
<th>$\psi$</th>
<th>$\sigma$</th>
<th>$\alpha$</th>
<th>$\gamma$</th>
<th>$\chi$</th>
<th>$\eta_h$</th>
<th>$\eta_f$</th>
<th>$\mu$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>7.0776*</td>
<td>1.8111*</td>
<td>0.2675*</td>
<td>0.0509*</td>
<td>0.0009*</td>
<td>0.5023*</td>
<td>0.5024*</td>
<td>2.9344*</td>
</tr>
<tr>
<td>$\rho_a$</td>
<td>$\rho_g$</td>
<td>$\text{std}(\zeta_{a,t})$</td>
<td>$\text{std}(\zeta_{g,t})$</td>
<td>$J - \text{stat}$</td>
<td>$\text{Ov. Id. Stat.}$</td>
<td>$\chi^2(16)$</td>
<td>$p - \text{value}$</td>
<td></td>
</tr>
<tr>
<td>0.9525*</td>
<td>0.8862*</td>
<td>0.0079*</td>
<td>0.0099*</td>
<td>10.0875</td>
<td>0.8035</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*: 99% significant

Parameter values are consistent with most estimates or calibrations reported in the literature and are significant. The inverse of the Frisch elasticity $\psi$ is equal to 7.0776 and lies on the upper bound of the range put forth by Canzoneri, Cumby and Diba (2006). This value is consistent with a sluggish response of labour supply to various shocks in the EMU. The intertemporal elasticity of substitution of private consumption is $\sigma = 1.8111$, close to standard values (see Benigno, 2004). This parameter governs both the intensity of the transmission of monetary policy through the sensitivity of consumption to the real interest rate and the arbitrage between leisure and consumption. Home bias parameters are $\gamma = 0.0509$ and $\alpha = 0.2675$ and determine the degree of trade openness of intermediate and final goods markets. These values are consistent with those found in Faia (2007) and with standard openness measures calculated using EMU data. The estimation of $\chi = 0.0009$ is not far from Schmitt-Grohé and Uribe (2003). It implies that households have to pay an average annual 0.36% interest rate premium to access financial markets. Nominal rigidities parameters are very close since $\eta_h = 0.5024$ and $\eta_f = 0.5023$. Our estimation is lower than usual estimations, as in Clarida, Gali and Gertler (1998), but matches the values put forth in Alvarez et al. (2006). Finally,
parameters governing shocks’ processes are $\rho_a = 0.9525$, $\rho_g = 0.8862$, $std(\zeta_{a,t}) = 0.79\%$ and $std(\zeta_{g,t}) = 0.99\%$. These estimations are consistent with most values found in the RBC literature.

4 Dynamic properties

In this section we study the dynamic properties of the economy when facing asymmetric productivity and public spending shocks.

Figure 4 plots the Impulse Response Functions (IRFs) to a positive unit productivity shock in the home country.

Fig. 2. IRFs to a unit productivity shock in country $h$

Output rises in both countries, while more substantially in country $h$, peaking at 0.7% for a 1% productivity shock. In country $h$, the remaining of productivity gains is used to reduce the labor effort, about 0.25% on impact. This effect arises because the wealth effect dominates in models with separable utility functions and without physical capital. The wealth effect is reinforced by the 0.17% drop of the PPI inflation in the home country.
The transmission of the shock in country $f$ draws on both trade flows and monetary policy. While agents in country $h$ sustain higher production and consumption levels, they generate intermediate and final trade flows within the monetary union, which induces a positive reaction of the output in country $f$ of about 0.3% on impact. Monetary policy also favors a positive transmission. By reacting to aggregate inflation, the central bank lowers its equilibrium nominal interest rate, which induces an increase of aggregate consumption and output in country $f$. The supply shock in country $h$ thus translates into a positive demand shock in country $f$, which generates some PPI inflation, peaking at 0.12% on impact and returning quickly to the steady state.

