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# Dynamic Regulation of Quality


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**Abstract**

We investigate the design of incentives for quality provision in a dynamic regulation setting in which maintenance efforts and quality shocks have durable effects. When the regulator contracts with a sequence of agents, asymmetries of information can lead to over-provision of quality, reflecting a dynamic rent extraction motive. When the regulator hires a single agent to manage quality, over-provision of quality can also be used by the regulator to strengthen dynamic incentives. We further show that for small levels of asymmetric information, the regulator may prefer contracting with a sequence of agents rather than hiring a single agent if high quality shocks are relatively unfrequent, provided all parties can commit to a long-term contract. When no such commitment is feasible, the fact that quality physically links periods together leads to a ratchet effect even under recurring private information, and shorter franchises are beneficial from a social viewpoint.

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## 1. Introduction

The provision of quality by public utilities or regulated firms more generally is one of the main objectives of regulatory activity. Besides securing basic services such as electricity supply or voice and data transmission, regulators attempt to incite regulated firms to provide adequate levels of service enhancements that affect final customers' welfare. For instance, the speed and clarity of transmissions are key dimensions of quality in the telecommunications sector, as is the reliability of supply in the energy sector.

The literature on the regulation of quality has mostly focused on static frameworks, with special emphasis on the issue of quality verifiability (see Laffont and Tirole (1991), Lewis and Sappington (1991, 1992), and the survey by Sappington (2005)). However, in many cases, the management of quality can be properly understood only in a dynamic context. For instance, the maintenance of a road or electricity network requires sustained efforts, while the network itself is subject to exogenous events which affect the quality of the service it provides to final customers. Similarly, the quality of water supply is affected by exogenous polluting activities, and requires continuing cleaning up efforts. In all these examples, the quality of the good or service provided evolves over time as a result of maintenance efforts and exogenous random shocks, and these efforts and shocks have long-lasting effects. As a result of this, quality is a durable characteristic which can be viewed as a capital stock: the current quality of service depends on its past levels. This paper analyzes the problem of optimal regulation of quality in such a dynamic environment. In this framework, we endeavor to shed light on the following questions: What is the best incentive scheme when the regulated firm's decisions affect the level of quality available both in the present and in the future? Does private information systematically lead to under-provision of quality, as in a static environment?

Our analysis focuses on the case where quality is verifiable. That is, it can be described ex ante in a contract and certified ex post in court, and, as a result of this, the regulator can directly impose a quality target on the regulated firm, or more generally reward or punish the firm directly as a function of quality improvements. This is most relevant for industries such as electricity, where the number and intensity of outages can be ascertained in an almost costless way, or water supply, in which the chemical composition of water provides an accurate measure of its quality for final customers.[1] While quality itself is verifiable, we assume that the factors governing its evolution over time cannot be verified separately, and are private information of the regulated firm. Thus, in line with Lewis and Sappington (1991), the regulator cannot determine the portion of overall quality that can be respectively attributed to the regulated firm's maintenance efforts and to the exogenous quality shocks. Quality shocks are assumed to be independently distributed across periods. Hence, the only link between periods is physical rather than informational. For simplicity, we consider a binomial

[1] As pointed out by De Fraja and Iozzi (2004), there have been in practice two types of regulatory responses to the problem of securing an adequate quality service (see also Armstrong, Cowan and Vickers (1994)). First, the imposition of quality standards, enforced through legal sanctions. Second, the imposition of a link between the firm's allowed revenues and prices and the quality of the service it provides. For instance, in the UK water industry, price cap adjustments are based on comparative performance indicators (OFWAT (2002)). Similarly, UK energy distribution companies receive financial compensations according to various quality indicators (OFGEM (2001)). Note that these two types of mechanisms require at least some dimensions of quality to be verifiable by a court. Example of such contracts are discussed in Sappington (2005, Section 4.3). De Fraja and Iozzi (2004) propose an extension of the Vogelsang and Finsinger (1979) dynamic model of price regulation that allows for such price-quality tradeoffs. They do not address the issue of quality durability, which is the objective of the present paper.

model in which only high and low quality shocks can occur.

Two regulatory frameworks are considered in turn. We first analyze the dynamics of quality in a stationary setup in which the regulator delegates the management of quality to a sequence of firms or agents, one for each period. This allows us to disentangle the question of quality dynamics from that of the provision of dynamic incentives, which is addressed in the second part of the paper. Each agent is protected by limited liability and hence must receive a non-negative utility in each state of nature (Laffont and Martimort (2002, §3.5)). That is, no contract can be enforced whereby a truthful agent could potentially incur losses. Within each period, the timing is as follows. After signing a contract with the regulator, which depends on the current level of quality, the agent in charge privately learns his type, that is, the current realization of the quality shock, and then privately chooses his maintenance effort. Transfers are then effected according to the achieved level of quality.

Because the agents must receive a non-negative utility in each state of nature, the fact that the quality shocks are privately observed by the agents leads to a sequence of non-degenerate moral hazard problems. The main difference with a static framework is that the social value of quality reflects not only the current social benefit of quality, but also its impact on the continuation game played by the regulator and the future agents. Accordingly, maintenance efforts and transfers will vary over time with the quality of the good or service provided. Using standard recursive techniques (Stokey and Lucas (1989)), we characterize the social value of quality under both symmetric and asymmetric information. A key result of our analysis is that the marginal social value of quality is strictly higher under asymmetric information than under symmetric information, reflecting a dynamic rent extraction effect. Indeed, the informational rent that the regulator must leave to the current agent in case of a high quality shock is decreasing in the current level of quality: when quality is high, the agency problem becomes less severe. This implies that, relative to a static environment, the regulator has an additional incentive to enhance quality, namely to reduce future informational rents. In particular, she will take advantage of a high quality shock to demand a higher effort from the current agent. As a result of this, there may be over-provision of quality relative to the symmetric information environment, typically following a sequence of high quality shocks. Sharper predictions are derived using a linear-quadratic specification of the model. It is shown that, while private information leads to a lower average growth rate of quality, it also increases the variance of quality. In the long run, the range of possible qualities is larger under asymmetric information than under symmetric information, and thus both over-provision and under-provision of quality can persist asymptotically.

We next turn to the case where the regulator delegates the management of quality to a single agent, which raises the issue of dynamic incentives. To deliver analytical results, we consider a two-period model. In each period, the agent privately learns the realization of the quality shock, and then chooses his effort. In line with the basic model, we assume that the agent must receive a non-negative utility at each date and in each state.[2] We mainly focus on the full commitment case, in which both the regulator and the agent can commit to a long-term contract, subject to incentive compatibility and limited liability constraints. Our main findings are as follows. First, the optimal long-term contract exhibits memory: the level of distortions in the second period depends on the type of the agent in the first period. With a sequence of agents, by contrast, distortions in the second period would

[2] One justification for this assumption is that the regulator is legally prevented from proposing contracts contingent on the agent's reported wealth.

depend on the past only through the level of quality inherited from the first period. Second, the regulator may now find it optimal to increase the agent's effort following a low quality shock in the first period. Indeed, doing so reduces the second period informational rent. This directly benefits the regulator, and also makes misreporting by the agent in case of a high quality shock less attractive from the first period perspective, thereby decreasing the cost of dynamic incentives. Interestingly, this effect may lead to over-provision of first period quality following a low quality shock. Finally, we compare the model with a single agent and full commitment to the model with a sequence of agents, specialized to the two-period case. A natural question is whether it is better for the regulator to hire a single agent, or to contract with a sequence of agents. The benefit of hiring a single agent is that the regulator can directly condition his continuation rent on his first period performance, as in the standard repeated moral hazard problem (Rogerson (1985)). The cost, by contrast, is that the agent correctly anticipates the impact of his first period actions on his future utility, thus making the first period incentive compatibility constraint more stringent than with a sequence of agents. In the linear-quadratic specification of the model, we show that, for small levels of asymmetric information, this second effect dominates when the probability of a high quality shock is below a threshold value. It is therefore optimal for the regulator to contract with a sequence of agents when high quality shocks are less likely to occur. These predictions are reversed when high quality shocks are relatively frequent.

This paper is in line with works that extend the analysis of incentives in regulation to a dynamic framework (Baron and Besanko (1984), Laffont and Tirole (1988, 1990), Lewis and Sappington (1997)). In these papers, the source of the dynamics is that the regulated firm's costs are correlated across periods, so that the regulator progressively learns about the efficiency of the firm.[3] Instead of this, the intertemporal link stressed in this paper is purely physical. This is similar to Lewis and Yildirim (2002), who study the optimal regulation of a firm who learns to use cost-reducing innovations over time.[4] They show that a light-handed regulation may encourage innovation, by allowing the firm to earn greater informational rents while providing greater service. Moreover, innovation may occur even in the absence of long-term agreements, provided private information is renewed in each period. Their model differs from ours in some important respects. First, private information in our model takes the form of quality shocks that directly affect future consumer surplus and thus have permanent effects, while it is embedded in their model in a transitory cost. Next, while Lewis and Yildirim (2002) only allow for sequences of spot contracts, we analyze the opposite case in which the regulator can commit to a long-term contract, subject to the constraint that the agent must receive a non-negative utility at each date and in each state.[5]

The paper is organized as follows. Section 2 presents the basic model. In Section 3, we analyze the dynamics of quality under both symmetric and asymmetric information when the regulator contracts with a sequence of short-lived agents. In Section 4, we consider the case of a single long-lived agent. Section 5 concludes. All proofs are in the appendices.

[3]Similarly, Battaglini (2005) characterizes the optimal contract between a monopolist and a consumer whose willingness to pay follows a Markov process, while general models of dynamic mechanism design with correlated types have been recently studied by Athey and Segal (2007) and Pavan, Segal and Toikka (2008). The emphasis on limited liability constraints distinguishes our framework from those studied in these papers.

[4]See Gaudet, Lasserre and Van Long (1995, 1996) and Gärtner (2004) for related models.

[5]We briefly consider the case of short-term contracts in Subsection 4.5. The main insight is that the absence of commitment generates a ratchet effect: an agent experiencing a high quality shock today anticipates that, as a result of this, quality tomorrow will be higher, which in turn leads to lower continuation rents.

## 2. The Basic Model

Our model departs in two ways from standard regulation models such as those studied by Baron and Myerson (1982) or Laffont and Tirole (1986). First, we consider a multi-period environment. Second, we assume that there exists a physical state variable that dynamically links periods to each other. This state variable is interpreted as the quality of a regulated good or service, such as a road or electricity network.

The basic model focuses on the dynamics of quality, leaving aside the complex issues related to the dynamic provision of incentives which are examined in Section 4. For this purpose, we consider a benevolent regulator who delegates the management of quality to a sequence of agents, one for each period. The per-period consumer surplus generated by a good or service of quality $q \geq 0$ is denoted $S(q)$. The function $S$ is bounded, continuously differentiable, strictly increasing and strictly concave over $\mathbb{R}_{++}$.

Quality evolves over time as a function of the agents' effort to maintain it and of exogenous shocks. Specifically, if quality at date $t = 0, 1, \ldots$ is $q_t$, then quality at date $t+1$ is

$$q_{t+1} = \delta q_t + e_t + \theta_t, \tag{1}$$

where $\delta \in (0,1)$ is a depreciation factor, $e_t$ is the maintenance effort exerted by the date $t$ agent, and $\theta_t$ is the date $t$ quality shock. The parameter $\delta$ measures the extent to which maintenance efforts and quality shocks have durable effects. The quality shocks $\{\theta_t\}_{t=0}^{\infty}$ are independently and identically distributed across periods, with support $\{\overline{\theta}, \underline{\theta}\}$ such that $\overline{\theta} > \underline{\theta} \geq 0$, and we let $\Delta\theta = \overline{\theta} - \underline{\theta}$. For any date $t = 0, 1, \ldots,$ we denote by $\nu \in (0,1)$ the probability that $\theta_t = \overline{\theta}$, and we let $\mathrm{E}_\theta = \mathrm{E}[\theta_t]$ and $\mathrm{Var}_\theta = \mathrm{Var}[\theta_t]$.

An agent exerting a maintenance effort $e$ incurs a disutility $\psi(e)$ in monetary units. The function $\psi$ is continuously differentiable, strictly increasing and strictly convex over $\mathbb{R}_+$, and satisfies $\psi(0) = \psi'(0) = 0$. It is analytically convenient to extend the function $\psi$ to the whole real line by setting $\psi = 0$ over $\mathbb{R}_-$. To guarantee that the regulator's objective function is concave, we also assume that $\psi$ has a convex derivative over $\mathbb{R}_+$.

Agents are compensated for their efforts by monetary transfers. Given effort level $e_t$ and monetary transfer $u_t$, the date $t$ agent's overall utility is then

$$U_t = u_t - \psi(e_t). \tag{2}$$

Each agent's outside opportunity is normalized to zero. In addition, we assume that, to accept working for the regulator, each agent must receive a non-negative utility in each state of nature. An interpretation of this limited liability constraint on rents is that agents have infinite risk aversion below zero wealth (Laffont and Martimort (2002, §3.5)).

As in Laffont and Tirole (1986), distortionary taxation inflicts a disutility \$$(1+\lambda)$ on consumers in order to levy \$1 for the state, where $\lambda > 0$ is the shadow cost of public funds. Hence, the date $t$ net consumer surplus is $S(q_t) - (1+\lambda)u_t$, so that by (2), the corresponding utilitarian social welfare is

$$S(q_t) - (1+\lambda)u_t + u_t - \psi(e_t) = S(q_t) - (1+\lambda)\psi(e_t) - \lambda U_t. \tag{3}$$

The regulator is far-sighted and discounts future payoffs with a discount factor $\beta \in (0,1)$. By (3), the expected discounted social welfare at date zero is thus equal to

$$\mathrm{E}\left[\sum_{t=0}^{\infty} \beta^t [S(q_t) - (1+\lambda)\psi(e_t) - \lambda U_t]\right]. \tag{4}$$

We assume throughout that quality is verifiable, so that the regulator can reward or punish the agents directly as a function of quality improvements. By (1), this means that, at each date, the sum of the agent's effort and of the quality shock is verifiable. Under symmetric information, efforts and shocks are themselves verifiable. Under asymmetric information, however, neither efforts nor shocks are verifiable, as in Lewis and Sappington (1991), while the shocks are privately observed by the agents.[6]

## 3. Regulating Quality with a Sequence of Agents

### *3.1. Regulation under Symmetric Information*

As a benchmark, we consider the symmetric information situation in which not only quality improvements, but also the extent to which these can be attributed to agents' efforts or to quality shocks are verifiable variables. As a result of this, regulatory contracts can be directly made contingent on agents' efforts. The existence of a shadow cost of public funds implies that agents receive no rent at the optimum, that is $u_t = \psi(e_t)$ for all $t = 0, 1, \ldots$. By (1)–(2) and (4), the regulator's problem is then to find a sequence of quality levels $\{q_t\}_{t=1}^{\infty}$, where each $q_t$ is contingent on the history of shocks $(\theta_0, \ldots, \theta_{t-1})$ up to date $t$, that solves

$$V^*(q_0) = \sup \left\{ \mathrm{E}\left[ \sum_{t=0}^{\infty} \beta^t [S(q_t) - (1+\lambda)\psi(q_{t+1} - \delta q_t - \theta_t)] \right] \right\},$$

given any initial quality level $q_0$. This is a standard dynamic programming problem that can be tackled by usual techniques. The symmetric information social value function $V^* : \mathbb{R}_+ \to \mathbb{R}$ is the unique bounded solution to the Bellman equation

$$V^*(q) = \max \left\{ S(q) - \nu(1+\lambda)\psi(\overline{e}) - (1-\nu)(1+\lambda)\psi(\underline{e}) + \nu\beta V^*(\overline{q}) + (1-\nu)\beta V^*(\underline{q}) \right\}, \quad (5)$$

where the controls $(\overline{e}, \overline{q}, \underline{e}, \underline{q})$ must satisfy the state transition constraints

$$\overline{q} = \delta q + \overline{e} + \overline{\theta}, \quad (6)$$

$$\underline{q} = \delta q + \underline{e} + \underline{\theta}, \quad (7)$$

and the feasibility constraints

$$\overline{q} \geq 0, \quad (8)$$

$$\underline{q} \geq 0. \quad (9)$$

Standard considerations (see for instance Stokey and Lucas (1989)) yield our first result.

**Lemma 1.** *The symmetric information social value function $V^*$ is bounded, continuously differentiable, strictly increasing and strictly concave over $\mathbb{R}_{++}$.*

An implication of $\psi'(0) = 0$ is that it is always optimal for the regulator to induce strictly positive effort on the part of the current agent, no matter the current level of quality or the

[6]This modelling assumption is naturally reminiscent of Laffont and Tirole's (1986) regulation model.

current quality shock. That is, the feasibility constraints (8)–(9) are never binding. Denote by $\overline{e}^*(q)$, $\overline{q}^*(q)$, $\underline{e}^*(q)$, $\underline{q}^*(q)$ the optimal choices in (5), which are uniquely determined. Then the first-order conditions for (5) read as:

$$\beta V^{*\prime}(\overline{q}^*(q)) = (1+\lambda)\psi'(\overline{q}^*(q) - \delta q - \overline{\theta}), \tag{10}$$

$$\beta V^{*\prime}(\underline{q}^*(q)) = (1+\lambda)\psi'(\underline{q}^*(q) - \delta q - \underline{\theta}). \tag{11}$$

Since $V^*$ is strictly concave and $\psi$ strictly convex over $\mathbb{R}_+$, it follows from (10)–(11) that $\overline{q}^*(q) > \underline{q}^*(q)$: for a given level of quality, a high quality shock today leads to a higher quality tomorrow than a low quality shock. Along with (6)–(7), (10)–(11) further imply that $\overline{e}^*(q) < \underline{e}^*(q)$: an agent facing a high quality shock exerts less effort than one facing a low quality shock, and therefore receives a lower transfer. Note that, as a result of this, $-(1+\lambda)\psi(\overline{e}^*(q)) + \beta V^*(\overline{q}^*(q)) > -(1+\lambda)\psi(\underline{e}^*(q)) + \beta V^*(\underline{q}^*(q))$, so that the regulator benefits from a high quality shock.

Using again the strict concavity of $V^*$ and the strict convexity of $\psi$ over $\mathbb{R}_+$, it is easy to check from (10)–(11) that the mappings $\overline{q}^*$ and $\underline{q}^*$ are strictly increasing, while the mappings $\overline{e}^*$ and $\underline{e}^*$ are strictly decreasing. Since $\psi'(0) = 0$ and $\lim_{q\to\infty} \overline{q}^*(q) = \lim_{q\to\infty} \underline{q}^*(q) = \infty$ by (6)–(7), and since $V^*$ is bounded, a further implication of (10)–(11) is that $\lim_{q\to\infty} \overline{e}^*(q) = \lim_{q\to\infty} \underline{e}^*(q) = 0$. These properties reflect the fact that the agents' maintenance efforts become less important from the regulator's viewpoint as quality improves. As a result of this, $\lim_{q\to\infty} V^*(q) = \lim_{q\to\infty} \frac{S(q)}{1-\beta}$.

### *3.2. Regulation under Asymmetric Information*

We now turn to the case in which neither agents' efforts nor quality shocks are verifiable. An asymmetry of information then arises because, once in charge, agents become privately informed of the current quality shock. Since they must receive a non-negative utility in each state, eliciting this information from them is socially costly.[7] The task of the regulator is to design appropriate incentive schemes to overcome this hidden information problem and the resulting moral hazard problem.

An incentive contract between the regulator and the date $t$ agent specifies a transfer-quality pair for each realization of the date $t$ quality shock, which will be henceforth referred to as the date $t$ agent's type. For a given level of quality $q$, a contract is thus a 4-tuple $(\overline{u}, \overline{q}, \underline{u}, \underline{q})$. Incentive compatibility requires that

$$\overline{u} - \psi(\overline{q} - \delta q - \overline{\theta}) \geq \underline{u} - \psi(\underline{q} - \delta q - \overline{\theta}), \tag{12}$$

$$\underline{u} - \psi(\underline{q} - \delta q - \underline{\theta}) \geq \overline{u} - \psi(\overline{q} - \delta q - \underline{\theta}). \tag{13}$$

Since the agent must receive a non-negative utility in each state, an incentive feasible contract must also satisfy the following limited liability constraints:

$$\overline{u} - \psi(\overline{q} - \delta q - \overline{\theta}) \geq 0, \tag{14}$$

$$\underline{u} - \psi(\underline{q} - \delta q - \underline{\theta}) \geq 0. \tag{15}$$

[7] In the absence of such an ex-post participation constraint, the regulator could achieve the symmetric information outcome through appropriate ex-ante contracting (D'Aspremont and Gérard-Varet (1979)).

