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Abstract

This paper proposes a theoretical framework to investigate the impact of military conflicts on business cycles, as well as defense policies through enrolment mechanisms. Our framework is a variation of a Real Business Cycle model first proposed by Hercowitz and Sampson (1991) that admits explicit solutions. We extend and estimate the initial model on US data to account for specific (potentially large) shocks that destroy the stock of capital. We consider two types of dynamics on the depreciation rate of capital: short–term shocks, that may be interpreted as riots and captured by a Moving Average specification, and mid–term shocks, that may be interpreted as wars and captured by a Markov Switching process. Destructions may be limited by enrolment policies, which allows to question the goals defense policies should aim at. First our model reproduces usual business cycle facts. Second, it allows to characterize the macroeconomic dynamics after shocks on the depreciation rate of capital. Finally, it provides a simple framework to quantify the welfare effects of alternative (simple) defense technologies.

Keywords: military policy, Real Business Cycle model, random coefficient autoregressive model.
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1 Introduction

Military and defense policies are a major source of central action in economics. As a first step, casual observation reveals that the budget of defense departments may reach a significant amount of the total public expenditures, and the amount of military forces may represent a significant fraction of the working–age population. For instance, according to Stockholm International Peace Research Institute, defense–related public expenditure account for 42% of the overall taxes in the USA. The figures are typically high for low income countries where most of the conflicts now take place. However, serious analysis of military expenditure cannot rely solely on accurate accounting. First, enlarging military expenditures is by no means neutral from the political viewpoint. Major conflicts – especially World Wars I and II – have been preceded in large investment in military sector. Also, military expenditures always raise sharply during major conflicts and they are a key feature of the so–called "economics of war". Finally any decent analysis of public policies should also questions its goals.

Various attempts have been made in the recent macroeconomic literature to treat specifically military/defense/peacekeeping policies. The question may then be raised whether this public effort should be incorporated in the total public expenditures or not (see Ramey (2008)). Another stream of literature aims at quantifying the impact of conflicts. Indeed, according to a "myth" wars may be "good for the economy" (often–quoted channels are increasing investment, less unemployment, boosted R&D ...). Most of the economic proponents to this view emphasize long–term effects. In the short term however, war episodes destroy lives and private – as well public – belongings. Another skeptical view toward the above "myth" comes from accurate measurements of direct costs of modern wars. Globalization and technical progress make wars more costly (Stigtlitz and Bilmes (2007) count the cost of Iraq war in trillions of dollars). Finally indirect costs may also be very high. Martin, Mayer and Thoenig (2008a, 2008b) examine the link between war episodes and international trade flows. Again, data show that major conflicts reduce international trade flows. Conflicts are therefore a possible source of economic downturns.

However, the main part of this economic literature remains empirical. The major goal of this paper is to propose a theoretical framework that is empirically consistent and that accounts for some of the major features of military issues. First, conflicts destroy capital and enrolment policies cut civilian working resources (possibly permanently if one accounts for human losses and/or severe physical damages). Moreover, the shocks related to war episodes cannot be considered as small with respect to the whole economy. Second, conflicts last for periods similar to economic cycles. Third, although several important economic mechanisms change during war episodes the ultimate needs and means of the population remain. Finally, defense expenditures are typically mostly – if not only – public. Of course, the model we propose is not able to account for other important features. In particular we do not tackle the difficult questions raised by the causes of war (for example, escalation problems are not considered). Also, the whole range of public interventions during wars (from specific monetary policies to subsidies in R&D technologies) are not considered. Finally the main part of the paper is written for a closed economy. A supplementary section presents some extensions to account for possible imported consumption. This extension is used to discuss some issues related to embargos or sieges. It should of course not be considered as a complete economic treatment of international military issues.

Our framework is a variation of a Real Business Cycle model proposed by Hercowitz and Sampson (1991). We choose this model because it admits explicit solutions. As an important consequence, shocks may be as large as desired. We extend the initial model to account for specific shocks that destroy capital stocks. These shocks may be viewed as destructions due to conflicts or possibly natural disasters. In our view, this second interpretation causes no problem as long as nation–specific considerations are absent from the model (in other words, whether destructions are caused by "enemies" or tsunami makes no difference in our model). We consider two types of dynamics : short–terms shocks are captured by a Moving Average specification and mid–term shocks by a Markov Switching process. Short–term shocks may be interpreted as riots (or natural disasters as explained above) whereas mid–term shocks are interpreted as wars. Part of the working force may be hired to limit capital destructions. This will allow us to question the goal defense policies should aim at.