Since wages and production prices drop in country $h$ and rise in country $f$, both intermediate and final terms-of-trade improve in country $h$. Final goods prices are sluggish, which implies an undershooting of final terms-of-trade with respect to the fluctuation of intermediate terms-of-trade. Finally, agents in country $h$ accumulate net foreign assets, implying an improvement of the current account peaking at 16-17% of steady state consumption on impact.

Figure 3 plots the IRFs to a positive unit public spending shock in the home country.

Fig. 3. IRFs to a unit public spending shock in country $h$

Output increases by 0.15% on impact in country $h$, implying a rise of both home and foreign labor supply, required to sustain the quantity of final goods demanded in coun-
try $h$. Private consumption drops steadily in both countries. The drop reaches 0.13% in country $h$, because of the crowding-out effect and because Ricardian equivalence holds. The drop is more gentle in country $f$, reaching 0.08% on impact. Since global demand drops in country $f$, output clearly falls by 0.07% on impact while returning very quickly to the steady state. Mechanisms behind the negative transmission of a public spending shocks in country $h$ build on a traditional beggar-thy-neighbor effect and highlight the role of home bias in public spending in a first approach. However, this effect also relies ($i$) on the fall of private consumption in country $h$, implying a drop of country $h$ imports from country $f$ and ($ii$) on the increase of nominal interest rate implied by the reaction of the central bank to the PPI inflation of country $h$. The positive demand shock in country $h$ thus translates into a negative demand shock in country $f$.

External adjustment implies a steady deterioration of final and intermediate terms-of-trade and an accumulation of net foreign liabilities in country $h$. The corresponding deficit of the current account peeks at 4.5-5% of steady state consumption on impact.

5 The welfare gains of trade integration

In this section, we measure the welfare gains arising from a deeper horizontal of vertical trade integration in the EMU.

5.1 Welfare indicators

We build an explicit welfare indicator using a second-order approximation of the aggregate utility function. The welfare measure is a standard discounted sum of utility flows,

$$\omega = -\frac{q}{2} \sum_{t=0}^{\infty} \beta^t E_0 \{ \ell_t \} + \text{t.i.p} + O \left( \| \xi^3 \| \right).$$

where $q = (1 - \kappa)^{-\sigma(1-\psi)}$, $t.i.p$ gathers terms independent of the problem and $O \left( \| \xi^3 \| \right)$ are terms of order 3 or higher. In this expression, the instant contribution to welfare losses $\ell_t$ is a quadratic function of deviations of key economic variables from their natural equilibrium path,

$$\ell_t = \frac{\theta}{2k^h} \left( \pi_{h,t} - \tilde{\pi}_{h,t} \right)^2 + \frac{\theta}{2k^f} \left( \pi_{f,t} - \tilde{\pi}_{f,t} \right)^2 + \frac{\sigma + \psi \left( 1 - \kappa \right)}{\left( 1 - \kappa \right)} \left( y^u_t - \tilde{y}^u_t \right)^2 + \left( 1 - \kappa \right) \psi \left( 1 - \kappa \right) \left( y^s_t - \tilde{y}^s_t \right)^2,$$

where $k^i = \left( \frac{1-\eta^i}{\eta^i} \right)^{(1-\eta^i)}$. In Eq. (7), a tilde denotes the path of variables in the natural equilibrium, defined as the equilibrium under flexible prices and complete and perfectly...
integrated asset markets, i.e. $\eta^h = \eta^f = 0$ and $\chi = 0$. Superscripts $u$ and $r$ respectively stand for aggregate and relative variables.

The welfare measure $\omega$ penalizes national PPI inflation rates, the aggregate output gap, the relative consumption gap, the relative hours gap and terms-of-trade gaps. The weights affected to national inflation rates are sensitive to the degree of price stickiness through the values of $k^i$. Parameter $k^i$ depends negatively on the degree of price rigidities, so that higher weights are deferred to inflation rates when prices are stickier.