It is easy to check that the optimal contract under symmetric information does not satisfy the incentive constraint (12) of the high type agent. As usual with this type of models, the incentive constraint (12) of the high type agent and the limited liability constraint (15) of the low type agent together imply the limited liability constraint (14) of the high type agent. To maximize the expected discounted social welfare, we shall momentarily neglect the incentive constraint (13) of the low type agent, and later check that the solution thus obtained satisfies this constraint.

Denote by $\overline{U}(q) = \overline{u} - \psi(\overline{q} - \delta q - \overline{\theta})$ and $\underline{U}(q) = \underline{u} - \psi(\underline{q} - \delta q - \underline{\theta})$ the rents left to the agent under the contract $(\overline{u}, \overline{q}, \underline{u}, \underline{q})$, given current quality $q$. The limited liability constraint (15) of the low type agent can be rewritten as:

$$\underline{U}(q) \geq 0, \tag{16}$$

while the incentive constraint (12) of the high type agent can be rewritten as:

$$\overline{U}(q) \geq \underline{U}(q) + \Phi(q, \underline{q}), \tag{17}$$

where the function $\Phi$ is defined by

$$\Phi(q, \underline{q}) = \psi(\underline{q} - \delta q - \underline{\theta}) - \psi(\underline{q} - \delta q - \overline{\theta}). \tag{18}$$

Intuitively, $\Phi(q, \underline{q})$ is the informational rent that must be left to the high type agent when the low type agent improves quality from $q$ to $\underline{q}$. Since $\psi$ is convex, $\Phi(q, \cdot)$ is increasing for any value of $q$, while $\Phi(\cdot, \underline{q})$ is decreasing for any value of $\underline{q}$.[8] Moreover, since $\psi'$ is convex, $\Phi(q, \cdot)$ is convex for any value of $q$, and $\Phi(\cdot, \underline{q})$ is convex for any value of $\underline{q}$. These properties will ensure that the regulator's objective function is concave, and that the social value function is strictly increasing and strictly concave.

Since leaving rents to the agent is socially costly, it will be optimal to let (16)–(17) be binding. That is, one will have

$$U_t = \mathrm{E}_t[\Phi(q_t, q_{t+1}) \,|\, \theta_t = \underline{\theta}] 1_{\{\theta_t = \overline{\theta}\}} \tag{19}$$

for all $t = 0, 1, \ldots$. By (1), (4) and (19), the regulator's problem is then to find a sequence of quality levels $\{q_t\}_{t=1}^{\infty}$, where each $q_t$ is contingent on the history of shocks $(\theta_0, \ldots, \theta_{t-1})$ up to date $t$, that solves

$$V^{**}(q_0) = \sup \left\{ \mathrm{E}\left[ \sum_{t=0}^{\infty} \beta^t \left[ S(q_t) - (1+\lambda)\psi(q_{t+1} - \delta q_t - \theta_t) - \frac{\lambda\nu}{1-\nu}\, \Phi(q_t, q_{t+1}) 1_{\{\theta_t = \underline{\theta}\}} \right] \right] \right\},$$

given any initial quality level $q_0$. In analogy with (5), the asymmetric information social value function $V^{**} : \mathbb{R}_+ \to \mathbb{R}$ is the unique bounded solution to the Bellman equation

$$\begin{aligned} V^{**}(q) \;=\; \max \big\{ & S(q) - \nu(1+\lambda)\psi(\overline{e}) - (1-\nu)(1+\lambda)\psi(\underline{e}) - \lambda\nu\Phi(q, \underline{q}) \\ & + \nu\beta V^{**}(\overline{q}) + (1-\nu)\beta V^{**}(\underline{q}) \big\}, \end{aligned} \tag{20}$$

subject to constraints (6)–(9). The following result parallels Lemma 1.

[8] However, for $\underline{q} < \delta q + \underline{\theta}$, these functions are constant and equal to zero.

**Lemma 2.** *The asymmetric information social value function $V^{**}$ is bounded, continuously differentiable, strictly increasing and strictly concave over $\mathbb{R}_{++}$.*

As in the symmetric information benchmark, the condition $\psi'(0) = 0$ implies that the feasibility constraints (8)–(9) are never binding: it is always optimal for the regulator to induce strictly positive effort on the part of the current agent, no matter his type or the current level of quality. Denote by $\overline{e}^{**}(q)$, $\overline{q}^{**}(q)$, $\underline{e}^{**}(q)$, $\underline{q}^{**}(q)$ the optimal choices in (20), which are uniquely determined. Then the first-order conditions for (20) read as:

$$\beta V^{**\prime}(\overline{q}^{**}(q)) = (1+\lambda)\psi'(\overline{q}^{**}(q) - \delta q - \overline{\theta}), \tag{21}$$

$$\beta V^{**\prime}(\underline{q}^{**}(q)) = (1+\lambda)\psi'(\underline{q}^{**}(q) - \delta q - \underline{\theta}) + \frac{\lambda\nu}{1-\nu}\,\Phi_2(q, \underline{q}^{**}(q)). \tag{22}$$

Since $V^{**}$ is strictly concave and $\psi$ strictly convex over $\mathbb{R}_+$, and since $\Phi(q,\cdot)$ is convex, it follows from (21)–(22) that $\overline{q}^{**}(q) > \underline{q}^{**}(q)$. This in turn implies that the neglected constraint (13) is satisfied by our candidate solution. Indeed, because (16)–(17) are binding, (13) is equivalent to $\Phi(q, \overline{q}^{**}(q)) \geq \Phi(q, \underline{q}^{**}(q))$, which holds as $\Phi(q,\cdot)$ is increasing. Because of the second term on the right-hand side of (22), which corresponds to the distortion due to asymmetric information, the comparison between the effort levels $\overline{e}^{**}(q)$ and $\underline{e}^{**}(q)$ is ambiguous, unlike in the symmetric information benchmark.

Using again the strict concavity of $V^{**}$ and the strict convexity of $\psi$ over $\mathbb{R}_+$, together with the convexity of $\psi'$ and the definition (18) of $\Phi$, it is easy to check from (21)–(22) that the mappings $\overline{q}^{**}$ and $\underline{q}^{**}$ are strictly increasing, while the mappings $\overline{e}^{**}$ and $\underline{e}^{**}$ are strictly decreasing. The strict convexity of $\psi$ over $\mathbb{R}_+$, along with the monotonicity of $\underline{e}^{**}$, also implies that the informational rent of a high type agent,

$$\Phi(q, \underline{q}^{**}(q)) = \psi(\underline{e}^{**}(q)) - \psi(\underline{e}^{**}(q) - \Delta\theta), \tag{23}$$

is a strictly decreasing function of quality $q$. It follows from (21)–(22) that, as in the symmetric information benchmark, $\lim_{q\to\infty} \overline{e}^{**}(q) = \lim_{q\to\infty} \underline{e}^{**}(q) = 0$. By (23), this implies that the informational rent of the high type agent vanishes as quality gets large, $\lim_{q\to\infty} \Phi(q, \underline{q}^{**}(q)) = 0$. The intuition for this result is that when quality improves, it becomes less important for the regulator to incite the low type agent to exert effort. In particular, $\lim_{q\to\infty} V^{**}(q) = \lim_{q\to\infty} \frac{S(q)}{1-\beta}$ as in the symmetric information benchmark.

*Remark.* Using the Envelope Theorem for (5) and (20), it is easy to check that both $V^{*\prime}$ and $V^{**\prime}$ are strictly greater than $S'$. It follows that, in both regulatory contexts, the regulator induces a higher quality than in the corresponding one-period version of the model.[9] This reflects that, because quality is durable in our model, the benefits of producing a higher quality today arise directly through an increase of future consumer surpluses, and, indirectly, through a reduced cost of supplying quality in the future.

### *3.3. Comparing the Two Regulatory Environments*

A key insight of our analysis is that the social value of quality depends on the regulatory environment. We argue in this section that, as a result of this, asymmetric information

[9]This result is in line with Lewis and Yildirim (2002, Proposition 3(i)).

typically leads to distortions in both types of agents' maintenance efforts relative to their symmetric information levels. To establish this point, we first present some general analytical results, and then examine a linear-quadratic specification of the model.

### *3.3.1. General Results*

To contrast the outcomes of the regulation game under symmetric and under asymmetric information, it is helpful to compare the social value functions $V^*$ and $V^{**}$. Formally, the only difference between the Bellman equations (5) and (20) that implicitly define $V^*$ and $V^{**}$ lies in the informational rent $\lambda\nu\Phi(q,\underline{q})$ that appears on the right-hand side of (20). As observed above, this rent is a decreasing function of the current quality level $q$. This suggests that under asymmetric information, an additional incentive to increase quality is to reduce future informational rents. Accordingly, the marginal social value of quality is strictly higher under asymmetric information than under symmetric information, as shown by the following result.

**Proposition 1.** *For any* $q > 0$, $V^{**\prime}(q) > V^{*\prime}(q)$.

Along with (10) and (21), Proposition 1 implies that $\overline{q}^{**}(q) > \overline{q}^{*}(q)$ for any $q > 0$. That is, for a given level of quality $q > 0$, and conditional on a high quality shock occurring, the regulator induces more effort from the agent under asymmetric information than under symmetric information, $\overline{e}^{**}(q) > \overline{e}^{*}(q)$. The intuition is straightforward: given a high quality shock, the marginal cost of effort is the same in both regulatory environments, while by Proposition 1, the marginal benefit of effort is higher under asymmetric information. In line with the dynamic rent extraction effect outlined above, the regulator therefore takes advantage of facing a high type agent today to build better quality for tomorrow. It should be noted that this over-provision of quality contrasts with the prediction of a one-period model, in which high type agents optimally exert the same level of effort under symmetric information as under asymmetric information. Because of the additional distortion term on the right-hand side of (22), it is not possible in general to rank the quality levels $\underline{q}^*(q)$ and $\underline{q}^{**}(q)$ conditional on a low shock to quality. The intuition is that asymmetric information raises both the marginal benefit and the marginal cost of exerting effort given a low quality shock. A robust prediction of our model is however that if sufficiently many high quality shocks occur, asymmetric information will lead to over-provision of quality relative to the symmetric information benchmark.

The differences between the two regulatory environments do not disappear in the long run, as can be shown by studying the asymptotic distribution of quality. Specifically, let $P^*$ and $P^{**}$ be the probability transition functions over quality levels respectively induced by (10)–(11) and (21)–(22). That is, for each $i \in \{*, **\}$ and for any $q \geq 0$, $P^i(q, \cdot)$ is the law of a random variable that takes the value $\overline{q}^i(q)$ with probability $\nu$, and the value $\underline{q}^i(q)$ with probability $1 - \nu$. Then the following holds.

**Proposition 2.** *$P^*$ and $P^{**}$ have unique invariant probability measures $\mu^*$ and $\mu^{**}$ with compact supports* $\operatorname{supp}\mu^*$ *and* $\operatorname{supp}\mu^{**}$ *such that* $\max\operatorname{supp}\mu^{**} > \max\operatorname{supp}\mu^*$.

For any initial quality level $q_0$, the distribution of quality will converge weakly to $\mu^*$ or $\mu^{**}$ depending on the regulatory environment. Since the upper bound of the support of the asymptotic distribution of quality is strictly larger under asymmetric information than under symmetric information, one will in the long run observe high quality levels in the former case that cannot be achieved in the latter case. Therefore over-provision of quality can persist in the long run under asymmetric information.

### *3.3.2. The Linear-Quadratic Case*

To obtain sharper predictions, we consider a special case of our model, in which the underlying surplus and cost functions are quadratic,

$$S(q) = aq - \frac{b}{2}\, q^2, \tag{24}$$

$$\psi(e) = \frac{c}{2}\, \max\{e, 0\}^2, \tag{25}$$

for some strictly positive parameters $a$, $b$ and $c$. It should be noted that the surplus function $S$ defined by (24) makes sense only as long as $q < \frac{a}{b}$. Along standard lines (Stokey and Lucas (1989, §4.4)), we first solve the Bellman equations (5) and (20) without taking into account this restriction. We then check under which conditions and over which ranges the resulting analytical solutions are economically meaningful. The appropriate parameter restrictions amount to make $\underline{\theta}$ and $\Delta\theta$ close enough to zero, see Appendix A for a precise statement.

The linear-quadratic specification (24)–(25) ensures that the social value functions $V^*$ and $V^{**}$ are themselves quadratic over the relevant ranges, that is, for each $i \in \{*, **\}$,

$$V^i(q) = A^i q - \frac{B^i}{2}\, q^2 + C^i, \tag{26}$$

for some strictly positive parameters $A^i$, $B^i$ and $C^i$, while the optimal policy functions $\overline{q}^*$, $\underline{q}^*$, $\overline{q}^{**}$ and $\underline{q}^{**}$ are linear over the relevant ranges,

$$\overline{q}^*(q) = \frac{\beta A^* + c(1+\lambda)(\overline{\theta} + \delta q)}{\beta B^* + c(1+\lambda)}, \tag{27}$$

$$\underline{q}^*(q) = \frac{\beta A^* + c(1+\lambda)(\underline{\theta} + \delta q)}{\beta B^* + c(1+\lambda)}, \tag{28}$$

$$\overline{q}^{**}(q) = \frac{\beta A^{**} + c(1+\lambda)(\overline{\theta} + \delta q)}{\beta B^{**} + c(1+\lambda)}, \tag{29}$$

$$\underline{q}^{**}(q) = \frac{\beta A^{**} - \frac{\lambda\nu}{1-\nu}\, c\Delta\theta + c(1+\lambda)(\underline{\theta} + \delta q)}{\beta B^{**} + c(1+\lambda)}. \tag{30}$$

To determine the optimal policy functions, what matters are the values of $A^*$, $B^*$, $A^{**}$ and $B^{**}$. These are easily obtained by inserting the policy functions (27)–(28) and (29)–(30) into the Bellman equations (5) and (20) and then using (26) to identify terms. An immediate result is that the coefficients $B^*$ and $B^{**}$ coincide, $B^* = B^{**} = B$. This reflects the property that, in the linear-quadratic specification, and over the relevant range, the informational rent $\Phi(q, \underline{q})$ is a linear function of $(q, \underline{q})$,

$$\Phi(q, \underline{q}) = c\Delta\theta\left(\underline{q} - \delta q - \frac{\overline{\theta} + \underline{\theta}}{2}\right). \tag{31}$$

It should be noted that, since the marginal social value of quality is higher under asymmetric information than under symmetric information, $A^{**} > A^*$.

An immediate implication of (27)–(30) is that asymmetric information increases the wedge between qualities following a high and a low quality shock, $\overline{q}^{**}(q) - \underline{q}^{**}(q) > \overline{q}^*(q) - \underline{q}^*(q)$. Our next result strengthens this insight.

**Proposition 3.** *In the linear-quadratic case, over the relevant range, the quality spread is larger under asymmetric information than under symmetric information,*

$$\overline{q}^{**}(q) > \overline{q}^*(q) > \underline{q}^*(q) > \underline{q}^{**}(q). \tag{32}$$

The intuition for this result can be grasped by comparing the objective functions of the regulator in (5) and (20). Under symmetric information, the marginal value of date $t+1$ quality is the same following a high or a low quality shock, namely $\beta(A^* - Bq_{t+1})$. By contrast, under asymmetric information, the need to concede an informational rent to high type agents introduces a wedge between the marginal value of quality following a high quality shock, $\beta(A^{**} - Bq_{t+1})$, and the marginal value of quality following a low quality shock, net of the incentive cost, $\beta(A^{**} - Bq_{t+1}) - \frac{\lambda\nu}{1-\nu} c\Delta\theta$. It turns out that

$$\beta A^* < \beta A^{**} < \beta A^* + \frac{\lambda\nu}{1-\nu} c\Delta\theta,$$

so that the higher marginal social benefit of quality due to asymmetric information does not fully offset the incentive cost given a low quality shock. As shown by (32), this simultaneously leads to over-provision of quality following a high quality shock, $\overline{q}^{**}(q) > \overline{q}^*(q)$, and to under-provision of quality following a low quality shock, $\underline{q}^*(q) > \underline{q}^{**}(q)$. While the first effect reflects a dynamic rent extraction motive, as discussed in Subsection 3.3.1, the second effect reflects a static rent extraction motive: the regulator induces less maintenance effort from a low type agent in order to reduce the rent she would have to leave today to a high type agent. Proposition 3 shows that this static rent extraction effect dominates the dynamic effect. As can be checked from (29)–(30), this may cause a high type agent to exert more effort than a low type agent given the same current level of quality, thus reversing the prediction of the symmetric information benchmark.

In light of these results, it is natural to investigate which of the static and the dynamic rent extraction motives dominates from an ex-ante perspective. To this end, we examine the time-series properties of quality under the two regulatory regimes. Denote by $\{q_t^*\}_{t=0}^{\infty}$ and $\{q_t^{**}\}_{t=0}^{\infty}$ the stochastic processes of quality under symmetric and asymmetric information respectively. Provided initial quality is low, these processes will remain in the appropriate ranges over which our analytical solutions to (5) and (20) are economically meaningful. One has the following result.

**Proposition 4.** *In the linear-quadratic case, quality grows at a lower expected rate under asymmetric information than under symmetric information,*

$$\mathrm{E}\,[q_{t+1}^* \,|\, q_t^* = q] > \mathrm{E}\,[q_{t+1}^{**} \,|\, q_t^{**} = q], \tag{33}$$

*while the variance of quality growth is higher under asymmetric information than under symmetric information,*

$$\mathrm{Var}\,[q_{t+1}^{**} \,|\, q_t^{**} = q] > \mathrm{Var}\,[q_{t+1}^* \,|\, q_t^* = q]. \tag{34}$$

From an ex-ante perspective, the static rent extraction motive therefore dominates the dynamic rent extraction motive, and asymmetric information leads to on average lower and more volatile quality growth. These properties carry over to the long run. For each $i \in \{*, **\}$, let $\mathrm{E}_{\mu^i} = \int q\, \mu^i(dq)$ and $\mathrm{Var}_{\mu^i} = \int (q - \mathrm{E}_{\mu^i})^2\, \mu^i(dq)$ be the expectation and variance of quality under the invariant measure $\mu^i$. Weak convergence of the distribution of quality to $\mu^i$ ensures that $\lim_{t\to\infty} \mathrm{E}\,[q_t^i] = \mathrm{E}_{\mu^i}$ and $\lim_{t\to\infty} \mathrm{Var}\,[q_t^i] = \mathrm{Var}_{\mu^i}$, no matter the initial quality level $q_0$. The following result provides the long run analogues of (33)–(34).

**Proposition 5.** *In the linear-quadratic case, in the long run, quality is on average lower under asymmetric information than under symmetric information,*

$$\mathrm{E}_{\mu^*} > \mathrm{E}_{\mu^{**}}, \tag{35}$$

*while its variance is higher under asymmetric information than under symmetric information,*

$$\mathrm{Var}_{\mu^{**}} > \mathrm{Var}_{\mu^*}. \tag{36}$$

## 4. Regulating Quality with a Single Agent

We have assumed so far that the regulator can contract with a sequence of agents. We now investigate what happens when there is a single agent the regulator can contract with over time. In contrast with the multiple agent case, the regulator can provide dynamic incentives to exert effort. However, the fact that the agent anticipates the impact of his actions on future quality and therefore on the continuation of the contract induces a further cost for the regulator. To analyze this trade-off, we first characterize the optimal contract with a single agent. We then compare regulation with a single agent and with multiple agents.