We compute the consumption equivalent welfare loss. As in Beetsma and Jensen (2005), it is defined as,

$$\Psi = 100 \cdot \left[ \frac{(1 - \beta)}{(1 - \kappa) (\sigma + \psi (1 - \kappa))} (\omega_1 - \omega_0) \right]^{\frac{1}{2}},$$

where $\omega_0$ measures the welfare for a given reference situation. $\Psi$ converts the welfare gains associated to a pareto-superior equilibrium $\omega_1$ into a sizable yardstick in terms of permanent increase of consumption for an unchanged labor effort.

5.2 Baseline scenario

Running simulations using the baseline estimation, Table 3 contrasts the welfare gains or losses ($\Psi$) arising from a deeper horizontal or vertical trade integration consistent with the evidence documented by Baldwin (2006), i.e. a 10% increase of $\alpha$ or $\gamma$. The table also details the evolution of the volatility of variables entering in the welfare loss function.

<table>
<thead>
<tr>
<th></th>
<th>$\Psi(%)$</th>
<th>$\hat{\pi}_{h,t}$</th>
<th>$\hat{\pi}_{f,t}$</th>
<th>$\hat{y}^c_t$</th>
<th>$\hat{s}_t$</th>
<th>$\hat{\sigma}_t$</th>
<th>$\hat{c}_t$</th>
<th>$\hat{n}_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Baseline</strong></td>
<td>–</td>
<td>0.188</td>
<td>0.190</td>
<td>0.047</td>
<td>0.731</td>
<td>1.925</td>
<td>1.002</td>
<td>0.308</td>
</tr>
<tr>
<td>$\alpha = 0.2397$</td>
<td>–1.81</td>
<td>0.186</td>
<td>0.187</td>
<td>0.047</td>
<td>0.665</td>
<td>1.963</td>
<td>1.008</td>
<td>0.311</td>
</tr>
<tr>
<td>Variation(%)</td>
<td>–1.50</td>
<td>–1.49</td>
<td>0.00</td>
<td>–9.12</td>
<td>1.98</td>
<td>0.64</td>
<td>1.17</td>
<td></td>
</tr>
<tr>
<td>$\gamma = 0.0704$</td>
<td>13.92</td>
<td>0.185</td>
<td>0.187</td>
<td>0.047</td>
<td>0.721</td>
<td>1.786</td>
<td>0.996</td>
<td>0.300</td>
</tr>
<tr>
<td>Variation(%)</td>
<td>–1.53</td>
<td>–1.54</td>
<td>–0.00</td>
<td>–1.38</td>
<td>–7.20</td>
<td>–0.56</td>
<td>–2.54</td>
<td></td>
</tr>
</tbody>
</table>

Note: variables with a hat denote deviations from natural equilibrium

On one hand, a 10% increase of $\gamma$ generates large welfare gains, equivalent to an average 13.92% increase of permanent consumption. The overall volatility of terms entering the loss function is clearly dampened. When vertical trade increases, the composition of
final goods produced becomes more similar, which implies that shocks affecting the production process of intermediate goods asymmetrically have more similar effects on output and marginal costs. This mechanism also contributes to lower the PPI national inflation rates, as illustrated by the new Keynesian Phillips curves. If marginal costs, the driving force behind the PPI inflation rates, are more correlated, then the PPI inflation rates are affected in the same way. The adequacy of the monetary policy to national inflation rates and business cycles increases, which improves its effectiveness and reduces the volatility of national inflation rates. Finally, by the same reasoning, the volatility of intermediate terms-of-trade, relative hours and relative consumptions gaps is reduced, which translates into aggregate welfare gains.

On the other hand, in the baseline scenario, a 10% increase of horizontal trade – measured by a 10% increase of $\alpha$, implies an average welfare loss equivalent to a 2.1% fall of permanent consumption.

A close examination of volatilities shows that the distance of national inflation rates and final terms-of-trade from their natural equilibrium path is clearly reduced (respectively by 1.865% in average and 9.12%), which has welfare improving consequences. Since the composition of the CPI inflation rates and private consumption bundles becomes more similar, for a given monetary policy rule, monetary policy becomes more effective and its ability to stabilize national PPI inflation rates increases. These lower national PPI inflation rates result in a lower pressure on final terms-of-trade, which clearly reduces their volatility.

However, while external adjustment relies less on the final terms-of-trade, the volatility of the current account is enhanced, which leads to welfare losses that more than compensate the previous welfare gains. These losses stem from the increased distance of hours and intermediate terms-of-trade to their natural level. The fact that agents use the current account more intensively to adjust asymmetric shocks implies that labor supplies and wages become less correlated and that the volatility of intermediate terms-of-trade, based on relative wages, increases.

These effects arise because debtor (resp. creditor) households need to improve (resp. lower) their labor supply in order to increase (resp. decrease) their earnings and repay their debts (resp. lower their savings) in the medium run. The intensity of this effect clearly depends on the level of costs levied by financial intermediaries, that increases the sensitivity of consumptions, labor efforts and equilibrium wages to variations of net foreign assets or liabilities. A corollary to the increased volatility of relative labor efforts and intermediate terms-of-trade is the increased volatility of relative consumptions. Because the composition of consumption bundles becomes more similar, relative consumptions should be less volatile. However, the consequences of the greater use of the current account dampen this first effect, implying that relative consumptions gaps are more volatile, which is also costly in terms of welfare.
Summing up, an increase in horizontal trade reduces the intensity of inefficiencies caused by nominal rigidities but magnifies inefficiencies related to the imperfect integration of financial markets. For the baseline scenario, welfare losses are higher than welfare gains.

Our results match those of other studies that quantify the welfare gains associated to the reduction of various distortions in the economy. Canzoneri et al. (2006) estimate that the welfare costs of nominal inertia can reach 4% to 5%, mostly depending on the degree of persistence in the economy. In our model, the value of the Frisch elasticity is low, the assumption of imperfect risk-sharing adds an important source of persistence, and the estimated persistence of shocks is quite high. The overall persistence is thus important and, in line with Canzoneri et al. (2006), nominal inertia implies important welfare costs in our model. Several studies also quantify the welfare gains of financial markets integration, building on higher risk-sharing and consumption smoothing. For example, Van Wincoop (1999) finds that the welfare gains from risk-sharing range from 1% to more than 7% of consumption. Those welfare gains could actually be much higher according to previous studies using alternative methods to measure financial markets integration (see Lewis, 1996). More recently, Demyanyk and Volosovych (2008) document that the welfare gains of financial markets integration range from 1% of permanent consumption for EMU members to more than 8% for new European Union members. In our model, both sources of welfare losses (nominal inertia and imperfect risk-sharing) are combined and yield important welfare losses. As suggested by Dotsey and Ireland (1996), combining various sources of frictions in the economy actually results in important welfare losses.

### 5.3 Sensitivity analysis

We investigate the robustness of our results to a wide range of parameters variations. The simulations have been run to evaluate the sensitivity of our results to the asymmetry in the pattern of nominal rigidities. Since these simulations show that asymmetries in the pattern of nominal rigidities do not play a significant role in generating our results, they are not reported but available.

Figure 4 reports the sensitivity of welfare gains or losses associated to a 10% increase in horizontal trade to different variations in the set of structural parameters.

Figure 4 once more highlights the interaction between two effects when horizontal trade integration increases: (i) welfare gains related to the lower costs of nominal rigidities and (ii) welfare losses caused by the increased volatility of the current account. Depending on parametrizations, the overall welfare effect of horizontal trade integration is either positive or negative.