### *4.1. The Model*

To simplify the analysis, we assume that the contractual relationship lasts only two periods: the agent exerts effort at dates 0 and 1 to provide quality at dates 1 and 2. The model remains otherwise the same as in Section 3.[10] Our setup differs in two ways from the dynamic regulation models previously studied in the literature (Baron and Besanko (1984), Freixas, Guesnerie and Tirole (1985), Laffont and Tirole (1988, 1990), Lewis and Sappington (1997)). First, the only link between periods is physical and not informational, as quality shocks are independently distributed across periods. This implies that the correlation between qualities across different periods is endogenous, and depends on past quality shocks and maintenance efforts, as well as on past contracts. Second, as in the basic model, the agent is protected by limited liability, so that only contracts that leave him with a non-negative utility at each date and in each state can be enforced.[11]

### *4.2. Regulation with Full Commitment*

Under asymmetric information, neither the agent's efforts nor the quality shocks are verifiable, but long-term regulatory contracts can be perfectly enforced, subject to the constraint that

[10] Note however that we no longer require the discount factor $\beta$ to be smaller than one. This may reflect the fact that the accounting period for future production stages exceeds that for the current production stage.

[11] Laffont and Martimort (2002, §8.1.3) study how this limited liability constraint affects the outcome of a repeated agency model in which the link between periods is purely informational.

the agent receives a non-negative utility at each date and in each state. Because of full commitment, the Revelation Principle applies. For each pair of reports $(\hat{\theta}_0, \hat{\theta}_1) \in \{\overline{\theta}, \underline{\theta}\}^2$ by the agent, a long-term contract specifies contingent transfer levels at dates 0 and 1, $\tilde{u}_0(\hat{\theta}_0)$ and $\tilde{u}_1(\hat{\theta}_0, \hat{\theta}_1)$, and contingent quality levels at dates 1 and 2, $\tilde{q}_1(\hat{\theta}_0)$ and $\tilde{q}_2(\hat{\theta}_0, \hat{\theta}_1)$.[12]

### *4.2.1. The Regulator's Problem*

We solve for the optimal contract in two steps. Consider first the date 1 contract. Since the regulator and the agent are risk-neutral and discount future utility at the same rate, there is no loss of generality in assuming that, whenever $\theta_1 = \underline{\theta}$, the agent receives no rent at date 1 no matter his date 0 report $\hat{\theta}_0$,

$$\tilde{u}_1(\hat{\theta}_0, \underline{\theta}) = \psi(\tilde{q}_2(\hat{\theta}_0, \underline{\theta}) - \delta \tilde{q}_1(\hat{\theta}_0) - \underline{\theta})$$

for each $\hat{\theta}_0 \in \{\overline{\theta}, \underline{\theta}\}$. Standard considerations imply that, given a date 0 report $\hat{\theta}_0$ , the rent that the agent receives at date 1 whenever $\theta_1 = \overline{\theta}$ must be given by $\Phi(\tilde{q}_1(\hat{\theta}_0), \tilde{q}_2(\hat{\theta}_0, \underline{\theta}))$. In turn, incentive compatibility at date 0 requires that

$$\begin{aligned} &\tilde{u}_0(\overline{\theta}) - \psi(\tilde{q}_1(\overline{\theta}) - \delta q_0 - \overline{\theta}) + \nu\beta\Phi(\tilde{q}_1(\overline{\theta}), \tilde{q}_2(\overline{\theta}, \underline{\theta})) \\ &\qquad\qquad \geq \tilde{u}_0(\underline{\theta}) - \psi(\tilde{q}_1(\underline{\theta}) - \delta q_0 - \overline{\theta}) + \nu\beta\Phi(\tilde{q}_1(\underline{\theta}), \tilde{q}_2(\underline{\theta}, \underline{\theta})), \end{aligned} \tag{37}$$

$$\begin{aligned} &\tilde{u}_0(\underline{\theta}) - \psi(\tilde{q}_1(\underline{\theta}) - \delta q_0 - \underline{\theta}) + \nu\beta\Phi(\tilde{q}_1(\underline{\theta}), \tilde{q}_2(\underline{\theta}, \underline{\theta})) \\ &\qquad\qquad \geq \tilde{u}_0(\overline{\theta}) - \psi(\tilde{q}_1(\overline{\theta}) - \delta q_0 - \underline{\theta}) + \nu\beta\Phi(\tilde{q}_1(\overline{\theta}), \tilde{q}_2(\overline{\theta}, \underline{\theta})). \end{aligned} \tag{38}$$

Since the agent must receive a non-negative utility at each date and in each state, an incentive feasible contract must also satisfy the following date 0 limited liability constraints:

$$\tilde{u}_0(\overline{\theta}) - \psi(\tilde{q}_1(\overline{\theta}) - \delta q_0 - \overline{\theta}) \geq 0, \tag{39}$$

$$\tilde{u}_0(\underline{\theta}) - \psi(\tilde{q}_1(\underline{\theta}) - \delta q_0 - \underline{\theta}) \geq 0. \tag{40}$$

An optimal contract is denoted by $(u_0^{\bullet\bullet}, q_1^{\bullet\bullet}, u_1^{\bullet\bullet}, q_2^{\bullet\bullet})$. It is not a priori clear which of the constraints (37)–(38) and (39)–(40) are binding at the optimum. This is because we require the optimal contract to satisfy not only intertemporal participation constraints, but also the more restrictive constraints (39)–(40) on date 0 utilities. As a result of this, one cannot for instance argue that the incentive constraint of the high type agent and the limited liability constraint of the low type agent together imply the limited liability constraint of the high type agent, in analogy with a standard model. Still, one can show that, in the optimal long-term contract, the usual monotonicity constraint on quantities applies, that is $q_1^{\bullet\bullet}(\overline{\theta}) \geq q_1^{\bullet\bullet}(\underline{\theta})$.

[12] We hereby restrict the analysis to deterministic mechanisms. Since the objective function of the regulator is not necessarily concave, she may increase social welfare by using stochastic mechanisms (see Laffont and Martimort (2002, §2.13)). Such mechanisms are however difficult to implement in practice, which is why we chose to rule them out. This problem does not arise in the linear-quadratic specification, because the function $\Phi$ is then linear over the relevant range, see (31).

Moreover, constraints (37) and (40) are binding, leading to the following expression for the date 0 rent of a high type agent:

$$U_0^{\bullet\bullet}(\overline{\theta}) = \Phi(q_0, q_1^{\bullet\bullet}(\underline{\theta})) + \nu\beta\big[\Phi(q_1^{\bullet\bullet}(\underline{\theta}), q_2^{\bullet\bullet}(\underline{\theta},\underline{\theta})) - \Phi(q_1^{\bullet\bullet}(\overline{\theta}), q_2^{\bullet\bullet}(\overline{\theta},\underline{\theta}))\big]. \tag{41}$$

We refer to Appendix B for the proof of this result. Equation (41) has a natural interpretation: the first term on the right-hand side is a standard static informational rent, while the second term represents the expected discounted gain in terms of date 1 rents from pretending to be a low type agent rather than a high type agent at date 0.

Hence, the total expected discounted rents at date 0 amount to

$$\begin{aligned} &\nu\big\{\Phi(q_0, q_1^{\bullet\bullet}(\underline{\theta})) + \nu\beta\big[\Phi(q_1^{\bullet\bullet}(\underline{\theta}), q_2^{\bullet\bullet}(\underline{\theta},\underline{\theta})) - \Phi(q_1^{\bullet\bullet}(\overline{\theta}), q_2^{\bullet\bullet}(\overline{\theta},\underline{\theta}))\big]\big\} \\ &\qquad + \nu^2\beta\Phi(q_1^{\bullet\bullet}(\overline{\theta}), q_2^{\bullet\bullet}(\overline{\theta},\underline{\theta})) + \nu(1-\nu)\beta\Phi(q_1^{\bullet\bullet}(\underline{\theta}), q_2^{\bullet\bullet}(\underline{\theta},\underline{\theta})) \qquad (42) \\ &= \nu\Phi(q_0, q_1^{\bullet\bullet}(\underline{\theta})) + \nu\beta\Phi(q_1^{\bullet\bullet}(\underline{\theta}), q_2^{\bullet\bullet}(\underline{\theta},\underline{\theta})). \end{aligned}$$

With this result at hand, one can simplify the regulator's problem as follows. For a given value of $q_0$, the pair of functions $(q_1^{\bullet\bullet}, q_2^{\bullet\bullet})$ solves

$$\begin{aligned} \max_{(\tilde{q}_1,\tilde{q}_2)} &\{\mathrm{E}\,[-(1+\lambda)\psi(\tilde{q}_1(\theta_0) - \delta q_0 - \theta_0) + \beta S(\tilde{q}_1(\theta_0)) \\ &- (1+\lambda)\beta\psi(\tilde{q}_2(\theta_0,\theta_1) - \delta\tilde{q}_1(\theta_0) - \theta_1) + \beta^2 S(\tilde{q}_2(\theta_0,\theta_1))] \qquad (43) \\ &- \lambda\nu\Phi(q_0, \tilde{q}_1(\underline{\theta})) - \lambda\nu\beta\Phi(\tilde{q}_1(\underline{\theta}), \tilde{q}_2(\underline{\theta},\underline{\theta}))\}, \end{aligned}$$

subject to the monotonicity constraint

$$\tilde{q}_1(\overline{\theta}) \geq \tilde{q}_1(\underline{\theta}), \tag{44}$$

and the constraint that the date 0 rent of a high agent be positive,

$$\Phi(q_0, \tilde{q}_1(\underline{\theta})) + \nu\beta\big[\Phi(\tilde{q}_1(\underline{\theta}), \tilde{q}_2(\underline{\theta},\underline{\theta})) - \Phi(\tilde{q}_1(\overline{\theta}), \tilde{q}_2(\overline{\theta},\underline{\theta}))\big] \geq 0. \tag{45}$$

While constraints (37) and (40) are necessarily binding in the optimal long-term contract, one cannot decide a priori on the status of constraints (44)–(45). Indeed, pooling over date 1 quality may occur, so that (38) or equivalently (44) may be binding. Furthermore, it is unclear that a high type agent receives a strictly positive rent at date 0, and thus that (39) or equivalently (45) are slack. In what follows, for sake of clarity, we only consider the case when the parameter values are such that both (44) and (45) are slack. It is worth noting that the main results remain unchanged when one or both constraints are binding. A detailed analysis of this model is provided in Appendix B.

*4.2.2. Date 2 Quality Levels*

We first investigate date 2 quality levels. As (45) can be neglected, there is no term of the form $\Phi(\tilde{q}_1(\overline{\theta}), \tilde{q}_2(\overline{\theta},\underline{\theta}))$ in the program (43)–(45) that determines $q_2^{\bullet\bullet}$. This reflects that, while increasing the date 1 rent $\Phi(\tilde{q}_1(\overline{\theta}), \tilde{q}_2(\overline{\theta},\underline{\theta}))$ by $\varepsilon > 0$ has a direct expected discounted cost $\lambda\nu^2\beta\varepsilon$ for the regulator, it also allows her to reduce the expected cost $\lambda\nu U_0^{\bullet\bullet}(\overline{\theta})$ of the date 0 rent by exactly the same amount, see (41). Thus the cost of providing incentives at date 1 given that the agent had a high type at date 0 is already taken into account in the rent left by the regulator to this agent at date 0, and is therefore perfectly internalized by her. By contrast, increasing the date 1 rent $\Phi(\tilde{q}_1(\underline{\theta}), \tilde{q}_2(\underline{\theta}, \underline{\theta}))$ by $\varepsilon > 0$ has a direct expected discounted cost $\lambda\nu(1-\nu)\beta\varepsilon$ at date 1, to which must be added an indirect expected cost $\lambda\nu^2\beta\varepsilon$ at date 0 due to the increase of the date 0 rent $U_0^{\bullet\bullet}(\overline{\theta})$, see again (41). This reflects that raising $\Phi(\tilde{q}_1(\underline{\theta}), \tilde{q}_2(\underline{\theta}, \underline{\theta}))$ makes a deviation more attractive for a high type agent at date 0. Given these observations, we obtain that there are no distortions of date 2 quality given the date 1 quality level if $\theta_0 = \overline{\theta}$ or $\theta_1 = \overline{\theta}$, while the date 2 quality if $(\theta_0, \theta_1) = (\underline{\theta}, \underline{\theta})$ is distorted downward according to

$$\beta S'(q_2^{\bullet\bullet}(\underline{\theta}, \underline{\theta})) = (1+\lambda)\psi'(q_2^{\bullet\bullet}(\underline{\theta}, \underline{\theta}) - \delta q_1^{\bullet\bullet}(\underline{\theta}) - \underline{\theta}) + \frac{\lambda\nu}{(1-\nu)^2}\,\Phi_2(q_1^{\bullet\bullet}(\underline{\theta}), q_2^{\bullet\bullet}(\underline{\theta}, \underline{\theta})). \quad (46)$$

The direct consequence of our limited liability constraints is that the optimal long-term contract exhibits memory, in the sense that the level of distortions at date 1 depends on the type of the agent at date 0. In particular, we have $q_2^{\bullet\bullet}(\overline{\theta}, \underline{\theta}) > q_2^{\bullet\bullet}(\underline{\theta}, \underline{\theta})$. This result is in line with the standard repeated moral hazard problem (Rogerson (1985)). It is due to the fact that, although the date 1 contract is signed at date 0 under symmetric information about the date 1 quality shock, the regulator cannot fully extract the agent's expected date 1 rent at date 0, unlike for instance in the Baron and Besanko (1984) regulation model with independent types. There are therefore two reasons for why $q_2^{\bullet\bullet}(\overline{\theta}, \underline{\theta}) > q_2^{\bullet\bullet}(\underline{\theta}, \underline{\theta})$ at the optimum. First, the physical link between periods and the fact that $q_1^{\bullet\bullet}(\overline{\theta}) \geq q_1^{\bullet\bullet}(\underline{\theta})$ together imply that it is less costly to enhance date 2 quality in state $(\overline{\theta}, \underline{\theta})$ than in state $(\underline{\theta}, \underline{\theta})$. Second, to this direct effect must be added the shadow cost of incentives, which is strictly higher following $\theta_0 = \underline{\theta}$ than following $\theta_0 = \overline{\theta}$.

### *4.2.3. Date 1 Quality Levels*

Having determined the optimal date 2 quality levels, one can move backward to date 1. We may have over-provision of date 1 quality following $\theta_0 = \underline{\theta}$ because $q_1^{\bullet\bullet}(\underline{\theta})$ does not only affect consumer surplus, but also the informational rents that must be left to the agent. As shown by (42), these rents consist of two terms. The first term is proportional to $\Phi(q_0, q_1^{\bullet\bullet}(\underline{\theta}))$, which is increasing in $q_1^{\bullet\bullet}(\underline{\theta})$. This reflects a standard static rent extraction motive. By contrast, the second term is proportional to $\Phi(q_1^{\bullet\bullet}(\underline{\theta}), q_2^{\bullet\bullet}(\underline{\theta}, \underline{\theta}))$, which is decreasing in $q_1^{\bullet\bullet}(\underline{\theta})$. As in the multiple agent model of Section 3, this reflects a dynamic rent extraction motive. The logic is slightly different, however. In the multiple agent model, the regulator typically wanted to increase the quality produced by a *high* type agent at date 0 only in order to decrease the rents that she had to concede at date 1, regardless of date 0 incentives. By contrast, in the single agent model, the regulator may want to increase the quality produced by a *low* type agent at date 0 in order to reduce the date 1 rents following a report $\hat{\theta}_0 = \underline{\theta}$, and thus make a deviation less attractive for a high type agent at date 0. The fact that, as discussed in Subsection 4.2.2, date 1 rents are treated asymmetrically by the regulator, depending on the type of the agent at date 0, explains why this dynamic rent extraction effect now affects the quality produced by a low type agent at date 0, and not directly that produced by a high type agent. As shown in Appendix B, this can lead to over-provision of date 1 quality by a low type agent at date 0.

### *4.3. Single versus Multiple Agents*

We now compare regulation with a single agent and with a sequence of agents. The key difference between the two setups is that, when the regulator contracts with a single agent, she is no longer constrained to use sequentially rational date 2 Markovian quality policies contingent on date 1 quality and the agent's date 1 type. In particular, the regulator commits to an efficient quality level when $(\theta_0, \theta_1) = (\overline{\theta}, \underline{\theta})$, whereas she would find it optimal to distort downward quality were she facing multiple agents in the same circumstances.[13] Conversely, as shown by (46), she commits to a higher level of distortions when $(\theta_0, \theta_1) = (\underline{\theta}, \underline{\theta})$, reflecting the cost of providing dynamic incentives.

Although general comparison results are hard to obtain, one can unambiguously rank date 1 quality levels in the two regulatory environments. Specifically, let $\overline{q}_1^{**}(q_0)$ and $\underline{q}_1^{**}(q_0)$ be the quality levels at date 1 following respectively a high and low quality shock at date 0 when there are multiple agents. Then the following holds.

**Proposition 6.** *The quality spread at date 1 is wider with a sequence of agents than with a single agent,*

$$\overline{q}_1^{**}(q_0) > q_1^{\bullet\bullet}(\overline{\theta}) > q_1^{\bullet\bullet}(\underline{\theta}) > \underline{q}_1^{**}(q_0). \tag{47}$$

This result reflects the difference in date 1 distortions that occur in the two regulatory environments. With a sequence of agents, as in the basic model of Section 3, there is over-provision of date 1 quality following a high quality shock at date 0. This is because the date 1 informational rent that has to be left when there is a further high quality shock at date 1 is not internalized by the regulator, as opposed to when there is a single agent. Increasing date 1 quality allows the regulator to reduce this rent, which leads to the first half of (47). By contrast, the date 1 distortions following a low quality shock at date 0 are larger with a single agent than with a sequence of agents, reflecting the cost of making a deviation for a high type agent less attractive at date 0. In turn, increasing date 1 quality allows the regulator to alleviate this cost, which leads to the second half of (47).

A natural question is whether it is better from the regulator's viewpoint to hire a single agent or to contract with a sequence of agents.[14] In our model, each option comes with costs and benefits. On the one hand, by hiring a single agent, the regulator can directly condition his date 1 compensation on his date 0 performance, while this occurs only indirectly through date 1 quality when there is a sequence of agents. This in turn allows the regulator to internalize some of the costs of providing incentives at date 1, as explained in Subsection 4.2.2. On the other hand, the fact that the same agent is rewarded over two periods raises the cost of providing incentives at date 0, because the agent anticipates the impact of his date 0 actions on his date 1 utility.

[13] The optimal distortion level is given by equation (B.36) in Appendix B, see also footnote 15.

[14] The answer to this question is not a priori obvious in our context because quality physically links periods together, and agents must receive a non-negative utility at each date and in each state. By contrast, in Baron and Besanko (1984), long-term contracts can be signed that need not satisfy this condition, and there is no physical intertemporal link. When types are independent across periods, sequential contracting with multiple agents is then suboptimal because it does not allow the regulator to extract the expected date 1 rent at date 0. This result still holds if, in addition, one modifies their model by imposing our restrictions on utilities: in the absence of a physical intertemporal link, the optimal contract with a sequence of agents is then simply the repetition of the optimal static contract, which can obviously be replicated in a long-term contract with a single agent.

Which of these effects prevails is a priori unclear. Part of the difficulty is that it is not possible to replicate the optimal contract with a sequence of agents in the environment with full commitment and a single agent. The problem lies with the incentive compatibility constraint (37). To see this, let $\underline{q}_2^{**}(q_1)$ be the optimal quality level at date 2 given a date 1 quality level $q_1$ and a low quality shock at date 1, in the model with a sequence of agents. As for (23), $\Phi(q_1, \underline{q}_2^{**}(q_1))$ is a decreasing function of $q_1$. Thus, since $\overline{q}_1^{**}(q_0) > \underline{q}_1^{**}(q_0)$,

$$\Phi(\overline{q}_1^{**}(q_0), \underline{q}_2^{**}(\overline{q}_1^{**}(q_0))) < \Phi(\underline{q}_1^{**}(q_0), \underline{q}_2^{**}(\underline{q}_1^{**}(q_0))). \tag{48}$$

Moreover, in the model with a sequence of agents, the date 1 analogue of (12) is binding at the optimum, as a date 1 agent facing a high quality shock is indifferent between reporting truthfully or lying. Along with (48), this implies that the optimal allocation and transfers in the model with a sequence of agents violate the incentive compatibility constraint (37). Intuitively, this is because short-lived agents do not internalize the fall in future rents that follows a high quality shock, while a long-lived agent does.