When portfolio management costs ($\chi$) fall under a certain threshold, between 0.09% and 0.1%, or when nominal rigidities are beyond 0.6, horizontal trade integration generates
welfare gains. This is the case either because the enhanced volatility of the current account becomes less costly or because the reduction of national PPI inflation rates generates higher welfare gains. These results clearly show that frictions on financial markets are a key assumption to generate our results. This assumption introduces welfare losses related to imperfect risk-sharing among members of the monetary union. The sensitivity analysis reveals that small frictions ($\chi = 0.09\%$ implies that households have to pay an average 0.36\% annual interest rate premium to access financial markets) are sufficient to mitigate the welfare gains of lower inflation rates when horizontal trade increases. One shall also notice that the welfare gains of lower national inflation rates clearly surpass welfare losses when nominal rigidities parameters take standard values of the literature (around 0.7-0.75).

The sensitivity of welfare gains/losses to variations of the elasticity of substitution between intermediate or final goods also illustrates the mechanism behind welfare gains or losses. As the elasticity of substitution between final goods ($\mu$) increases, changes in the volatility of final terms-of-trade implied by an enhanced horizontal trade integration are lower. In the equilibrium, the volatility of PPI inflation rates is thus reduced, while the impact of $\mu$ on the volatility of the current account is clearly positive (see the equation governing net foreign assets dynamics in Table B.1). Welfare gains related to lower national inflation rates are thus dampened, while welfare losses caused by the increased volatility of the current account increase. As a consequence,
net welfare gains of horizontal trade integration depend negatively on the elasticity of substitution between final goods. On the contrary, as the elasticity of substitution between intermediate goods (\(\phi\)) increases, intermediate terms-of-trade are less required to fluctuate to reach the equilibrium on intermediate goods markets, *ceteris paribus*. As a consequence, the rise of the volatility of intermediate terms-of-trade, relative hours and relative consumptions gaps are reduced when horizontal trade increases, which has a positive impact on welfare gains.

Finally, the sensitivity of welfare gains to the inverse of the Frisch elasticity (\(\psi\)) and the risk-aversion parameter (\(\sigma\)) is also investigated. When the intertemporal elasticity of labor supply (\(\psi\)) increases, the volatility of hours decreases in the equilibrium. Since welfare losses related to financial frictions depend on the response of labor supply – as documented in the previous subsection, lower responses of labor supplies imply lower overall welfare losses or higher overall welfare gains when horizontal trade integration increases. The effect of the risk-aversion parameter is somehow surprising. The risk-aversion parameter governs the willingness of households to smooth their consumption over time when undergoing unexpected asymmetric shocks, which is associated with an increased use of financial markets, and should lead to higher welfare losses. However, Figure 4 tells us that these aspects are more than compensated by the drop of the volatility of terms-of-trade and of national inflation rates. Risk-aversion is thus found to have a positive impact on the welfare gains generated by an increase in horizontal trade integration.

Second, figure 5 reports the sensitivity of welfare gains or losses associated to a 10% increase in vertical trade to different variations in the set of structural parameters.

Welfare gains generated by a 10% deeper vertical trade integration are clearly increasing with the degree of nominal rigidities (\(\eta\)), since the reduction of national inflation rates is both enhanced and more weighted in the loss function.

These gains are barely sensitive to the level of portfolio management costs (\(\chi\)), which confirms that financial markets do not play an important role when trade integration is vertical.

The welfare gains of a deeper vertical trade integration also clearly decrease with the degree of substitutability between goods. While the decrease is important when the substitutability of final goods (\(\mu\)) increases (from more than 15% of permanent consumption when \(\mu = 1\) to 10% when \(\mu = 15\)), welfare gains decline even more when the substitutability of intermediate goods (\(\phi\)) increases (from 14% of permanent consumption when \(\phi = 1\) to 0% when \(\phi = 15\)). In general, higher substitutability lowers the required variations of terms-of-trade volatility when vertical trade increases. As a consequence, as substitutability increases, changes in intermediate and final terms-of-trade volatility become very small when trade integration increases, which impacts negatively on welfare gains. This effect is much stronger for the substitutability between intermediate goods because nominal rigidities bear on final goods prices while
intermediate goods prices are flexible. When the substitutability between intermediate goods increases, the volatility of intermediate terms-of-trade gaps tend to become unaffected and the impact of vertical trade integration on welfare vanishes. Because final terms-of-trade are staggered, the welfare gains of vertical trade integration do not completely fade away.

Finally, an increase of the intertemporal elasticity of substitution of labor supply ($\psi$) has a negative impact on the welfare gains of vertical trade integration for most plausible values ($\psi > 1$). In the case of vertical trade integration, the volatility of the current account decreases. The softening effect of an increase of $\psi$ on the volatility of labor supplies thus affects welfare gains in the opposite direction. Indeed, in the equilibrium, higher values of $\psi$ imply lower volatilities of hours worked, which slows down the process of reduction of regional differences in hours and lowers welfare gains of vertical trade integration.

6 Conclusion

This paper shows that horizontal and vertical trade integration have different outcomes in terms of welfare in a monetary union characterized by business cycles asymmetries
and inflation differentials. In both cases, a deeper trade integration reduces inflation differentials by favoring a better diffusion of shocks from one country to another, through increased trade flows. This increased macroeconomic interdependence improves the effectiveness of the common monetary policy. Equilibrium national inflation rates are thus less volatile which generates welfare gains.

However, horizontal trade integration improves the volatility of the current account while vertical trade integration reduces the overall need for external adjustment in case of asymmetric shocks. As a consequence, horizontal trade integration implies welfare losses that might exceed the previous welfare gains.

For the baseline estimation presented in this paper, horizontal trade integration produces welfare losses equivalent to an average 1.8% drop of permanent consumption and vertical trade integration generates welfare gains that amount to an average 13.9% of permanent consumption. However, an extensive sensitivity analysis indicates that the relative importance of financial markets frictions and nominal rigidities plays a key role in the pattern of welfare gains or losses.

The main conclusion of the paper is that financial frictions, as well as their interactions with real and nominal rigidities should carefully be taken into account when analyzing business cycles asymmetries in open economies and/or monetary unions.

References


APPENDIX

A Iceberg costs and home bias in consumption

We firstly express relative consumptions in a framework à la Obstfeld et Rogoff (2000). Consumption bundles are defined as,

\[
C = \left( (C_h) \frac{\mu - 1}{\mu} + (C_f) \frac{\mu - 1}{\mu} \right) \frac{\mu}{\mu - 1},
\]

\[
C^* = \left( (C_h^*) \frac{\mu - 1}{\mu} + (C_f^*) \frac{\mu - 1}{\mu} \right) \frac{\mu}{\mu - 1},
\]

with,

\[
P = \left[ (P_h) (1 - \mu) \right] \frac{1}{\mu} \quad \text{and} \quad P^* = \left[ (P_h^*) (1 - \mu) \right] \frac{1}{\mu} \]

\[
P_h = (1 - c) P_h^*, \quad P_f = (1 - c) P_f,
\]

where \( c \in [0, 1] \) and \( \mu \) is the elasticity of substitution between home and foreign goods. Households solve,

\[
\begin{cases}
\min_{C_h, C_f} P_h C_h + P_f C_f \\
\text{s.t. } \left[ (C_h) \frac{\mu - 1}{\mu} + (C_f) \frac{\mu - 1}{\mu} \right] \frac{\mu}{\mu - 1} = 1
\end{cases}
\]

from which we derive the following first order conditions,

\[
P_h - \lambda \left( C_h \right)^{\frac{1}{\mu}} \frac{1}{\mu} = 0,
\]

\[
P_f - \lambda \left( C_f \right)^{\frac{1}{\mu}} \frac{1}{\mu} = 0.
\]

Combining both relations, we get,

\[
P_h^{1-\mu} + P_f^{1-\mu} - \lambda^{1-\mu} \left[ (C_f) \frac{\mu - 1}{\mu} + (C_h) \frac{\mu - 1}{\mu} \right] = 0,
\]

\[
\lambda = \left[ P_h^{1-\mu} + P_f^{1-\mu} \right] \frac{1}{\mu} = P.
\]
Home demands for home and foreign goods are then,
\[ C_h = \left( \frac{P_h}{P} \right)^{-\mu} C, \]
\[ C_f = \left( \frac{P_f}{P} \right)^{-\mu} C. \]