While a general comparison result seems hard to obtain, one can use the linear-quadratic specification to compare the levels of welfare obtained in both regulatory environments.

**Proposition 7.** *Consider the linear-quadratic case, and fix all the parameters of the model except $\nu$. When $q_0$, $\underline{\theta}$ and $\Delta\theta$ are close enough to zero and $\lambda < 1$, there exists a threshold $\nu^* \in (0, 1)$ such that ex-ante social welfare is higher in the optimal contract with a sequence of agents than in the optimal contract with a single agent if $\nu < \nu^*$.*

The intuition for this result is that, when the probability of a high quality shock is low, a regulator contracting with a single agent faces a high probability of having to distort heavily the date 2 allocation in order to maintain the incentives of the agent at date 1. Moreover, the benefits from fully internalizing the cost of providing incentives to a high type agent at date 1 are small since a high quality shock is relatively unlikely to occur. In these circumstances, the regulator would be better off decoupling incentives in the two periods by rather contracting with a sequence of agents.

### *4.4. No Commitment and the Ratchet Effect*

Let us briefly consider the case where there is a single agent, but neither the regulator nor the agent can commit to a long-term contract. At date 1, the regulator offers the same contract than with a sequence of agents. At date 0, and for an initial level of quality $q_0$, a contract is simply a 4-tuple $(\overline{u}_0, \overline{q}_1, \underline{u}_0, \underline{q}_1)$. Incentive compatibility at date 0 requires that

$$\begin{aligned}&\overline{u}_0 - \psi(\overline{q}_1 - \delta q_0 - \overline{\theta}) + \nu\beta\Phi(\overline{q}_1, \underline{q}_{2,2}^{**}(\overline{q}_1)) \\ &\qquad \geq \underline{u}_0 - \psi(\underline{q}_1 - \delta q_0 - \overline{\theta}) + \nu\beta\Phi(\underline{q}_1, \underline{q}_{2,2}^{**}(\underline{q}_1)),\end{aligned} \tag{49}$$

$$\begin{aligned}&\underline{u}_0 - \psi(\underline{q}_1 - \delta q_0 - \underline{\theta}) + \nu\beta\Phi(\underline{q}_1, \underline{q}_{2,2}^{**}(\underline{q}_1)) \\ &\qquad \geq \overline{u}_0 - \psi(\overline{q}_1 - \delta q_0 - \underline{\theta}) + \nu\beta\Phi(\overline{q}_1, \underline{q}_{2,2}^{**}(\overline{q}_1)).\end{aligned} \tag{50}$$

Because $\psi$ is convex, an immediate implication of (49)–(50) is that $\overline{q}_1 \geq \underline{q}_1$. In this context, if a low type agent misrepresents his type at date 0, he will still contract with the regulator at date 1 and earn a positive rent on average: a "take-the-money-and-run" strategy is not profitable, reflecting the fact that types are independent across periods. This represents a key difference between this setup and a standard dynamic adverse selection model with perfectly correlated types (Laffont and Tirole (1988)).

Since the agent must receive a non-negative utility in each state, an incentive feasible contract must also satisfy the following date 0 limited liability constraints:

$$\overline{u}_0 - \psi(\overline{q}_1 - \delta q_0 - \overline{\theta}) \geq 0, \tag{51}$$

$$\underline{u}_0 - \psi(\underline{q}_1 - \delta q_0 - \underline{\theta}) \geq 0. \tag{52}$$

Given that, in analogy with the basic model, the date 1 rent is a decreasing function of date 1 quality, it is easy to check that (49) and (52) imply (51) provided that $\overline{q}_1 \geq \underline{q}_1$. Adding this as a constraint to the regulator's problem, one can then solve for the optimal date 0 contract in a completely standard way. The incentive compatibility constraint (49) of the high type agent is binding, as well as the limited liability constraint (52) of the low type agent. Straightforward manipulations imply that the incentive compatibility constraint (50) of the low type agent can be rewritten as $\Phi(q_0, \overline{q}_1) \geq \Phi(q_0, \underline{q}_1)$, which is satisfied when $\overline{q}_1 \geq \underline{q}_1$.

Although the complications that usually arise in models of dynamic adverse selection are absent in this model, the lack of commitment still generates a ratchet effect. Indeed, since the regulator cannot commit to a compensation scheme at date 1, a high type agent anticipates at date 0 that, by revealing his type, he will reduce his date 1 rent, since

$$\Phi(\overline{q}_1, \underline{q}_2^{**}(\overline{q}_1)) \leq \Phi(\underline{q}_1, \underline{q}_2^{**}(\underline{q}_1)).$$

That is, the agent anticipates that being efficient today will increase the level of quality tomorrow and therefore jeopardize his continuation rent. The corresponding date 0 rent of a high type agent is therefore given by

$$\Phi(q_0, \underline{q}_1) + \nu\beta\Big[\Phi(\underline{q}_1, \underline{q}_2^{**}(\underline{q}_1)) - \Phi(\overline{q}_1, \underline{q}_2^{**}(\overline{q}_1))\Big].$$

To reduce this rent, it is typically optimal for the regulator to induce a pooling outcome at date 0 when $\beta$ is large.

It is straightforward to compare social welfare with a single agent and no commitment to social welfare with a sequence of agents. Indeed, the optimal date 1 policies are the same in both problems, while the incentive compatibility constraint at date 0 is more stringent with a single agent, due to the ratchet effect. Therefore, when no commitment is feasible, it is better from a social viewpoint to contract with a sequence of agents rather than to hire a single agent. This contrasts with the findings of Lewis and Yildirim (2002, Proposition 4), who show in a learning-by-doing model that when supply costs decrease with past production, the regulator will prefer dealing with a single supplier rather than relying on less durable franchises. The reason is that, in their model, the regulated firm's informational rents are higher when supply costs falls and the regulator demands more service, while these rents are lower in our model when quality increases.

## 5. Concluding Remarks

We have explored the design of incentives for quality provision in a dynamic regulation framework in which maintenance efforts and quality shocks have long-lasting effects. We considered in turn two regulatory frameworks, one with a sequence of short-term agents, and one with a single long-term agent. When the regulator contracts with a sequence of agents, asymmetries of information can result in over-provision of quality under optimal regulation, reflecting a dynamic rent extraction motive. This contrasts with the standard prediction of a static model of quality provision. In the linear-quadratic example, asymmetries of information lead to higher (lower) maintenance efforts in the case of a high (low) quality shock, which translates into more volatile quality growth than under symmetric information. When the regulator hires a single agent to manage quality, over-provision of quality can also occur, but through a different mechanism: optimal regulation can lead to over-provision of quality following a low quality shock in order to deter deviations from agents facing high quality shocks. In the linear-quadratic example, the regulator may prefer contracting with a sequence of agents rather than hiring a single agent to avoid the cost of providing dynamic incentives. This provides an argument in favor of shorter franchises even in the case in which all parties can commit to a long-term contract.

This paper abstracts from several important features of quality regulation, which should be investigated in future work. We have considered a project of fixed size, implicitly focusing on quality as the sole dimension of differentiation. It would be interesting to extend the analysis to the case where consumers care both about the quantity and the quality of output, in order to explore the trade-off between the level and the quality of service, as well as the interaction between the regulated firm's prices and changes in quality. Another meaningful extension of the analysis would be to relax the assumption that quality is verifiable by allowing only imperfect signals of quality to be ascertained in court.

Finally, the techniques and insights developed in this paper might be also applied to the study of other dynamic agency relationships in which the cost of providing incentives is affected by an endogenous state variable as in our model. For instance, the owner of a durable good such as a flat or a building might be concerned with the maintenance efforts exerted by the successive renters, while the latter may differ with respect to the kind of use they will make of the property—they may be for instance careful or careless. The state of the property would then vary over time as a function of the successive renters' types and maintenance efforts, while the value of the property would be endogenously determined by the whole sequence of rental contracts offered by the owner. As in our model, a succession of careful renters may increase the state of the property over and above the first-best level. In this context, it might be interesting to endogenize the sequence of renters' types by studying the incentives of different types of renters to select properties of different qualities, and the impact of this self-selection mechanism on the value of property over time. These important questions are left for future research.

## Appendix A: Regulation with a Sequence of Agents

*Proof of Proposition 1.* Denote by $T^*$ and $T^{**}$ the Bellman operators associated to (5) and (20) respectively. The following lemma holds.

**Lemma A.1.** *Let $f$ and $g$ be two real-valued functions over $\mathbb{R}_+$ that are both bounded, strictly increasing, strictly concave and continuously differentiable over $\mathbb{R}_{++}$. Then, if $g' \geq f'$ over $\mathbb{R}_{++}$, $(T^{**}g)' > (T^*f)'$ over $\mathbb{R}_{++}$.*

*Proof.* Standard considerations (see for instance Stokey and Lucas (1989, §4)) imply that $T^*f$ and $T^{**}g$ are bounded, strictly increasing, strictly concave and continuously differentiable over $\mathbb{R}_{++}$. The condition $\psi'(0) = 0$ ensures that the optimal effort levels remain strictly positive. Fix $q > 0$. The first-order conditions corresponding to the program that defines $(T^*f)(q)$ are

$$\beta f'(\overline{q}_f^*(q)) \;=\; (1+\lambda)\psi'(\overline{q}_f^*(q) - \delta q - \overline{\theta}), \tag{A.1}$$

$$\beta f'(\underline{q}_f^*(q)) \;=\; (1+\lambda)\psi'(\underline{q}_f^*(q) - \delta q - \underline{\theta}). \tag{A.2}$$

By the Envelope Theorem,

$$(T^*f)'(q) = S'(q) + \nu\delta(1+\lambda)\psi'(\overline{q}_f^*(q) - \delta q - \overline{\theta}) + (1-\nu)\delta(1+\lambda)\psi'(\underline{q}_f^*(q) - \delta q - \overline{\theta}).$$

Using (A.1)–(A.2), this can be rewritten as:

$$(T^*f)'(q) = S'(q) + \nu\beta\delta f'(\overline{q}_f^*(q)) + (1-\nu)\beta\delta f'(\underline{q}_f^*(q)). \tag{A.3}$$

Similarly, the first-order conditions corresponding to the program that defines $(T^{**}g)(q)$ are

$$\beta g'(\overline{q}_g^{**}(q)) \;=\; (1+\lambda)\psi'(\overline{q}_g^{**}(q) - \delta q - \overline{\theta}), \tag{A.4}$$

$$\beta g'(\underline{q}_g^{**}(q)) \;=\; (1+\lambda)\psi'(\underline{q}_g^{**}(q) - \delta q - \underline{\theta}) + \frac{\lambda\nu}{1-\nu}\,\Phi_2(q, \underline{q}_g^{**}(q)). \tag{A.5}$$

By the Envelope Theorem,

$$(T^{**}g)'(q) = S'(q) + \nu\delta(1+\lambda)\psi'(\overline{q}_g^{**}(q) - \delta q - \overline{\theta}) + (1-\nu)\delta(1+\lambda)\psi'(\underline{q}_g^{**}(q) - \delta q - \overline{\theta}) - \nu\lambda\Phi_1(q, \underline{q}_g^{**}(q)).$$

Using (A.4)–(A.5) together with $\Phi_1 = -\delta\Phi_2$, this can be rewritten as:

$$(T^{**}g)'(q) = S'(q) + \nu\beta\delta g'(\overline{q}_g^{**}(q)) + (1-\nu)\beta\delta g'(\underline{q}_g^{**}(q)). \tag{A.6}$$

Using the fact that $f$ and $g$ are strictly concave, that $\psi$ is strictly convex over $\mathbb{R}_+$ and that $g' \geq f'$, one can check from (A.1) and (A.4) that $g'(\overline{q}_g^{**}(q)) \geq f'(\overline{q}_f^*(q))$. Since $\Phi_2(q, \underline{q}_g^{**}(q)) > 0$, it follows in a similar way from (A.2) and (A.5) that $g'(\underline{q}_g^{**}(q)) > f'(\underline{q}_f^*(q))$. Therefore, by (A.3) and (A.6), one obtains that $(T^{**}g)'(q) > (T^*f)'(q)$, which implies the result since $q$ is arbitrarily chosen. □

We are now ready to complete the proof of Proposition 1. The fact that $T^*$ and $T^{**}$ are contractions with unique fixed points $V^*$ and $V^{**}$ over the space of bounded continuous functions defined over $\mathbb{R}_+$ ensures that $V^* = \lim_{n\to\infty} T^{*n}S$ and $V^{**} = \lim_{n\to\infty} T^{**n}S$ pointwise over $\mathbb{R}_+$. Since $V^*$ and $V^{**}$ are bounded, concave and differentiable over $\mathbb{R}_{++}$, this implies that $V^{*\prime} = \lim_{n\to\infty}(T^{*n}S)'$ and $V^{**\prime} = \lim_{n\to\infty}(T^{**n}S)'$ pointwise over $\mathbb{R}_{++}$ (Rockafellar (1970, Theorem 25.7)). By Lemma A.1,

$(T^{**n}S)' > (T^{*n}S)'$ over $\mathbb{R}_{++}$ for each $n \in \mathbb{N} \setminus \{0\}$. Taking limits, it follows that $V^{**\prime} \geq V^{*\prime}$ over $\mathbb{R}_{++}$. Applying Lemma A.1 again and using the fact that $T^*V^* = V^*$ and $T^{**}V^{**} = V^{**}$, one obtains that $V^{**\prime} > V^{*\prime}$ over $\mathbb{R}_{++}$, as claimed. ■

*Proof of Proposition 2.* For each $i \in \{*, **\}$, the effort mappings $\overline{e}^i$ and $\underline{e}^i$ are strictly decreasing, or, equivalently, the mappings $q \mapsto \overline{q}^i(q) - \delta q$ and $q \mapsto \underline{q}^i(q) - \delta q$ are strictly decreasing. Since $\delta \in (0, 1)$ and both $\overline{q}^i(0)$ and $\underline{q}^i(0)$ are strictly positive, and since the mappings $\overline{q}^i$ and $\underline{q}^i$ are strictly increasing with $\overline{q}^i > \underline{q}^i$, this implies that there are exactly two points $q^{i+} > q^{i-} > 0$ such that $\overline{q}^i(q^{i+}) = q^{i+}$ and $\underline{q}^i(q^{i-}) = q^{i-}$. Note that $q^{**+} > q^{*+}$ since $\overline{q}^{**} > \overline{q}^*$ over $\mathbb{R}_{++}$. It is easy to verify that both $[0, q^{i-})$ and $(q^{i+}, \infty]$ are transient sets for the transition function $P^i$, so that one can restrict the analysis to $[q^{i-}, q^{i+}]$. Because the mappings $\overline{q}^i$ and $\underline{q}^i$ are strictly increasing and continuous, $P^i$ is monotone and satisfies the Feller property. Moreover, since $\lim_{n\to\infty} \overline{q}^{in}(q^{i-}) = q^{i+}$ and $\lim_{n\to\infty} \underline{q}^{in}(q^{i+}) = q^{i-}$, it follows that for any $q \in (q^{i-}, q^{i+})$, there exists an integer $n \geq 1$ such that $P^{in}(q^{i-}, [q, q^{i+}]) \geq \nu^n$ and $P^{in}(q^{i+}, [q^{i-}, q]) \geq (1-\nu)^n$. Thus $P^i$ satisfies the mixing Assumption 12.1 in Stokey and Lucas (1989) over $[q^{i-}, q^{i+}]$. As a result of this, there exists a unique probability measure $\mu^i$ over this interval that is invariant under $P^i$ in the sense that $\mu^i(A) = \int P^i(q, A)\, \mu^i(dq)$ for any Borel subset $A$ (Stokey and Lucas (1989, Theorem 12.12)). We now prove that $\max \operatorname{supp} \mu^i = q^{i+}$, which concludes the proof as $q^{**+} > q^{*+}$. Suppose instead that $q^{i+} > \max \operatorname{supp} \mu^i$. Then, since $\overline{q}^i(q) > q$ for any $q < q^{i+}$, $\overline{q}^i(q) > \max \operatorname{supp} \mu^i$ for any $q$ close enough to $\max \operatorname{supp} \mu^i$. By definition of $\operatorname{supp} \mu^i$, it follows that $\mu^i((\overline{q}^i)^{-1}((\max \operatorname{supp} \mu^i, q^{i+}))) > 0$. However, as $\mu^i$ is invariant under $P^i$, one then obtains that

$$\mu^i((\max \operatorname{supp} \mu^i, q^{i+})) = \int P^i(q, (\max \operatorname{supp} \mu^i, q^{i+}))\, \mu^i(dq) \geq \nu \mu^i((\overline{q}^i)^{-1}((\max \operatorname{supp} \mu^i, q^{i+}))) > 0,$$

a contradiction. Therefore $\max \operatorname{supp} \mu^i = q^{i+}$, as claimed, and similarly $\min \operatorname{supp} \mu^i = q^{i-}$. Finally, note that Theorem 12.12 in Stokey and Lucas (1989) also ensures that for any initial quality level, the distribution of quality converges weakly to $\mu^i$. This concludes the proof. ■

*Proof of Proposition 3.* We first solve (5) and (20) without restrictions on quality levels, and verify that (32) holds. Using (27)–(28) and (29)–(30) together with (26), and identifying terms in $q^2$ in (5) and (20), we first obtain that $B^* = B^{**} = B$, where $B$ is the positive solution to

$$B = b + \frac{\beta\delta^2 c(1+\lambda)B}{\beta B + c(1+\lambda)}. \tag{A.7}$$

Next, identifying terms in $q$ in (5) and (20), we obtain that

$$A^* = \frac{a[\beta B + c(1+\lambda)] - \beta\delta c B(1+\lambda)\mathrm{E}_\theta}{\beta B + c(1+\lambda)(1-\beta\delta)} \tag{A.8}$$

and

$$A^{**} = \frac{a[\beta B + c(1+\lambda)] - \beta\delta c B(1+\lambda)\mathrm{E}_\theta + \nu\beta\delta c B\lambda\Delta\theta}{\beta B + c(1+\lambda)(1-\beta\delta)}. \tag{A.9}$$

Hence $A^{**} > A^*$, and since $B^* = B^{**} = B$, it follows from (27)–(29) that $\overline{q}^{**}(q) > \overline{q}^*(q) > \underline{q}^*(q)$, as expected. Next, from (28) and (30), $\underline{q}^*(q) > \underline{q}^{**}(q)$ if and only if $\beta A^* > \beta A^{**} - \frac{\lambda\nu}{1-\nu}\, c\Delta\theta$. Using (A.8)–(A.9), this condition can be rewritten as:

$$\frac{\beta B + c(1+\lambda)(1-\beta\delta)}{1-\nu} > \beta^2\delta B,$$

which clearly holds as $(\beta, \delta, \nu) \in (0,1)^3$ and $B > 0$. Hence $\underline{q}^*(q) > \underline{q}^{**}(q)$, and (32) follows. We now provide parameter and range restrictions under which the social value functions $V^*$ and $V^{**}$ thus obtained are economically meaningful. Define $q^{*+}$ and $q^{**+}$ as in the proof of Proposition 2. By construction, the interval $[0, q^{**+}]$ is invariant under both the transition functions $P^*$ and $P^{**}$. We shall therefore assume that $q_0$ initially belongs to $[0, q^{**+}]$ and restrict our attention to this interval. A necessary condition for our solutions to (5) and (20) to be economically meaningful over $[0, q^{**+}]$ is that $q^{**+} < \frac{a}{b}$. Using (29) to solve for $q^{**+}$, and taking advantage of (A.7) and (A.9), one can check after straightforward algebraic manipulations that a sufficient condition for this to occur is

$$\frac{\overline{\theta} - \beta\delta \mathrm{E}_\theta}{(1-\delta)(1-\beta\delta)} + \frac{\nu\beta\delta\lambda\Delta\theta}{(1+\lambda)(1-\delta)(1-\beta\delta)} < \frac{a}{b}. \tag{A.10}$$