Defining domestic terms of trade as \( T = \frac{P^*_f}{P_h} \) and expressing relative consumptions as a function of \( T \) and \( c \), we get,
\[ \frac{C_h}{C_f} = \left( \frac{P_f}{P_h} \right)^{\mu} = \left( \frac{P^*_f}{P_h (1 - c)} \right)^{\mu} = \left( \frac{T}{1 - c} \right)^{\mu}. \] (A.1)

In a framework with home bias in consumption, we mimic this process. Consumption bundles are,
\[ C = \left[ (1 - \alpha)^{\frac{1}{\mu}} (C_h)^{\frac{\mu - 1}{\mu}} + \alpha^{\frac{1}{\mu}} (C_f)^{\frac{\mu - 1}{\mu}} \right]^{\frac{1}{\mu - 1}}, \]
\[ C^* = \left[ (1 - \alpha)^{\frac{1}{\mu}} (C_f)^{\frac{\mu - 1}{\mu}} + \alpha^{\frac{1}{\mu}} (C_h)^{\frac{\mu - 1}{\mu}} \right]^{\frac{1}{\mu - 1}}, \]
and corresponding price indexes are,
\[ P = \left[ (1 - \alpha) (P_h)^{1-\mu} + \alpha (P_f)^{1-\mu} \right]^{\frac{1}{1-\mu}}, \]
\[ P^* = \left[ (1 - \alpha) (P_f)^{1-\mu} + \alpha (P_h)^{1-\mu} \right]^{\frac{1}{1-\mu}}. \]

Households solve,
\[
\begin{align*}
\min_{C_h, C_f} & \quad P_h C_h + P_f C_f \\
\text{s.t.} & \quad \left[ (1 - \alpha)^{\frac{1}{\mu}} (C_h)^{\frac{\mu - 1}{\mu}} + \alpha^{\frac{1}{\mu}} (C_f)^{\frac{\mu - 1}{\mu}} \right]^{\frac{1}{\mu - 1}} = 1
\end{align*}
\]
from which the following first order conditions are obtained,
\[ P_h - \lambda (1 - \alpha)^{\frac{1}{\mu}} (C_h)^{-\frac{1}{\mu}} \sum_{i=1}^{\frac{1}{\mu}} = 0, \]
\[ P_f - \lambda \alpha^{\frac{1}{\mu}} (C_f)^{-\frac{1}{\mu}} \sum_{i=1}^{\frac{1}{\mu}} = 0. \]
Combining, we get,

\[(1 - \alpha) P_h^{1-\mu} + \alpha P_f^{1-\mu} - \lambda^{1-\mu} \left[ (1 - \alpha) \frac{1}{\mu} (C_f)_{\mu-1}^{\frac{1}{\mu}} + \alpha \frac{1}{\mu} (C_h)_{\mu-1}^{\frac{1}{\mu}} \right] = 0, \]

and,

\[\lambda = \left[ (1 - \alpha) P_h^{1-\mu} + \alpha P_f^{1-\mu} \right]^{\frac{1}{\mu}} = P.\]

Home demands for home and foreign goods are then,

\[C_h = (1 - \alpha) \left( \frac{P_h}{P} \right)^{\mu-1} C,\]
\[C_f = \alpha \left( \frac{P_f}{P} \right)^{\mu-1} C.\]

Posing \(T = \frac{P_f}{P_h} = \frac{P_f}{P_h}^*\) (since the law of one price holds in this monetary union setting), relative consumptions are,

\[\frac{C_h}{C_f} = \frac{(1 - \alpha)}{\alpha} T^{\mu}. \quad (A.2)\]

In Eqs. (A.1) et (A.2), \(T\) stands for domestic terms of trade. We deduce the following relation between trade costs and home bias,

\[c = 1 - \left[ \frac{\alpha}{(1 - \alpha)} \right]^{\frac{1}{\mu}} \text{ or } \alpha = \frac{(1 - c)^{\mu}}{1 + (1 - c)^{\mu}}. \quad (A.3)\]

The relation between trade costs and home bias is checked, therefore.
B A log–linear approximation of the model

Table B.1
A log–linear approximation of the model

Euler equations and labor supply

\[
\sigma E_t \left\{ c_{t+1}^h - c_t^h \right\} = r_{t+1} - E_t \left\{ (1 - \alpha) \pi_{h,t+1} + \alpha \pi_{f,t+1} \right\} - \chi (1 - \kappa) \frac{\psi}{\bar{\psi}} b_{t+1}^h
\]

\[
\sigma E_t \left\{ c_{t+1}^f - c_t^f \right\} = r_{t+1} - E_t \left\{ (1 - \alpha) \pi_{f,t+1} + \alpha \pi_{h,t+1} \right\} + \chi (1 - \kappa) \frac{\psi}{\bar{\psi}} b_{t+1}^h
\]

\[
\psi n_t^h + \sigma c_t^h = w_t^h - p_{h,t} - \alpha s_t
\]

\[
\psi n_t^f + \sigma c_t^f = w_t^f - p_{f,t} + \alpha s_t
\]

Inflation and terms-of-trade

\[
\pi_{h,t} = \beta E_t \left\{ \pi_{h,t+1} \right\} + \frac{(1 - \eta h \beta)(1 - \eta h)}{(1 - \gamma)(1 - \mu)} \left( (1 - \gamma) \left( w_t^h - a_t^h \right) + \gamma \left( w_t^f - a_t^f \right) - p_{h,t} \right)
\]

\[
\pi_{f,t} = \beta E_t \left\{ \pi_{f,t+1} \right\} + \frac{(1 - \eta f \beta)(1 - \eta f)}{(1 - \gamma)(1 - \mu)} \left( (1 - \gamma) \left( w_t^f - a_t^f \right) + \gamma \left( w_t^h - a_t^h \right) - p_{f,t} \right)
\]

\[
\pi_{h,t} = p_{h,t} - p_{h,t-1}
\]

\[
\pi_{f,t} = p_{f,t} - p_{f,t-1}
\]

\[
s_t - s_{t-1} = \pi_{f,t} - \pi_{h,t}
\]

\[
\sigma_t = \left( w_t^f - a_t^f \right) - \left( w_t^h - a_t^h \right)
\]

Goods market clearing

\[
y_t^h = (1 - \kappa) \left( (1 - \alpha) c_t^h + \alpha c_t^f + 2\alpha \mu (1 - \alpha) s_t \right) + \kappa g_t^h
\]

\[
y_t^f = (1 - \kappa) \left( (1 - \alpha) c_t^f + \alpha c_t^h - 2\alpha \mu (1 - \alpha) s_t \right) + \kappa g_t^f
\]

\[
a_t^h + n_t^h = (1 - \gamma) y_t^h + \gamma y_t^f + 2\phi \gamma (1 - \gamma) \sigma_t
\]

\[
a_t^f + n_t^f = (1 - \gamma) y_t^f + \gamma y_t^h - 2\phi \gamma (1 - \gamma) \sigma_t
\]

Current account

\[
b_{t+1}^h = \beta^{-1} b_t^h = \alpha \left[ c_t^f - c_t^h + (2\mu (1 - \alpha) - 1) s_t \right] + \frac{\gamma}{1 - \gamma} \left[ y_t^f - y_t^h + (2\phi (1 - \gamma) - 1) \sigma_t \right]
\]

Interest rate rule

\[
r_{t+1} = \rho + \beta \varphi \left( \frac{1}{2} \pi_{h,t} + \frac{1}{2} \pi_{f,t} \right)
\]