We shall hereafter assume that (A.10) holds. This condition immediately implies that

$$(1-\delta)\frac{a}{b} > \overline{\theta}. \tag{A.11}$$

Using (A.11) together with (A.7)–(A.9), one can in turn check that

$$\frac{A^{**}}{B} > \frac{A^*}{B} > \frac{a}{b}. \tag{A.12}$$

The inequalities (A.12) capture the intuitive fact that, as long as the surplus function $S$ is increasing, so are the social value functions $V^*$ and $V^{**}$. In particular, the latter are strictly increasing over $[0, q^{**+}]$. We now investigate under which conditions efforts remain strictly positive over this interval. It is easy to check from (27) and (29) that for each $i \in \{*, **\}$ and $q \in [0, q^{**+}]$,

$$\overline{e}^i(q) = \frac{\beta B}{\beta B + c(1+\lambda)}\left(\frac{A^i}{B} - \delta q - \overline{\theta}\right) > \frac{\beta B}{\beta B + c(1+\lambda)}\left[(1-\delta)\frac{a}{b} - \overline{\theta}\right] > 0,$$

where the first inequality follows from (A.12) and the fact that $q^{**+} < \frac{a}{b}$, and the second from (A.11). Hence efforts conditional on a high quality shock are strictly positive, and since $\underline{e}^*(q) > \overline{e}^*(q)$, so are efforts conditional on a low quality shock under symmetric information. To conclude, one must check that $\underline{e}^{**}(q) \geq \Delta\theta$ for all $q \in [0, q^{**+}]$, so that we are justified in using the quadratic specification of $\psi$ when writing down the informational rent of the high type agent, see (31). From (30), one has

$$\underline{e}^{**}(q) = \frac{\beta B}{\beta B + c(1+\lambda)}\left(\frac{A^{**}}{B} - \delta q - \underline{\theta}\right) - \frac{\lambda\nu c\Delta\theta}{(1-\nu)[\beta B + c(1+\lambda)]}.$$

Proceeding as above, we obtain that for each $q \in [0, q^{**+}]$,

$$\frac{\beta B}{\beta B + c(1+\lambda)}\left(\frac{A^{**}}{B} - \delta q - \underline{\theta}\right) > \frac{\beta B}{\beta B + c(1+\lambda)}\left[(1-\delta)\frac{a}{b} - \underline{\theta}\right].$$

Hence a sufficient condition for $\underline{e}^{**}(q) \geq \Delta\theta$ to hold for all $q \in [0, q^{**+}]$ is that

$$\left[(1-\delta)\frac{a}{b} - \underline{\theta}\right]\beta B > \left[\frac{\lambda\nu c}{1-\nu} + \beta B + c(1+\lambda)\right]\Delta\theta. \tag{A.13}$$

Note from (A.7) that $B$ is independent of $\overline{\theta}$, $\underline{\theta}$ and $\Delta\theta$. Hence, given (A.11), (A.13) typically holds if $\Delta\theta$ is close enough to zero. It is straightforward to find parameter values such that (A.10) and (A.13) simultaneously hold. ■

*Proof of Proposition 4.* Fix some $t = 0, 1, \ldots$. From (27)–(30), $\mathrm{E}\,[q^*_{t+1} \,|\, q^*_t = q] > \mathrm{E}\,[q^{**}_{t+1} \,|\, q^{**}_t = q]$ if and only if $\beta A^* > \beta A^{**} - \lambda\nu c\Delta\theta$. Using (A.8)–(A.9), this condition can be rewritten as:

$$\beta B + c(1+\lambda)(1-\beta\delta) > \beta^2\delta B,$$

which clearly holds as $(\beta, \delta, \nu) \in (0,1)^3$ and $B > 0$. Therefore (33) follows. As for (34), it is an immediate consequence of (32). For further reference, note that

$$\mathrm{Var}\,[q^*_{t+1}\,|\,q^*_t = q] \;=\; \left[\frac{c(1+\lambda)}{\beta B + c(1+\lambda)}\right]^2 \mathrm{Var}_\theta, \tag{A.14}$$

$$\mathrm{Var}\,[q^{**}_{t+1}\,|\,q^{**}_t = q] \;=\; \left[\frac{c(1+\lambda)}{\beta B + c(1+\lambda)}\right]^2 \mathrm{Var}_\theta + \frac{\lambda\nu^2 c^2 \Delta\theta^2[\nu\lambda + 2(1-\nu)(1+\lambda)]}{(1-\nu)[\beta B + c(1+\lambda)]^2}, \tag{A.15}$$

as is easy to check from (27)–(30). ■

*Proof of Proposition 5.* From (27)–(28) and (29)–(30), it follows that for each $t = 0, 1, \ldots,$

$$\mathrm{E}\,[q^*_{t+1}] \;=\; \frac{\beta A^* + c(1+\lambda)(\mathrm{E}_\theta + \delta \mathrm{E}\,[q^*_t])}{\beta B + c(1+\lambda)},$$

$$\mathrm{E}\,[q^{**}_{t+1}] \;=\; \frac{\beta A^{**} - \lambda\nu c\Delta\theta + c(1+\lambda)(\mathrm{E}_\theta + \delta \mathrm{E}\,[q^{**}_t])}{\beta B + c(1+\lambda)}.$$

Taking limits as $t$ goes to infinity, one obtains that

$$\mathrm{E}_{\mu^*} \;=\; \frac{\beta A^* + c(1+\lambda)\mathrm{E}_\theta}{\beta B + c(1+\lambda)(1-\delta)},$$

$$\mathrm{E}_{\mu^{**}} \;=\; \frac{\beta A^{**} - \lambda\nu c\Delta\theta + c(1+\lambda)\mathrm{E}_\theta}{\beta B + c(1+\lambda)(1-\delta)},$$

from which (35) follows as $\beta A^* > \beta A^{**} - \lambda\nu c\Delta\theta$, see the proof of Proposition 4. Next, for each $i \in \{*, **\}$ and $t = 0, 1, \ldots,$

$$\begin{aligned}
\mathrm{E}\,[\mathrm{Var}\,[q^i_{t+1}\,|\,q^i_t]] \;&=\; \mathrm{E}\,[(q^i_{t+1})^2] - \mathrm{E}\,[\mathrm{E}\,[q^i_{t+1}\,|\,q^i_t]^2] \\
&=\; \mathrm{E}\,[(q^i_{t+1})^2] - \mathrm{E}\left[\left[\mathrm{E}\,[q^i_{t+1}] + \frac{c\delta(1+\lambda)(q^i_t - \mathrm{E}[q^i_t])}{\beta B + c(1+\lambda)}\right]^2\right] \\
&=\; \mathrm{Var}\,[q^i_{t+1}] - \left[\frac{c\delta(1+\lambda)}{\beta B + c(1+\lambda)}\right]^2 \mathrm{Var}\,[q^i_t],
\end{aligned}$$

where the second equality follows from (27)–(28) and (29)–(30). Using (A.14)–(A.15), this yields

$$\mathrm{Var}\,[q^*_{t+1}] \;=\; \left[\frac{c(1+\lambda)}{\beta B + c(1+\lambda)}\right]^2 (\mathrm{Var}_\theta + \delta^2 \mathrm{Var}\,[q^*_t]),$$

$$\mathrm{Var}\,[q^{**}_{t+1}] \;=\; \left[\frac{c(1+\lambda)}{\beta B + c(1+\lambda)}\right]^2 (\mathrm{Var}_\theta + \delta^2 \mathrm{Var}\,[q^{**}_t]) + \frac{\lambda\nu^2 c^2 \Delta\theta^2[\nu\lambda + 2(1-\nu)(1+\lambda)]}{(1-\nu)[\beta B + c(1+\lambda)]^2}$$

for each $t = 0, 1, \ldots$. Taking limits as $t$ goes to infinity, one obtains that

$$\mathrm{Var}_{\mu^*} \;=\; \frac{c^2(1+\lambda)^2 \mathrm{Var}_\theta}{[\beta B + c(1+\lambda)]^2 - c^2\delta^2(1+\lambda)^2},$$

$$\mathrm{Var}_{\mu^{**}} \;=\; \frac{c^2(1+\lambda)^2 \mathrm{Var}_\theta}{[\beta B + c(1+\lambda)]^2 - c^2\delta^2(1+\lambda)^2} + \frac{\lambda\nu^2 c^2 \Delta\theta^2[\nu\lambda + 2(1-\nu)(1+\lambda)]}{(1-\nu)\{[\beta B + c(1+\lambda)]^2 - c^2\delta^2(1+\lambda)^2\}},$$

from which (36) follows. Hence the result. ■

## Appendix B: Regulation with a Single Agent: A Detailed Analysis

In this appendix, we offer a detailed analysis of the regulation problem with a single agent. We first provide a justification for the formulation (43)–(45) of the regulator's problem. Proceeding backwards, we then use this formulation to study the optimal provision of quality at date 2 and at date 1. We conclude with a comparison between the single agent case and the multiple agent case.

### *B.1. A Reformulation of the Regulator's Problem*

Using date 1 and 2 qualities and date 0 rents, denoted by $\tilde{U}_0(\theta) = \widetilde{u}_0(\theta) - \psi(\tilde{q}_1(\theta_0) - \delta q_0 - \theta_0)$, as control variables, the regulator's problem can be written as follows:

$$\max_{(\tilde{q}_1,\tilde{q}_2,\tilde{U}_0)} \{\mathrm{E}\,[-(1+\lambda)\psi(\tilde{q}_1(\theta_0) - \delta q_0 - \theta_0) + \beta S(\tilde{q}_1(\theta_0))$$

$$-(1+\lambda)\beta\psi(\tilde{q}_2(\theta_0,\theta_1) - \delta\tilde{q}_1(\theta_0) - \theta_1) + \beta^2 S(\tilde{q}_2(\theta_0,\theta_1))] \tag{B.1}$$

$$-\lambda\nu\tilde{U}_0(\overline{\theta}) - \lambda(1-\nu)\tilde{U}_0(\underline{\theta}) - \lambda\nu^2\beta\Phi(\tilde{q}_1(\overline{\theta}),\tilde{q}_2(\overline{\theta},\underline{\theta})) - \lambda\nu(1-\nu)\beta\Phi(\tilde{q}_1(\underline{\theta}),\tilde{q}_2(\underline{\theta},\underline{\theta}))\},$$

subject to the constraints

$$\tilde{U}_0(\overline{\theta}) + \nu\beta\Phi(\tilde{q}_1(\overline{\theta}),\tilde{q}_2(\overline{\theta},\underline{\theta})) \geq \tilde{U}_0(\underline{\theta}) + \Phi(q_0,\tilde{q}_1(\underline{\theta})) + \nu\beta\Phi(\tilde{q}_1(\underline{\theta}),\tilde{q}_2(\underline{\theta},\underline{\theta})), \tag{B.2}$$

$$\tilde{U}_0(\underline{\theta}) + \nu\beta\Phi(\tilde{q}_1(\underline{\theta}),\tilde{q}_2(\underline{\theta},\underline{\theta})) \geq \tilde{U}_0(\overline{\theta}) - \Phi(q_0,\tilde{q}_1(\overline{\theta})) + \nu\beta\Phi(\tilde{q}_1(\overline{\theta}),\tilde{q}_2(\overline{\theta},\underline{\theta})), \tag{B.3}$$

$$\tilde{U}_0(\overline{\theta}) \geq 0, \tag{B.4}$$

$$\tilde{U}_0(\underline{\theta}) \geq 0. \tag{B.5}$$

Let $(q_1^{\bullet\bullet}, q_2^{\bullet\bullet}, U_0^{\bullet\bullet})$ be the solution to this problem and define the equilibrium levels of effort as:

$$e_0^{\bullet\bullet}(\theta_0) = q_1^{\bullet\bullet}(\theta_0) - \delta q_0 - \theta_0, \tag{B.6}$$

$$e_1^{\bullet\bullet}(\theta_0,\theta_1) = q_2^{\bullet\bullet}(\theta_0,\theta_1) - \delta q_1^{\bullet\bullet}(\theta_0) - \theta_1 \tag{B.7}$$

for each $(\theta_0,\theta_1) \in \{\overline{\theta},\underline{\theta}\}^2$.

**Proposition B.1.** *In the optimal long-term contract, $q_1^{\bullet\bullet}(\overline{\theta}) \geq q_1^{\bullet\bullet}(\underline{\theta})$, and constraints (37) and (40), or equivalently constraints (B.2) and (B.5), are binding. Therefore, the date 0 rent of a high type agent is given by (41).*

*Proof.* One first has the following lemma.

**Lemma B.1.** $q_1^{\bullet\bullet}(\overline{\theta}) \geq q_1^{\bullet\bullet}(\underline{\theta})$.

*Proof.* An immediate implication of (B.2)–(B.3) is that $\Phi(q_0, q_1^{\bullet\bullet}(\overline{\theta})) \geq \Phi(q_0, q_1^{\bullet\bullet}(\underline{\theta}))$. Since $\Phi(q,\underline{q})$ is strictly increasing in $\underline{q}$ for $\underline{q} > \delta q + \underline{\theta}$, we need only to show that $q_1^{\bullet\bullet}(\overline{\theta}) > \delta q_0 + \underline{\theta}$. Suppose the contrary holds, and consider the following modification of $(q_1^{\bullet\bullet}, q_2^{\bullet\bullet}, U_0^{\bullet\bullet})$. First, keep $\tilde{q}_1(\underline{\theta})$, $\tilde{q}_2$, and

$\tilde{U}_0(\underline{\theta})$ the same as in the original contract, so that (B.5) is preserved. Second, let $\tilde{q}_1(\overline{\theta}) = \delta q_0 + \underline{\theta} + \varepsilon$ for some $\varepsilon > 0$, and let $\tilde{U}_0(\overline{\theta}) = U_0^{\bullet\bullet}(\overline{\theta}) + \nu\beta\Phi(q_1^{\bullet\bullet}(\overline{\theta}), q_2^{\bullet\bullet}(\overline{\theta}, \underline{\theta})) - \nu\beta\Phi(\tilde{q}_1(\overline{\theta}), q_2^{\bullet\bullet}(\overline{\theta}, \underline{\theta}))$ so as to maintain the same level of intertemporal rents for a good type agent at date 0. As a result of this, (B.2) is preserved. Next, since $\tilde{q}_1(\overline{\theta}) > q_1^{\bullet\bullet}(\overline{\theta})$ and $\Phi$ is increasing with respect to its second argument, (B.3) is also preserved. Finally, since $\tilde{q}_1(\overline{\theta}) > q_1^{\bullet\bullet}(\overline{\theta})$ and $\Phi$ is decreasing with respect to its first argument, one obtains that $\tilde{U}_0(\overline{\theta}) \geq U_0^{\bullet\bullet}(\overline{\theta})$ so that (B.4) is preserved. The new contract is therefore incentive feasible, and yields the same expected intertemporal rent to the agent as the original contract. The condition $\psi'(0) = 0$ then implies that, for $\varepsilon$ close enough to zero, this contract yields a strictly higher expected social welfare at date 0 than the original contract, a contradiction. □

It turns out that which constraints are binding at the optimum depends on the sign of the following quantities:

$$\Delta_r = \Phi(q_0, q_1^{\bullet\bullet}(\underline{\theta})) + \nu\beta\big[\Phi(q_1^{\bullet\bullet}(\underline{\theta}), q_2^{\bullet\bullet}(\underline{\theta}, \underline{\theta})) - \Phi(q_1^{\bullet\bullet}(\overline{\theta}), q_2^{\bullet\bullet}(\overline{\theta}, \underline{\theta}))\big], \tag{B.8}$$

$$\Delta_i = -\Phi(q_0, q_1^{\bullet\bullet}(\overline{\theta})) + \nu\beta\big[\Phi(q_1^{\bullet\bullet}(\overline{\theta}), q_2^{\bullet\bullet}(\overline{\theta}, \underline{\theta})) - \Phi(q_1^{\bullet\bullet}(\underline{\theta}), q_2^{\bullet\bullet}(\underline{\theta}, \underline{\theta}))\big]. \tag{B.9}$$

$\Delta_r$ is the date 0 rent that must be left to a high type agent whenever (37) and (40) are the binding constraints. For (39) to be satisfied, one must then have $\Delta_r \geq 0$. Similarly, $\Delta_i$ is the date 0 rent that must be left to a low type agent whenever (38) and (39) are the binding constraints. For (40) to be satisfied, one must then have $\Delta_i \geq 0$. The following lemma holds.

**Lemma B.2.** *$\Delta_r$ and $\Delta_i$ cannot be both non-negative unless they are both equal to zero. Moreover, at least one of the following assertions is true*:

*(i)* $\Delta_r \geq 0 \geq \Delta_i$, *in which case (37) and (40) are binding*;

*(ii)* $\Delta_i \geq 0 \geq \Delta_r$, *in which case (38) and (39) are binding.*

*Proof.* We first establish that $\Delta_r$ and $\Delta_i$ cannot be both non-negative unless they are both equal to zero. Suppose the contrary holds. Then, by (B.8)–(B.9),

$$\Phi(q_0, q_1^{\bullet\bullet}(\underline{\theta})) \geq \nu\beta\big[\Phi(q_1^{\bullet\bullet}(\overline{\theta}), q_2^{\bullet\bullet}(\overline{\theta}, \underline{\theta})) - \Phi(q_1^{\bullet\bullet}(\underline{\theta}), q_2^{\bullet\bullet}(\underline{\theta}, \underline{\theta}))\big] \geq \Phi(q_0, q_1^{\bullet\bullet}(\overline{\theta})),$$

with at least one strict inequality. This however implies that $\Phi(q_0, q_1^{\bullet\bullet}(\underline{\theta})) > \Phi(q_0, q_1^{\bullet\bullet}(\overline{\theta}))$, which contradicts Lemma B.1. This implies the claim.

Suppose next that $\Delta_r \geq 0$ at the optimum contract. Observing that one can rewrite (37) as $u_0^{\bullet\bullet}(\overline{\theta}) - \psi(e_0^{\bullet\bullet}(\overline{\theta})) \geq u_0^{\bullet\bullet}(\underline{\theta}) - \psi(e_0^{\bullet\bullet}(\underline{\theta})) + \Delta_r$ using (B.6) and (B.8), it necessarily follows that (40) is binding, since otherwise reducing $u_0^{\bullet\bullet}(\overline{\theta})$ and $u_0^{\bullet\bullet}(\underline{\theta})$ by some small amount $\varepsilon > 0$ would preserve all constraints and increase social welfare. Similarly, (37) must be binding since otherwise reducing $u_0^{\bullet\bullet}(\overline{\theta})$ by some small amount $\varepsilon > 0$ would preserve all constraints and increase social welfare. Similar arguments imply that if $\Delta_i \geq 0$ at the optimum contract, then (38) and (39) must be binding.

To conclude the proof, we must only check that $\Delta_r$ and $\Delta_i$ cannot be both strictly negative. Suppose the contrary holds. Then, observing that one can rewrite (37) as $u_0^{\bullet\bullet}(\overline{\theta}) - \psi(e_0^{\bullet\bullet}(\overline{\theta})) \geq u_0^{\bullet\bullet}(\underline{\theta}) - \psi(e_0^{\bullet\bullet}(\underline{\theta})) + \Delta_r$ using (B.8), and (38) as $u_0^{\bullet\bullet}(\underline{\theta}) - \psi(e_0^{\bullet\bullet}(\underline{\theta})) \geq u_0^{\bullet\bullet}(\overline{\theta}) - \psi(e_0^{\bullet\bullet}(\overline{\theta})) + \Delta_i$ using (B.9), it is clear that (37) and (40) cannot be both binding, as this would violate (39) since $\Delta_r < 0$, and that (38) and (39) cannot be both binding, as this would violate (40) since $\Delta_i < 0$. Now, suppose that (39) is slack. Then (40) must be binding, since otherwise reducing $u_0^{\bullet\bullet}(\overline{\theta})$ and $u_0^{\bullet\bullet}(\underline{\theta})$ by some small amount $\varepsilon > 0$ would preserve all constraints and increase social welfare. Thus, as (37) and (40) cannot be both binding, (37) must be slack. But (37) and (39) cannot both be slack, since otherwise reducing $u_0^{\bullet\bullet}(\overline{\theta})$ by some small amount $\varepsilon > 0$ would preserve all constraints and increase social welfare. Hence, (39) must be binding, contrary to the assumption. A similar reasoning implies that (40) must be binding as well. Since (37) and (40) cannot be both binding, and similarly for (38) and (39), this implies that (37)–(38) are slack. Therefore, for a given value of $q_0$, the function $q_2^{\bullet\bullet}$ solves

$$\max_{\tilde{q}_2} \{\mathrm{E}\,[-(1+\lambda)\psi(\tilde{q}_2(\theta_0,\theta_1) - \delta q_{2,1}(\theta_0) - \theta_1) + \beta S(\tilde{q}_2(\theta_0,\theta_1))]$$

$$- \lambda\nu^2\Phi(q_1^{\bullet\bullet}(\overline{\theta}), \tilde{q}_2(\overline{\theta},\underline{\theta})) - \lambda\nu(1-\nu)\Phi(q_1^{\bullet\bullet}(\underline{\theta}), \tilde{q}_2(\underline{\theta},\underline{\theta}))\}.$$

It follows that $q_2^{\bullet\bullet}(\overline{\theta},\underline{\theta}) = \underline{\mathcal{Q}}_{1,2}(q_1^{\bullet\bullet}(\overline{\theta}))$ and $q_2^{\bullet\bullet}(\underline{\theta},\underline{\theta}) = \underline{\mathcal{Q}}_{1,2}(q_1^{\bullet\bullet}(\underline{\theta}))$, where for each $q_1 \geq 0$, $\underline{\mathcal{Q}}_{1,2}(q_1)$ is implicitly defined by[15]

$$\beta S'(\underline{\mathcal{Q}}_{1,2}(q_1)) = (1+\lambda)\psi'(\underline{\mathcal{Q}}_{1,2}(q_1) - \delta q_1 - \underline{\theta}) + \frac{\lambda\nu}{1-\nu}\,\Phi_2(q_1, \underline{\mathcal{Q}}_{1,2}(q_1)). \tag{B.10}$$

The condition $\psi'(0) = 0$ ensures that $\underline{\mathcal{E}}_{1,1}(q_1) = \underline{\mathcal{Q}}_{1,2}(q_1) - \delta q_1 - \underline{\theta}$ remains strictly positive for each $q_1 \geq 0$. Using now standard arguments, it is easy to check from (B.10) that the function $\underline{\mathcal{E}}_{1,1}$ is strictly decreasing, which implies in turn that the informational rent

$$\Phi(q_1, \underline{\mathcal{Q}}_{1,2}(q_1)) = \psi(\underline{\mathcal{E}}_{1,1}(q_1)) - \psi(\underline{\mathcal{E}}_{1,1}(q_1) - \Delta\theta)$$

is a strictly decreasing function of $q_1$. Since $\Delta_r < 0$ and $q_1^{\bullet\bullet}(\overline{\theta}) \geq q_1^{\bullet\bullet}(\underline{\theta})$ by Lemma B.1, one has

$$\begin{aligned}
0 &\leq \Phi(q_0, q_1^{\bullet\bullet}(\underline{\theta})) \\
&< \nu\beta\big[\Phi(q_1^{\bullet\bullet}(\overline{\theta}), q_2^{\bullet\bullet}(\overline{\theta},\underline{\theta})) - \Phi(q_1^{\bullet\bullet}(\underline{\theta}), q_2^{\bullet\bullet}(\underline{\theta},\underline{\theta}))\big] \\
&= \nu\beta\Big[\Phi(q_1^{\bullet\bullet}(\overline{\theta}), \underline{\mathcal{Q}}_{1,2}(q_1^{\bullet\bullet}(\overline{\theta}))) - \Phi(q_1^{\bullet\bullet}(\underline{\theta}), \underline{\mathcal{Q}}_{1,2}(q_1^{\bullet\bullet}(\underline{\theta})))\Big] \\
&\leq 0,
\end{aligned}$$

a contradiction. Hence the result. □

To determine which of cases (i) or (ii) of Lemma B.2 holds, we first need the following result.

**Lemma B.3.** $\Delta_i \leq 0$.

*Proof.* Suppose by way of contradiction that $\Delta_i > 0$. Then, by Lemma B.2, (38) and (39) are binding. Moreover, a low type agent is left with a strictly positive rent $\Delta_i$ at date 0, so that (40) is slack. Therefore, for a given value of $q_0$, the function $q_{2,2}$ solves

$$\max_{\tilde{q}_2} \big\{\mathrm{E}\,[-(1+\lambda)\psi(\tilde{q}_2(\theta_0,\theta_1) - q_{2,1}(\theta_0) - \theta_1) + \beta S(\tilde{q}_2(\theta_0,\theta_1))] - \lambda\nu\Phi(q_1^{\bullet\bullet}(\overline{\theta}), \tilde{q}_2(\overline{\theta},\underline{\theta}))\big\}.$$

It follows that $q_2^{\bullet\bullet}(\overline{\theta},\underline{\theta}) = \underline{\mathcal{Q}}_{2,2}(q_1^{\bullet\bullet}(\overline{\theta}))$ and $q_2^{\bullet\bullet}(\underline{\theta},\underline{\theta}) = \underline{\mathcal{Q}}_{3,2}(q_1^{\bullet\bullet}(\underline{\theta}))$, where for each $q_1 \geq 0$, $\underline{\mathcal{Q}}_{2,2}(q_1)$ and $\underline{\mathcal{Q}}_{3,2}(q_1)$ are implicitly defined by

[15] Note that (B.10) is formally analogous to the first-order condition (22) for $\underline{q}^{**}(q)$ derived in the stationary model with multiple agents. Indeed, it is easy to see that (B.10) gives the sequentially rational date 2 optimal quality policy given that $\theta_1 = \underline{\theta}$, see (B.36).

$$\beta S'(\underline{\mathcal{Q}}_{2,2}(q_1)) \;=\; (1+\lambda)\psi'(\underline{\mathcal{Q}}_{2,2}(q_1) - \delta q_1 - \underline{\theta}) + \frac{\lambda}{1-\nu}\,\Phi_2(q_1, \underline{\mathcal{Q}}_{2,2}(q_1)). \tag{B.11}$$

$$\beta S'(\underline{\mathcal{Q}}_{3,2}(q_1)) \;=\; (1+\lambda)\psi'(\underline{\mathcal{Q}}_{3,2}(q_1) - \delta q_1 - \underline{\theta}), \tag{B.12}$$

The condition $\psi'(0) = 0$ ensures that $\underline{\mathcal{E}}_{2,1}(q_1) = \underline{\mathcal{Q}}_{2,2}(q_1) - \delta q_1 - \underline{\theta}$ and $\underline{\mathcal{E}}_{3,1}(q_1) = \underline{\mathcal{Q}}_{3,2}(q_1) - \delta q_1 - \underline{\theta}$ remain strictly positive for each $q_1 \geq 0$. Since one clearly has $\underline{\mathcal{Q}}_{2,2}(q_1) < \underline{\mathcal{Q}}_{3,2}(q_1)$ by (B.11)–(B.12), this implies that $\Phi(q_1, \underline{\mathcal{Q}}_{2,2}(q_1)) < \Phi(q_1, \underline{\mathcal{Q}}_{3,2}(q_1))$ for each $q_1 \geq 0$. Using now standard arguments, it is easy to check from (B.12) that the function $\underline{\mathcal{E}}_{3,1}$ is strictly decreasing, which implies in turn that the informational rent

$$\Phi(q_1, \underline{\mathcal{Q}}_{3,2}(q_1)) = \psi(\underline{\mathcal{E}}_{3,1}(q_1)) - \psi(\underline{\mathcal{E}}_{3,1}(q_1) - \Delta\theta)$$

is a strictly decreasing function of $q_1$. Since $\Delta_i > 0$ and $q_1^{\bullet\bullet}(\underline{\theta}) \geq q_1^{\bullet\bullet}(\underline{\theta})$ by Lemma B.1, one has

$$\begin{aligned}
0 \;&\leq\; \Phi(q_0, q_1^{\bullet\bullet}(\overline{\theta})) \\
&<\; \nu\beta\big[\Phi(q_1^{\bullet\bullet}(\overline{\theta}), q_2^{\bullet\bullet}(\overline{\theta}, \underline{\theta})) - \Phi(q_1^{\bullet\bullet}(\underline{\theta}), q_2^{\bullet\bullet}(\underline{\theta}, \underline{\theta}))\big] \\
&=\; \nu\beta\Big[\Phi(q_1^{\bullet\bullet}(\overline{\theta}), \underline{\mathcal{Q}}_{2,2}(q_1^{\bullet\bullet}(\overline{\theta}))) - \Phi(q_1^{\bullet\bullet}(\underline{\theta}), \underline{\mathcal{Q}}_{3,2}(q_1^{\bullet\bullet}(\underline{\theta})))\Big] \\
&<\; \nu\beta\Big[\Phi(q_1^{\bullet\bullet}(\overline{\theta}), \underline{\mathcal{Q}}_{3,2}(q_1^{\bullet\bullet}(\overline{\theta}))) - \Phi(q_1^{\bullet\bullet}(\underline{\theta}), \underline{\mathcal{Q}}_{3,2}(q_1^{\bullet\bullet}(\underline{\theta})))\Big] \\
&\leq\; 0,
\end{aligned}$$

a contradiction. Hence the result. □

Finally, the following lemma rules out the case $\Delta_i = 0 > \Delta_r$.

**Lemma B.4.** $\Delta_r \geq 0$.

*Proof.* Suppose by way of contradiction that $\Delta_r < 0$. Then, by Lemmas B.2 and B.3, $\Delta_i = 0$, so that (38)–(40) are binding. Observing that one can rewrite (37) as $u_0^{\bullet\bullet}(\overline{\theta}) - \psi(e_0^{\bullet\bullet}(\overline{\theta})) \geq u_0^{\bullet\bullet}(\underline{\theta}) - \psi(e_0^{\bullet\bullet}(\underline{\theta})) + \Delta_r$ using (B.8), it necessarily follows that (37) is slack. Now turn to the formulation (B.1)–(B.5) of the regulator's problem. We know from the proof of Lemma B.1 that $q_1^{\bullet\bullet}(\overline{\theta}) > \delta q_0 + \underline{\theta}$. It is easy to check that this implies that (B.2)–(B.5) satisfy the Kuhn-Tucker constraint qualification conditions at the optimum. Denote by $\eta_1$, $\eta_2$, $\eta_3$ and $\eta_4$ the corresponding multipliers. As (B.2) is slack at the optimum, $\eta_1 = 0$. It follows that $q_2^{\bullet\bullet}(\overline{\theta}, \underline{\theta}) = \underline{\mathcal{Q}}_{4,2}(q_1^{\bullet\bullet}(\overline{\theta}))$ and $q_2^{\bullet\bullet}(\underline{\theta}, \underline{\theta}) = \underline{\mathcal{Q}}_{5,2}(q_1^{\bullet\bullet}(\underline{\theta}))$, where for each $q_1 \geq 0$, $\underline{\mathcal{Q}}_{4,2}(q_1)$ and $\underline{\mathcal{Q}}_{5,2}(q_1)$ are implicitly defined by

$$\beta S'(\underline{\mathcal{Q}}_{4,2}(q_1)) \;=\; (1+\lambda)\psi'(\underline{\mathcal{Q}}_{4,2}(q_1) - \delta q_1 - \underline{\theta}) + \frac{\lambda\nu + \eta_2}{1-\nu}\,\Phi_2(q_1, \underline{\mathcal{Q}}_{4,2}(q_1)), \tag{B.13}$$

$$\beta S'(\underline{\mathcal{Q}}_{5,2}(q_1)) \;=\; (1+\lambda)\psi'(\underline{\mathcal{Q}}_{5,2}(q_1) - \delta q_1 - \underline{\theta}) + \frac{[\lambda(1-\nu) - \eta_2]\nu}{(1-\nu)^2}\,\Phi_2(q_1, \underline{\mathcal{Q}}_{5,2}(q_1)). \tag{B.14}$$

Optimizing with respect to $\tilde{U}_0(\underline{\theta})$ yields $\eta_2 + \eta_4 = \lambda(1-\nu)$, so that $\eta_2 \leq \lambda(1-\nu)$. The condition $\psi'(0) = 0$ then ensures that $\underline{\mathcal{E}}_{5,1}(q_1) = \underline{\mathcal{Q}}_{5,2}(q_1) - \delta q_1 - \underline{\theta}$ remains strictly positive for each $q_1 \geq 0$.

Since one clearly has $\underline{\mathcal{Q}}_{4,2}(q_1) \leq \underline{\mathcal{Q}}_{5,2}(q_1)$ by (B.13)–(B.14), $\Phi(q_1, \underline{\mathcal{Q}}_{4,2}(q_1)) \leq \Phi(q_1, \underline{\mathcal{Q}}_{5,2}(q_1))$ for each $q_1 \geq 0$. Using now standard arguments, it is easy to check from (B.14) that the function $\underline{\mathcal{E}}_{5,1}$ is strictly decreasing, which implies in turn that the informational rent

$$\Phi(q_1, \underline{\mathcal{Q}}_{5,2}(q_1)) = \psi(\underline{\mathcal{E}}_{5,1}(q_1)) - \psi(\underline{\mathcal{E}}_{5,1}(q_1) - \Delta\theta)$$

is a strictly decreasing function of $q_1$. Since $\Delta_r < 0$ and $q_1^{\bullet\bullet}(\overline{\theta}) \geq q_1^{\bullet\bullet}(\underline{\theta})$ by Lemma B.1, one has

$$\begin{aligned}
0 &\leq \Phi(q_0, q_1^{\bullet\bullet}(\underline{\theta})) \\
&< \nu\beta\big[\Phi(q_1^{\bullet\bullet}(\overline{\theta}), q_2^{\bullet\bullet}(\overline{\theta}, \underline{\theta})) - \Phi(q_1^{\bullet\bullet}(\underline{\theta}), q_2^{\bullet\bullet}(\underline{\theta}, \underline{\theta}))\big] \\
&= \nu\beta\Big[\Phi(q_1^{\bullet\bullet}(\overline{\theta}), \underline{\mathcal{Q}}_{4,2}(q_1^{\bullet\bullet}(\overline{\theta}))) - \Phi(q_1^{\bullet\bullet}(\underline{\theta}), \underline{\mathcal{Q}}_{5,2}(q_1^{\bullet\bullet}(\underline{\theta})))\Big] \\
&\leq \nu\beta\Big[\Phi(q_1^{\bullet\bullet}(\overline{\theta}), \underline{\mathcal{Q}}_{5,2}(q_1^{\bullet\bullet}(\overline{\theta}))) - \Phi(q_1^{\bullet\bullet}(\underline{\theta}), \underline{\mathcal{Q}}_{5,2}(q_1^{\bullet\bullet}(\underline{\theta})))\Big] \\
&\leq 0,
\end{aligned}$$

a contradiction. Hence the result. □

To conclude the proof of Proposition B.1, simply observe that Lemmas B.3 and B.4 imply that case (i) of Lemma B.2 obtains. Thus (37) and (40) are binding, and the result follows. ■

Proposition B.1 allows us to reformulate the regulator's problem as in (43)–(45). As pointed out at the end of Subsection 4.2.1, one cannot a priori determine whether constraint (45), or equivalently constraint (39), is binding or not. We shall say that we are in the regular case when parameter values are such that this constraint is slack, and in the irregular case otherwise.

### *B.2. Date 2 Quality Levels*

The following result offers a characterization of the optimal provision of quality at date 2.

**Proposition B.2.** *In the optimal long-term contract, there are no distortions of date 2 quality given the date 1 quality level if $\theta_1 = \overline{\theta}$. If $\theta_1 = \underline{\theta}$, the level of distortions at date 1 is strictly higher following $\theta_0 = \underline{\theta}$ than following $\theta_0 = \overline{\theta}$. In particular, $q_2^{\bullet\bullet}(\overline{\theta}, \underline{\theta}) > q_2^{\bullet\bullet}(\underline{\theta}, \underline{\theta})$.*

*Proof.* We first prove that $q_2^{\bullet\bullet}(\overline{\theta}, \underline{\theta}) > q_2^{\bullet\bullet}(\underline{\theta}, \underline{\theta})$. Let $\xi_2$ be the multiplier associated to (45). Then, from (43)–(45), the first-order conditions for date 2 quality levels are given by

$$\beta S'(q_2^{\bullet\bullet}(\overline{\theta}, \overline{\theta})) = (1+\lambda)\psi'(q_2^{\bullet\bullet}(\overline{\theta}, \overline{\theta}) - \delta q_1^{\bullet\bullet}(\overline{\theta}) - \overline{\theta}), \tag{B.15}$$

$$\beta S'(q_2^{\bullet\bullet}(\underline{\theta}, \overline{\theta})) = (1+\lambda)\psi'(q_2^{\bullet\bullet}(\underline{\theta}, \overline{\theta}) - \delta q_1^{\bullet\bullet}(\underline{\theta}) - \overline{\theta}), \tag{B.16}$$

$$\beta S'(q_2^{\bullet\bullet}(\overline{\theta}, \underline{\theta})) = (1+\lambda)\psi'(q_2^{\bullet\bullet}(\overline{\theta}, \underline{\theta}) - \delta q_1^{\bullet\bullet}(\overline{\theta}) - \underline{\theta}) + \frac{\xi_2}{1-\nu}\,\Phi_2(q_1^{\bullet\bullet}(\overline{\theta}), q_2^{\bullet\bullet}(\overline{\theta}, \underline{\theta})), \tag{B.17}$$

$$\beta S'(q_2^{\bullet\bullet}(\underline{\theta}, \underline{\theta})) = (1+\lambda)\psi'(q_2^{\bullet\bullet}(\underline{\theta}, \underline{\theta}) - \delta q_1^{\bullet\bullet}(\underline{\theta}) - \underline{\theta}) + \frac{(\lambda - \xi_2)\nu}{(1-\nu)^2}\,\Phi_2(q_1^{\bullet\bullet}(\underline{\theta}), q_2^{\bullet\bullet}(\underline{\theta}, \underline{\theta})). \tag{B.18}$$

The first part of the result follows directly from (B.15)–(B.16). We next prove that $q_2^{\bullet\bullet}(\overline{\theta},\underline{\theta}) > q_2^{\bullet\bullet}(\underline{\theta},\underline{\theta})$. In the regular case, this follows from substituting $\xi_2 = 0$ into (B.17)–(B.18) and using the fact that $q_1^{\bullet\bullet}(\overline{\theta}) \geq q_1^{\bullet\bullet}(\underline{\theta})$ by Lemma B.1. Consider next the irregular case. Two subcases must be distinguished. Suppose first that (B.3) or equivalently (44) is binding. From the proof of Lemma B.1, we know that $q_1^{\bullet\bullet}(\overline{\theta}) > \delta q_0 + \underline{\theta}$, and hence $\Phi(q_0, q_1^{\bullet\bullet}(\underline{\theta})) = \Phi(q_0, q_1^{\bullet\bullet}(\overline{\theta})) > 0$. Since $U_0^{\bullet\bullet}(\overline{\theta}) = 0$, it follows that $\Phi(q_1^{\bullet\bullet}(\overline{\theta}), q_2^{\bullet\bullet}(\overline{\theta},\underline{\theta})) > \Phi(q_1^{\bullet\bullet}(\underline{\theta}), q_2^{\bullet\bullet}(\underline{\theta},\underline{\theta}))$, which implies that $q_2^{\bullet\bullet}(\overline{\theta},\underline{\theta}) > q_2^{\bullet\bullet}(\underline{\theta},\underline{\theta})$ as claimed. Suppose next that (B.3) or equivalently (44) is slack. Since $U_0^{\bullet\bullet}(\overline{\theta}) = 0$, $\Phi(q_1^{\bullet\bullet}(\overline{\theta}), q_2^{\bullet\bullet}(\overline{\theta},\underline{\theta})) \geq \Phi(q_1^{\bullet\bullet}(\underline{\theta}), q_2^{\bullet\bullet}(\underline{\theta},\underline{\theta}))$. As (44) is slack, we need only to prove that this implies that $q_2^{\bullet\bullet}(\overline{\theta},\underline{\theta}) - q_2^{\bullet\bullet}(\underline{\theta},\underline{\theta}) \geq \delta[q_1^{\bullet\bullet}(\overline{\theta}) - q_1^{\bullet\bullet}(\underline{\theta})]$. Again, this will be the case if $q_2^{\bullet\bullet}(\overline{\theta},\underline{\theta}) > \delta q_1^{\bullet\bullet}(\overline{\theta}) + \underline{\theta}$. Suppose the contrary holds, and consider the following modification of $(q_1^{\bullet\bullet}, q_2^{\bullet\bullet}, U_0^{\bullet\bullet})$. First, keep $\tilde{q}_1$, $\tilde{q}_2(\overline{\theta},\overline{\theta})$, $\tilde{q}_2(\underline{\theta},\overline{\theta})$, $\tilde{q}_2(\underline{\theta},\underline{\theta})$ and $\tilde{U}_0$ the same as in the original contract, so that (B.4)–(B.5) are preserved. Second, let $\tilde{q}_2(\overline{\theta},\underline{\theta}) = \delta q_1^{\bullet\bullet}(\overline{\theta}) + \underline{\theta} + \varepsilon$ for some $\varepsilon > 0$. Since $\tilde{q}_2(\overline{\theta},\underline{\theta}) > q_2^{\bullet\bullet}(\overline{\theta},\underline{\theta})$ and $\Phi$ is increasing in its second argument, (B.2) is preserved. Moreover, since (B.3) is slack, it is preserved in the new contract if $\varepsilon$ is close enough to zero, which ensures that the new contract is incentive feasible. The condition $\psi'(0) = 0$ then implies that, for $\varepsilon$ close enough to zero, this contract yields a strictly higher expected social welfare at date 0 than the original contract, a contradiction. Thus $q_2^{\bullet\bullet}(\overline{\theta},\underline{\theta}) > \delta q_1^{\bullet\bullet}(\overline{\theta}) + \underline{\theta}$, which implies that $q_2^{\bullet\bullet}(\overline{\theta},\underline{\theta}) > q_2^{\bullet\bullet}(\underline{\theta},\underline{\theta})$ as claimed. Note that $(\Phi(q_1^{\bullet\bullet}(\overline{\theta}), q_2^{\bullet\bullet}(\overline{\theta},\underline{\theta})), \Phi(q_1^{\bullet\bullet}(\underline{\theta}), q_2^{\bullet\bullet}(\underline{\theta},\underline{\theta}))) \neq (0,0)$, so that (44)–(45) satisfy the Kuhn-Tucker constraint qualification conditions at the optimum.

We now show that the level of distortions at date 1 is strictly higher following $\theta_0 = \underline{\theta}$ than following $\theta_0 = \overline{\theta}$. Given the first-order conditions (B.17)–(B.18), we need to prove that $(\lambda - \xi_2)\nu > \xi_2(1-\nu)$ or equivalently $\lambda\nu > \xi_2$. If $\xi_2 = 0$ this is immediate. Suppose next that $\xi_2 > 0$, so that (45) is binding. Then, proceeding as above, one obtains that $q_2^{\bullet\bullet}(\overline{\theta},\underline{\theta}) - \delta q_1^{\bullet\bullet}(\overline{\theta}) \geq q_2^{\bullet\bullet}(\underline{\theta},\underline{\theta}) - \delta q_1^{\bullet\bullet}(\underline{\theta})$. Since $\psi'$ is convex, it therefore follows from (18) that $\Phi_2(q_1^{\bullet\bullet}(\overline{\theta}), q_2^{\bullet\bullet}(\overline{\theta},\underline{\theta})) \geq \Phi_2(q_1^{\bullet\bullet}(\underline{\theta}), q_2^{\bullet\bullet}(\underline{\theta},\underline{\theta}))$. Suppose now that $\xi_2 \geq \lambda\nu$. Then the right-hand side of (B.17) is greater than or equal to the right-hand side of (B.18). This however is impossible since $q_2^{\bullet\bullet}(\overline{\theta},\underline{\theta}) > q_2^{\bullet\bullet}(\underline{\theta},\underline{\theta})$ and $S$ is strictly concave. Thus $\lambda\nu > \xi_2$, and the result follows. ■

### *B.3. Date 1 Quality Levels*

We focus now on the monotonicity constraint (44), and we determine under which circumstances pooling over date 1 quality may occur.

**Proposition B.3.** *If $q_0$ is close enough to zero, pooling over date 1 quality can occur in the optimal long-term contract only if $\beta\delta > 1$.*

*Proof.* Suppose that pooling over date 1 qualities occurs in equilibrium, and let $\xi_1$ and $\xi_2$ be the multipliers associated to (44)–(45). Using the Envelope Theorem, the first-order conditions for $q_1^{\bullet\bullet}(\overline{\theta}) = q_1^{\bullet\bullet}(\underline{\theta}) = q_1^p$ can be written as follows:

$$\beta S'(q_1^p) = (1+\lambda)\psi'(q_1^p - \delta q_0 - \overline{\theta}) - \beta\delta(1+\lambda)\mathrm{E}\big[\psi'(q_2^{\bullet\bullet}(\overline{\theta},\theta_1)) - \delta q_1^p - \theta_1)\big] - \frac{\xi_1}{\nu} + \xi_2\beta\Phi_1(q_1^p, q_2^{\bullet\bullet}(\overline{\theta},\underline{\theta})), \tag{B.19}$$

$$\beta S'(q_1^p) = (1+\lambda)\psi'(q_1^p - \delta q_0 - \underline{\theta}) - \beta\delta(1+\lambda)\mathrm{E}[\psi'(q_2^{\bullet\bullet}(\underline{\theta},\theta_1)) - \delta q_1^p - \theta_1)] + \frac{\lambda\nu}{1-\nu}\,\Phi_2(q_0, q_1^p) + \frac{\lambda\nu\beta}{1-\nu}\,\Phi_1(q_1^p, q_2^{\bullet\bullet}(\underline{\theta},\underline{\theta})), \tag{B.20}$$

$$+\frac{\xi_1}{1-\nu}-\frac{\xi_2}{1-\nu}\,\Phi_2(q_0,q_1^p)-\frac{\xi_2\nu\beta}{1-\nu}\,\Phi_1(q_1^p,q_2^{\bullet\bullet}(\underline{\theta},\underline{\theta})),$$

The following lemma holds.

**Lemma B.5.** *If $q_0$ is close enough to zero, $q_1^p-\delta q_0>q_2^{\bullet\bullet}(\underline{\theta},\underline{\theta})-\delta q_1^p$.*

*Proof.* Using the definition of $\Phi$ and notably the fact that $\Phi_1=-\delta\Phi_2$, one can rewrite (B.19) as:

$$\beta S'(q_1^p) = (1+\lambda)\psi'(q_1^p-\delta q_0-\underline{\theta})-(1+\lambda)\Phi_2(q_0,q_1^p)-\beta\delta(1+\lambda)\mathrm{E}\big[\psi'(q_2^{\bullet\bullet}(\overline{\theta},\theta_1))-\delta q_1^p-\theta_1)\big] \tag{B.21}$$
$$-\frac{\xi_1}{\nu}-\xi_2\beta\delta\Phi_2(q_1^p,q_2^{\bullet\bullet}(\overline{\theta},\underline{\theta})).$$

Since $S$ is concave and $\psi$ is convex, and since $\lambda\nu>\xi_2$ as shown in the proof of Proposition B.2, it follows from (B.18) and (B.21) that if $q_2^{\bullet\bullet}(\underline{\theta},\underline{\theta})-\delta q_1^p\geq q_1^p-\delta q_0$, then $q_1^p\geq q_2^{\bullet\bullet}(\underline{\theta},\underline{\theta})$ and therefore $q_0\geq q_1^p$. But if $q_0$ is close to zero, this is inconsistent with a positive effort being exerted at date 0 by a high type agent, in contradiction to (B.19). Hence $q_1^p-\delta q_0>q_2^{\bullet\bullet}(\underline{\theta},\underline{\theta})-\delta q_1^p$, as claimed. □

We are now ready to complete the proof. Using (B.15)–(B.16) together with the definition of $\Phi$, one can rearrange (B.19)–(B.20) to obtain

$$\xi_1\left(\frac{1}{\nu}+\frac{1}{1-\nu}\right) = -(1-\nu)\beta\delta(1+\lambda)\big[\psi'(q_2^{\bullet\bullet}(\overline{\theta},\underline{\theta})-\delta q_1^p-\underline{\theta})-\psi'(q_2^{\bullet\bullet}(\underline{\theta},\underline{\theta})-\delta q_1^p-\underline{\theta})\big]$$
$$-\left(1+\lambda+\frac{\lambda\nu}{1-\nu}\right)\Phi_2(q_0,q_1^p)+\frac{\lambda\nu\beta\delta}{1-\nu}\,\Phi_2(q_1^p,q_2^{\bullet\bullet}(\underline{\theta},\underline{\theta})) \tag{B.22}$$
$$-\xi_2\beta\delta\Phi_2(q_1^p,q_2^{\bullet\bullet}(\overline{\theta},\underline{\theta}))+\frac{\xi_2}{1-\nu}\,\Phi_2(q_0,q_1^p)-\frac{\xi_2\nu\beta\delta}{1-\nu}\,\Phi_2(q_1^p,q_2^{\bullet\bullet}(\underline{\theta},\underline{\theta})).$$

Since $q_2^{\bullet\bullet}(\overline{\theta},\underline{\theta})>q_2^{\bullet\bullet}(\underline{\theta},\underline{\theta})$ by Proposition B.2, and since the functions $\psi$ and $\Phi(q_1^p,\cdot)$ are convex, it therefore follows that

$$\frac{\xi_1}{\nu}<(\lambda\nu-\xi_2)[\beta\delta\Phi_2(q_1^p,q_2^{\bullet\bullet}(\underline{\theta},\underline{\theta}))-\Phi_2(q_0,q_1^p)]. \tag{B.23}$$

By Lemma B.5, $q_1^p-\delta q_0>q_2^{\bullet\bullet}(\underline{\theta},\underline{\theta})-\delta q_1^p$ for $q_0$ close enough to zero. Hence, since a high type agent exerts a positive effort at date 0, $\Phi_2(q_0,q_1^p)>\Phi_2(q_1^p,q_2^{\bullet\bullet}(\underline{\theta},\underline{\theta}))$. Moreover, $\lambda\nu>\xi_2$ as shown in the proof of Proposition B.2. Hence (B.23) implies that if $\beta\delta\leq 1$, then $\xi_1<0$, a contradiction. Thus pooling over date 1 qualities can occur only if $\beta\delta>1$, as claimed. ■

In order to show that pooling over date 1 quality can indeed occur at the equilibrium in the circumstances delineated in Proposition B.2, we turn to the linear-quadratic specification (24)–(25). As in the basic model, special care must be taken in specifying parameters for the model such that the solution is economically meaningful. In particular, it turns out that for date 1 quality levels to remain smaller than $\frac{a}{b}$, the discount factor $\beta$ cannot be too large for a given value of $\delta$. Specifically, it can be shown that if $q_0$, $\underline{\theta}$ and $\Delta\theta$ are close enough to zero, it must be the case that $\beta<\overline{\beta}(\delta)$, where $\overline{\beta}(\delta)$ is the positive solution to

$$\frac{\beta^2\delta(1-\delta)b}{\beta b+c(1+\lambda)}=1. \tag{B.24}$$

Note that $\overline{\beta}(\delta)$ goes to infinity when $\delta$ goes to one, and thus one can choose $\beta\delta$ to be greater than one as required by Proposition B.2. One then has the following result.

**Proposition B.4.** *In the linear-quadratic case, for any $\delta$ close enough to one, there exists a $\underline{\beta}(\delta) \in (0, \overline{\beta}(\delta))$ such that, for any $\beta \in [\underline{\beta}(\delta), \overline{\beta}(\delta)]$, and for any $q_0$, $\underline{\theta}$ and $\Delta\theta$ close enough to zero, pooling over date 1 quality occurs in the optimal long-term contract and the regular case obtains.*

*Proof.* To construct the optimal contract, we first conjecture that (45) is slack at the optimum, which we will verify ex-post. As a result of this, the multiplier $\xi_2$ of (45) is equal to zero. From (B.15)–(B.18), we obtain

$$q_2^{\bullet\bullet}(\overline{\theta}, \overline{\theta}) = \frac{\beta a + c(1+\lambda)[\overline{\theta} + \delta q_1^{\bullet\bullet}(\overline{\theta})]}{\beta b + c(1+\lambda)}, \tag{B.25}$$

$$q_2^{\bullet\bullet}(\underline{\theta}, \overline{\theta}) = \frac{\beta a + c(1+\lambda)[\overline{\theta} + \delta q_1^{\bullet\bullet}(\underline{\theta})]}{\beta b + c(1+\lambda)}, \tag{B.26}$$

$$q_2^{\bullet\bullet}(\overline{\theta}, \underline{\theta}) = \frac{\beta a + c(1+\lambda)[\underline{\theta} + \delta q_1^{\bullet\bullet}(\overline{\theta})]}{\beta b + c(1+\lambda)}, \tag{B.27}$$

$$q_2^{\bullet\bullet}(\underline{\theta}, \underline{\theta}) = \frac{\beta a - \frac{\lambda\nu}{(1-\nu)^2} c\Delta\theta + c(1+\lambda)[\underline{\theta} + \delta q_1^{\bullet\bullet}(\underline{\theta})]}{\beta b + c(1+\lambda)}. \tag{B.28}$$

For (B.25)–(B.28) to hold, one must ensure that whatever $q_1^{\bullet\bullet}(\overline{\theta})$ and $q_1^{\bullet\bullet}(\underline{\theta})$ might be, provided they remain smaller than $\frac{a}{b}$, the date 1 efforts stay positive, and that $e_1^{\bullet\bullet}(\overline{\theta}, \underline{\theta}) \geq \Delta\theta$ and $e_1^{\bullet\bullet}(\underline{\theta}, \underline{\theta}) \geq \Delta\theta$, so that we are justified in using the quadratic specification of $\psi$ when writing down the informational rents $\Phi(q_1^{\bullet\bullet}(\overline{\theta}), q_2^{\bullet\bullet}(\overline{\theta}, \underline{\theta}))$ and $\Phi(q_1^{\bullet\bullet}(\underline{\theta}), q_2^{\bullet\bullet}(\underline{\theta}, \underline{\theta}))$, see (31). It is easy to check that a sufficient condition for this to be true is that (A.11) holds and that

$$\left[(1-\delta)\frac{a}{b} - \underline{\theta}\right]\beta b > \left[\frac{\lambda\nu c}{(1-\nu)^2} + \beta b + c(1+\lambda)\right]\Delta\theta, \tag{B.29}$$

which, given (A.11), typically holds if $\Delta\theta$ is close enough to zero. To show that pooling over date 1 quality can occur, let us evaluate the multiplier $\xi_1$ of (44) using (B.22). This yields

$$\xi_1 = -c\Delta\theta\nu(1-\nu)\varphi_\delta(\beta), \tag{B.30}$$

where

$$\varphi_\delta(\beta) = 1 + \lambda + \frac{\lambda\nu}{1-\nu}\left[\frac{\beta\delta c(1+\lambda)}{\beta b + c(1+\lambda)} + 1 - \beta\delta\right]. \tag{B.31}$$

For each $\delta > 0$, the function $\varphi_\delta$ is strictly decreasing, with $\varphi_\delta(0) > 0$ and $\lim_{\beta\to\infty} \varphi_\delta(\beta) = -\infty$. Let $\underline{\beta}(\delta) = \varphi_\delta^{-1}(0)$. Clearly $\underline{\beta}(\delta) > 1$ and $\lim_{\delta\to 1} \underline{\beta}(\delta) < \infty$. Thus $\xi_1 > 0$ if $\beta > \underline{\beta}(\delta)$. For the corresponding allocation to be economically meaningful, $q_1^{\bullet\bullet}(\overline{\theta}) = q_1^{\bullet\bullet}(\underline{\theta}) = q_1^p$ must remain smaller than $\frac{a}{b}$. For $q_0$, $\underline{\theta}$ and $\Delta\theta$ close enough to zero, it is easy to check this will be the case if $\beta < \overline{\beta}(\delta)$, where $\overline{\beta}(\delta)$ is the positive solution to (B.24). From (B.24), one has

$$\varphi_\delta(\overline{\beta}(\delta)) = 1 + \lambda + \frac{\lambda\nu}{1-\nu}\left[\frac{c(1+\lambda)}{\overline{\beta}(\delta)(1-\delta)b} + 1 - \overline{\beta}(\delta)\delta\right], \tag{B.32}$$

as well as $\lim_{\delta\to 1} \overline{\beta}(\delta) = \infty$ and $\lim_{\delta\to 1} \overline{\beta}(\delta)(1-\delta) = 1$. Thus (B.32) implies that $\lim_{\delta\to 1} \varphi_\delta(\overline{\beta}(\delta)) = -\infty$. Moreover, $\overline{\beta}(\delta) > \underline{\beta}(\delta)$ whenever $\delta$ is close enough to one. Thus, pooling over date 1 quality can occur if $\beta \in [\underline{\beta}(\delta), \overline{\beta}(\delta)]$.[16] To check that this can indeed be the case, one needs only to find

[16]Consider by contrast what happens when $\delta$ is close to zero. From (B.24), one has $\lim_{\delta\to 0} \overline{\beta}(\delta) = \infty$ and $\lim_{\delta\to 0} \overline{\beta}(\delta)\delta = 1$. It follows from (B.32) that $\lim_{\delta\to 0} \varphi_\delta(\overline{\beta}(\delta)) = 1 + \lambda$. Since $\varphi_\delta$ is decreasing, we obtain that, for $\delta$ close enough to zero, $\varphi_\delta(\beta) > 0$ for any $\beta \leq \overline{\beta}(\delta)$, and thus pooling over date 1 quality cannot occur.

parameter restrictions such that $U_0^{\bullet\bullet}(\overline{\theta}) > 0$, so that (45) is slack as postulated. From (31), (41), and (B.27)–(B.28), one has

$$U_0^{\bullet\bullet}(\overline{\theta}) = c\Delta\theta\left(q_1^p - \delta q_0 - \frac{\overline{\theta}+\underline{\theta}}{2}\right) - (c\Delta\theta)^2 \frac{\lambda\nu^2\beta}{(1-\nu)^2[\beta b + c(1+\lambda)]},$$

which is strictly positive for $q_0$, $\underline{\theta}$ and $\Delta\theta$ close enough to zero. Since $\varphi_\delta$ is independent from $q_0$, $\underline{\theta}$ and $\Delta\theta$, the result follows. ■

Pooling over date 1 quality leads to over-provision of date 1 quality following both a high and a low quality shock. To understand why, consider the case of a high quality shock at date 0, and suppose that we are in the regular case, as in Proposition B.4. We know from (B.15) and (B.17) that, in this case, there will be no distortions of date 2 quality levels given the date 1 quality level $q_1^{\bullet\bullet}(\overline{\theta})$. In turn, $q_1^{\bullet\bullet}(\overline{\theta})$ satisfies the first-order condition

$$\beta S'(q_1^{\bullet\bullet}(\overline{\theta})) + \beta\delta(1+\lambda)\mathrm{E}\big[\psi'(e_1^{\bullet\bullet}(\overline{\theta},\theta_1))\big] = (1+\lambda)\psi'(e_0^{\bullet\bullet}(\overline{\theta})) - \frac{\xi_1}{\nu}, \tag{B.33}$$

where $\xi_1$ is the multiplier associated to (44). Under symmetric information, denoting by $q_1^*(\theta_0)$ the optimal contingent quality level at date 1 after a quality shock $\theta_0$ at date 0, and by $e_0^*(\theta_0)$ and $e_1^*(\theta_0,\theta_1)$ the corresponding optimal contingent effort levels at dates 0 and 1, $q_1^*(\overline{\theta})$ satisfies the following first-order condition:

$$\beta S'(q_1^*(\overline{\theta})) + \beta\delta(1+\lambda)\mathrm{E}\big[\psi'(e_1^*(\overline{\theta},\theta_1))\big] = (1+\lambda)\psi'(e_0^*(\overline{\theta})). \tag{B.34}$$

It follows from (B.33) and (B.34) that a high type agent at date 0 always produces at least as much quality as in the symmetric information benchmark, $q_1^{\bullet\bullet}(\overline{\theta}) \geq q_1^*(\overline{\theta})$, and strictly more if $\xi_1 > 0$. Thus pooling over date 1 quality leads to over-provision of date 1 quality when $\theta_0 = \overline{\theta}$ relative to the symmetric information benchmark. This is a fortiori true when $\theta_0 = \underline{\theta}$, since $q_1^*(\overline{\theta}) > q_1^*(\underline{\theta})$. Therefore, by continuity, we obtain that there may be over-provision of date 1 quality when $\theta_0 = \underline{\theta}$ relative to the symmetric information benchmark, even if no pooling over date 1 quality actually takes place.

Finally, it should be emphasized that pooling over date 1 quality does not prevent the regulator from screening the agent's type at date 0. Rather, screening is achieved through date 0 transfers $u_0^{\bullet\bullet}(\hat{\theta}_0)$ and date 1 continuation rents $\Phi(q_1^{\bullet\bullet}(\hat{\theta}_0), q_2^{\bullet\bullet}(\hat{\theta}_0,\underline{\theta}))$. Specifically, let $q_1^p = q_1^{\bullet\bullet}(\overline{\theta}) = q_1^{\bullet\bullet}(\underline{\theta})$. Then, one has

$$\begin{aligned} u_0^{\bullet\bullet}(\overline{\theta}) &= \psi(q_1^p - \delta q_0 - \overline{\theta}) + \Phi(q_0, q_1^p) + \nu\beta\big[\Phi(q_1^p, q_2^{\bullet\bullet}(\underline{\theta},\underline{\theta})) - \Phi(q_1^p, q_2^{\bullet\bullet}(\overline{\theta},\underline{\theta}))\big] \\ &= u_0^{\bullet\bullet}(\underline{\theta}) + \nu\beta\big[\Phi(q_1^p, q_2^{\bullet\bullet}(\underline{\theta},\underline{\theta})) - \Phi(q_1^p, q_2^{\bullet\bullet}(\overline{\theta},\underline{\theta}))\big] \\ &< u_0^{\bullet\bullet}(\underline{\theta}), \end{aligned}$$

where we have taken advantage from the fact that (37) and (40) are binding, and that $q_2^{\bullet\bullet}(\overline{\theta},\underline{\theta}) > q_2^{\bullet\bullet}(\underline{\theta},\underline{\theta})$ by Proposition B.2. At date 0, a high type agent therefore receives a lower transfer than a low type agent, $u_0^{\bullet\bullet}(\overline{\theta}) < u_0^{\bullet\bullet}(\underline{\theta})$, in exchange for a higher date 1 continuation rent, $\Phi(q_1^p, q_2^{\bullet\bullet}(\overline{\theta},\underline{\theta})) > \Phi(q_1^p, q_2^{\bullet\bullet}(\underline{\theta},\underline{\theta}))$. Note that $q_2^{\bullet\bullet}(\overline{\theta},\underline{\theta}) > q_2^{\bullet\bullet}(\underline{\theta},\underline{\theta})$ although $q_1^{\bullet\bullet}(\overline{\theta}) = q_1^{\bullet\bullet}(\underline{\theta})$. Thus, unlike in the multiple agent model, date 1 quality does not constitute a sufficient statistic for the continuation of the contractual relationship: while a high type and a low type agent having delivered the same level of date 1 quality have the same preferences at date 1, they are treated differently in the continuation contract. The reason of this apparent paradox is that the regulator does not treat the date 1 rents of a former high type agent in the same way as those of a former low type agent. This endogenously leads to a sorting condition that makes it optimal for her to screen the agent at date 0, in spite of pooling over date 1 quality.

### *B.4. Single versus Multiple Agents*

*Proof of Proposition 6.* For each $t = 1, 2$, let $\overline{q}_t^{**}$ and $\underline{q}_t^{**}$ be the functions mapping date $t-1$ quality into date $t$ quality following respectively a high and low quality shock at date $t-1$ in the model with a sequence of agents. For any $q_1 \geq 0$, it is easy to check that

$$\beta S'(\overline{q}_2^{**}(q_1)) = (1+\lambda)\psi'(\overline{q}_2^{**}(q_1) - \delta q_1 - \overline{\theta}), \tag{B.35}$$

$$\beta S'(\underline{q}_2^{**}(q_1)) = (1+\lambda)\psi'(\underline{q}_2^{**}(q_1) - \delta q_1 - \underline{\theta}) + \frac{\lambda\nu}{1-\nu}\,\Phi_2(q_1, \underline{q}_2^{**}(q_1)), \tag{B.36}$$

in analogy with (21)–(22). By (B.15)–(B.16) and (B.35), $q_2^{\bullet\bullet}(\overline{\theta}, \overline{\theta}) = \overline{q}_2^{**}(q_1^{\bullet\bullet}(\overline{\theta}))$ and $q_2^{\bullet\bullet}(\underline{\theta}, \overline{\theta}) = \overline{q}_2^{**}(q_1^{\bullet\bullet}(\underline{\theta}))$. Since it is assumed that the regular case obtains in the optimal long-term contract with a single agent, (B.12) and (B.17) with $\xi_2 = 0$ imply that $q_2^{\bullet\bullet}(\overline{\theta}, \underline{\theta}) = \underline{\mathcal{Q}}_{3,2}(q_1^{\bullet\bullet}(\overline{\theta}))$. Similarly, it follows from (46) that $q_2^{\bullet\bullet}(\underline{\theta}, \underline{\theta}) = \underline{\mathcal{Q}}_{6,2}(q_1^{\bullet\bullet}(\underline{\theta}))$, where for each $q_1 \geq 0$, $\underline{\mathcal{Q}}_{6,2}(q_1)$ is implicitly defined by

$$\beta S'(\underline{\mathcal{Q}}_{6,2}(q_1)) = (1+\lambda)\psi'(\underline{\mathcal{Q}}_{6,2}(q_1) - \delta q_1 - \underline{\theta}) + \frac{\lambda\nu}{(1-\nu)^2}\,\Phi_2(q_1, \underline{\mathcal{Q}}_{6,2}(q_1)). \tag{B.37}$$

We first compare $\overline{q}_1^{**}(q_0)$ and $q_1^{\bullet\bullet}(\overline{\theta})$. It is helpful to define two functions $\overline{\Omega}^{**}$ and $\overline{\Omega}^{\bullet\bullet}$ as follows:

$$\begin{aligned}\overline{\Omega}^{**}(q_1) &= (1+\lambda)\psi'(q_1 - \delta q_0 - \overline{\theta}) - \nu\beta\delta(1+\lambda)\psi'(\overline{q}_2^{**}(q_1) - \delta q_1 - \overline{\theta}) \\ &\quad - (1-\nu)\beta\delta(1+\lambda)\psi'(\underline{q}_2^{**}(q_1) - \delta q_1 - \underline{\theta}) - \lambda\nu\beta\delta\Phi_2(q_1, \underline{q}_2^{**}(q_1)), \\ \overline{\Omega}^{\bullet\bullet}(q_1) &= (1+\lambda)\psi'(q_1 - \delta q_0 - \overline{\theta}) - \nu\beta\delta(1+\lambda)\psi'(\overline{q}_2^{**}(q_1) - \delta q_1 - \overline{\theta}) \\ &\quad - (1-\nu)\beta\delta(1+\lambda)\psi'(\underline{\mathcal{Q}}_{3,2}(q_1) - \delta q_1 - \underline{\theta}).\end{aligned}$$

Both $\overline{\Omega}^{**}$ and $\overline{\Omega}^{\bullet\bullet}$ are increasing, and the difference $\overline{\Omega}^{**}(q_1) - \overline{\Omega}^{\bullet\bullet}(q_1)$ is proportional to

$$(1+\lambda)\psi'(\underline{\mathcal{Q}}_{3,2}(q_1) - \delta q_1 - \underline{\theta}) - (1+\lambda)\psi'(\underline{q}_2^{**}(q_1) - \delta q_1 - \underline{\theta}) - \frac{\lambda\nu}{1-\nu}\,\Phi_2(q_1, \underline{q}_2^{**}(q_1)),$$

which is negative from the first-order conditions (B.12) and (B.36) that define $\underline{\mathcal{Q}}_{3,2}(q_1)$ and $\underline{q}_2^{**}(q_1)$ for a given value of $q_1$. Now, from the Envelope Theorem, it is easy to check that

$$\beta S'(\overline{q}_1^{**}(q_0)) = \overline{\Omega}^{**}(\overline{q}_1^{**}(q_0)), \tag{B.38}$$

$$\beta S'(q_1^{\bullet\bullet}(\overline{\theta})) = \overline{\Omega}^{\bullet\bullet}(q_1^{\bullet\bullet}(\overline{\theta})). \tag{B.39}$$

Since $S$ is strictly concave and $\overline{\Omega}^{**}$ and $\overline{\Omega}^{\bullet\bullet}$ are increasing, it follows that $\overline{q}_1^{**}(q_0) > q_1^{\bullet\bullet}(\overline{\theta})$, as claimed.

We next compare $\underline{q}_1^{**}(q_0)$ and $q_1^{\bullet\bullet}(\underline{\theta})$. It is helpful to define two functions $\underline{\Omega}^{**}$ and $\underline{\Omega}^{\bullet\bullet}$ as follows:

$$\underline{\Omega}^{**}(q_1) \;=\; (1+\lambda)\psi'(q_1-\delta q_0-\underline{\theta}) - \nu\beta\delta(1+\lambda)\psi'(\overline{q}_2^{**}(q_1)-\delta q_1-\overline{\theta})$$

$$- (1-\nu)\beta\delta(1+\lambda)\psi'(\underline{q}_2^{**}(q_1(q_1)-\delta q_1-\underline{\theta}) - \lambda\nu\beta\delta\Phi_2(q_1,\underline{q}_2^{**}(q_1(q_1)) + \frac{\lambda\nu}{1-\nu}\,\Phi_2(q_0,q_1),$$

$$\underline{\Omega}^{\bullet\bullet}(q_1) \;=\; (1+\lambda)\psi'(q_1-\delta q_0-\underline{\theta}) - \nu\beta\delta(1+\lambda)\psi'(\overline{q}_2^{**}(q_1)-\delta q_1-\overline{\theta})$$

$$- (1-\nu)\beta\delta(1+\lambda)\psi'(\underline{\mathcal{Q}}_{6,2}(q_1)-\delta q_1-\underline{\theta}) + \frac{\lambda\nu}{1-\nu}\,\Phi_2(q_0,q_1) + \frac{\lambda\nu\beta}{1-\nu}\,\Phi_1(q_1,\underline{\mathcal{Q}}_{6,2}(q_1)).$$

Both $\underline{\Omega}^{**}$ and $\underline{\Omega}^{\bullet\bullet}$ are increasing, and the difference $\underline{\Omega}^{**}(q_1)-\underline{\Omega}^{\bullet\bullet}(q_1)$ is proportional to

$$(1+\lambda)\psi'(\underline{\mathcal{Q}}_{6,2}(q_1)-\delta q_1-\underline{\theta}) + \frac{\lambda\nu}{(1-\nu)^2}\,\Phi_2(q_1,\underline{\mathcal{Q}}_{6,2}(q_1))$$

$$-\;(1+\lambda)\psi'(\underline{q}_2^{**}(q_1)-\delta q_1-\underline{\theta}) - \frac{\lambda\nu}{1-\nu}\,\Phi_2(q_1,\underline{q}_2^{**}(q_1)),$$

which is positive from the first-order conditions (B.36)–(B.37) that define $\underline{q}_2^{**}(q_1)$ and $\underline{\mathcal{Q}}_{6,2}(q_1)$ for a given value of $q_1$. Now, from the Envelope Theorem, it is easy to check that

$$\beta S'(\underline{q}_1^{**}(q_0)) \;=\; \underline{\Omega}^{**}(\underline{q}_1^{**}(q_0)), \tag{B.40}$$

$$\beta S'(q_1^{\bullet\bullet}(\underline{\theta})) \;=\; \underline{\Omega}^{\bullet\bullet}(q_1^{\bullet\bullet}(\underline{\theta})). \tag{B.41}$$

Since $S$ is strictly concave and $\underline{\Omega}^{**}$ and $\underline{\Omega}^{\bullet\bullet}$ are increasing, it follows that $q_1^{\bullet\bullet}(\underline{\theta}) > \underline{q}_1^{**}(q_0)$, as claimed. The result follows. ■

*Proof of Proposition 7.* Denote by $V^{**}(q_0)$ the ex-ante social welfare obtained in the optimal contract with a sequence of agents, given an initial quality level $q_0$. One has

$$V^{**}(q_0) \;=\; S(q_0) + \mathrm{E}\,[-(1+\lambda)\psi(q_1^{**}(q_0)-\delta q_0-\theta_0) + \beta S(q_1^{**}(q_0))$$

$$-(1+\lambda)\beta\psi(q_2^{**}(q_1^{**}(q_0)) - \delta q_1^{**}(q_0)-\theta_1) + \beta^2 S(q_2^{**}(q_1^{**}(q_0)))]$$

$$-\lambda\nu\Phi(q_0,\underline{q}_1^{**}(q_0)) - \lambda\nu^2\beta\Phi(\overline{q}_1^{**}(q_0),\underline{q}_2^{**}(\overline{q}_1^{**}(q_0)))$$

$$-\lambda\nu(1-\nu)\beta\Phi(\underline{q}_1^{**}(q_0),\underline{q}_2^{**}(\underline{q}_1^{**}(q_0))),$$

with obvious notation for the terms inside the expectation. Similarly, denote by $V^{\bullet\bullet}(q_0)$ the ex-ante social welfare obtained in the optimal contract with a single agent, given an initial quality level $q_0$. One has

$$V^{\bullet\bullet}(q_0) \;=\; S(q_0) + \mathrm{E}\,[-(1+\lambda)\psi(q_1^{\bullet\bullet}(\theta_0)-\delta q_0-\theta_0) + \beta S(q_1^{\bullet\bullet}(\theta_0))$$

$$-(1+\lambda)\beta\psi(q_2^{\bullet\bullet}(\theta_0,\theta_1)-\delta q_1^{\bullet\bullet}(\theta_0)-\theta_1) + \beta^2 S(q_2^{\bullet\bullet}(\theta_0,\theta_1))]$$

$$-\lambda\nu\Phi(q_0,q_1^{\bullet\bullet}(\underline{\theta})) - \lambda\nu\beta\Phi(q_1^{\bullet\bullet}(\underline{\theta}),q_2^{\bullet\bullet}(\underline{\theta},\underline{\theta})).$$

To compare $V^{**}(q_0)$ and $V^{\bullet\bullet}(q_0)$ in the linear-quadratic case, we need to compute the quality levels obtained in each regulatory environment. It follows from (B.35)–(B.36) that

$$\overline{q}_2^{**}(q_1) = \frac{\beta a + c(1+\lambda)(\overline{\theta} + \delta q_1)}{\beta b + c(1+\lambda)},$$

$$\underline{q}_2^{**}(q_1) = \frac{\beta a - \frac{\lambda\nu}{1-\nu} c\Delta\theta + c(1+\lambda)(\underline{\theta} + \delta q_1)}{\beta b + c(1+\lambda)},$$

while the levels of quality $q_2^{\bullet\bullet}(\theta_0, \theta_1)$ are given by (B.25)–(B.28). Let

$$D = \beta b + c(1+\lambda) + \frac{\beta^2\delta^2 bc(1+\lambda)}{\beta b + c(1+\lambda)}$$

$$N = \beta a + c(1+\lambda)\delta q_0 + \frac{\beta^2\delta(a - b\mathrm{E}_\theta)c(1+\lambda)}{\beta b + c(1+\lambda)}$$

From (B.38)–(B.41), one can verify that

$$\overline{q}_1^{**}(q_0) = \frac{1}{D}\left[N + c(1+\lambda)\overline{\theta} + \frac{\lambda\nu\beta^2\delta b}{\beta b + c(1+\lambda)} c\Delta\theta\right],$$

$$\underline{q}_1^{**}(q_0) = \frac{1}{D}\left\{N + c(1+\lambda)\underline{\theta} + \left[\frac{\lambda\nu\beta^2\delta b}{\beta b + c(1+\lambda)} - \frac{\lambda\nu}{1-\nu}\right] c\Delta\theta\right\},$$

$$q_1^{\bullet\bullet}(\overline{\theta}) = \frac{1}{D}\left[N + c(1+\lambda)\overline{\theta}\right],$$

$$q_1^{\bullet\bullet}(\underline{\theta}) = \frac{1}{D}\left\{N + c(1+\lambda)\underline{\theta} + \frac{1}{1-\nu}\left[\frac{\lambda\nu\beta^2\delta b}{\beta b + c(1+\lambda)} - \lambda\nu\right] c\Delta\theta\right\}.$$

It is easy to check that $\mathrm{E}\left[q_1^{**}(q_0)\right] = \mathrm{E}\left[q_1^{\bullet\bullet}(\theta_0)\right]$ and $\mathrm{E}\left[q_2^{**}(q_1^{**}(q_0))\right] = \mathrm{E}\left[q_2^{\bullet\bullet}(\theta_0, \theta_1)\right]$. Tedious algebra then leads to the following expression:

$$V^{**}(q_0) - V^{\bullet\bullet}(q_0) = \frac{\lambda\beta}{D}\left(\frac{\nu}{1-\nu}\right)^2 \left[\frac{c\Delta\theta}{\beta b + c(1+\lambda)}\right]^2 P(\nu), \tag{B.42}$$

where $P(\nu) = A_1\nu^3 + A_2\nu^2 + A_3\nu + A_4$ is a third-degree polynomial in $\nu$ with coefficients given by

$$A_1 = \beta^2\delta^2 b(1-\lambda)[\beta b + c(1+\lambda)],$$

$$A_2 = \beta\delta b[\beta b + c(1+\lambda)] - 2\beta^2\delta^2 bc(1-\lambda^2) + \beta^3\delta^2 b^2\left(\frac{5\lambda}{2} - 2\right),$$

$$A_3 = -\frac{3c^2\lambda(1+\lambda)^2}{2} - \beta bc(\delta\lambda^2 + 3\lambda^2 + 3\lambda\delta + 3\lambda + 2\delta) - \beta^2 b^2\left(\delta\lambda + \frac{3\lambda}{2} + 2\delta\right)$$
$$- \beta^2\delta^2 bc\left(\frac{5\lambda^2}{2} + \frac{3\lambda}{2} - 1\right) - \beta^3\delta^2 b^2\left(\frac{3\lambda}{2} - 1\right),$$

$$A_4 = \beta\delta b(1+\lambda)[\beta b + c(1+\lambda)].$$

The expression (B.42) is valid when $q_0$, $\underline{\theta}$ and $\Delta\theta$ are close enough to zero, as in Proposition B.4. Note that $A_4 > 0$, so that $P(0) > 0$. Moreover, it is easy to verify that

$$A_1 + A_2 + A_3 + A_4 = -\frac{3\lambda}{2}\left\{[\beta b + c(1+\lambda)]^2 + \beta^2\delta^2 bc(1+\lambda)\right\} < 0,$$

so that $P(1) < 0$. When $\lambda < 1$, $A_1 > 0$, so that $\lim_{\nu\to-\infty} P(\nu) = -\infty$ and $\lim_{\nu\to\infty} P(\nu) = \infty$. Since $P(0) > 0$ and $P(1) < 0$, it follows that $P$ has exactly one root in $(-\infty, 0)$, one in $(0,1)$ and one in $(1,\infty)$. Denoting by $\nu^*$ the unique root of $P$ in $(0,1)$, the result follows. ■

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