Our model is able to reproduce some interesting facts. First, our model reproduces usual responses to productivity shocks (consumption, working hours and investment grow after booms). Second, investment grows as an immediate response to conflicts but the response is negative in the mid–term. On the contrary, production and working hours typically fall after destructions. Also our model is able to quantify the long–term welfare gains induced by perpetual peace.

The paper is organized as follows. Section 2 proposes a presentation of the related literature. Section 3 details the model. Section 4 proposes an inference strategy to calibrate the model. Section 5 is devoted to the analysis of Impulse Response Functions as well as welfare analsis. Section 6 presents some possible extensions of the baseline model. Section 7 concludes.

2 Related literature

Using post–war data for developed countries has become customary in current macroeconometric literature. Indeed, the usefulness of the neoclassical framework during war episode is questionable. First, war may induce very large shocks in the economy. This is at odd with the usual practice to build DSGE models as linear approximations around a deterministic steady state. Second, modern macroeconometric literature typically treats government spending as unanticipated "shocks" whereas central policies may account for more than half of the GDP during major conflicts.

On the other hand, many empirical works have emphasized the importance of a careful look at military and conflict data in macroeconomics. For instance, Ramey (2008) shows that much of the variability of the shocks in public expenditure is explained by military expenditures. The main reason is that other sources of public expenditures (education, health, ...) are much more stable over time. Hence, once trends are removed, unexpected shocks in public expenditures are mainly military. Moreover the "narrative approach" (Ramey and Shapiro (1998)) emphasizes the role of wars/peace episodes in the identification of unexpected shocks. Ramey (2008) argues that distinction between military and civilian public expenditures induces major differences in the analysis of private response unanticipated central policy shocks.

In addition, much empirical effort has been devoted to precise accounting of the costs of war. As one can expect, there is a considerable disagreement among authors (see Arias and Ardila (2003) for a discussion). Most empirical contributions show that net effects of wars (including destructions and possible positive effects induced by larger public expenditures) are negative. For instance, Collier (1999) argues that civil war might cause a 2.2 percentage point loss in the annual growth rate. In a welfare analyze, Hess (2003) obtains an average cost of conflicts equal to 102.3 USD per person.

Adopting an alternative perspective, Martin et al. (2008b) identify the impact of international trade on the occurrence of armed conflicts. Using an extensive dataset on bilateral trade and armed conflicts, they show that increasing bilateral trade flows (through bilateral trade agreements for example) significantly reduces the probability of armed conflict with the corresponding trade partner without increasing the probability of conflicts with other trade partners. In this sense, trade openness could be seen as a peace–promoting technology. However, they show that multilateral trade openness reduces the bilateral trade dependence and thus the cost of bilateral armed conflicts, which increases the probability of war. This mixed evidence shows that trade openness and armed conflicts are closely related in the data but that both the sign and the magnitude of the relation depend on the specific characteristics of trade flows and agreements.

Finally, many contributions study particular conflicts to understand the precise linkage between military engagements and economic activities. For instance, the massive role played by engineering and logistics problems has been made clear for World War II while specialists of World War I put forth the major impact of human losses. Many debates are still open as well on the role of economic "repairs" due to allies by Germany and their consequences in the failure of the Weimar Republic.

The model we propose here does not address all these issues extensively. It is however intended to fill the existing gap between empirical contributions and macroeconomic models dealing with the economic gains/costs of conflicts. We argue that Real Business Cycle (RBC hereafter) models are the proper tool to handle some of these questions. First, these models are well–suited to analyze both the long and short–run consequences of macroeconomic shocks. In particular, they rely on explicit transmission mechanisms that explain propagation of these shocks in the economy. Moreover, as McGrattan and Ohanian (2008) make clear, these models are able to capture many economic phenomena of WWII (despite the fact that much of the empirical literature is based on post–war data). One of the reason may be the following: even during war episodes the very basic needs – consumption – and means of production – capital and labor – remain and RBC models precisely rely on these basic means and needs.

3 The model

The representative household maximizes,

$$E_0 \sum_{t=0}^{\infty} \beta^t \log(C_t - h(N_t)^{1+\mu}), \tag{1}$$

where β is the discount factor, C_t is the level of consumption chosen by the representative household and N_t is the total amount of working time supplied by the representative household in period t. Part of this time will be used in a military "service". More precisely, when households supply the N_t hours of work at period t and the capital installed at date t is K_{t-1}, the level of production Y_t is given by,

$$Y_t = A_t K_{t-1}^{\alpha}((1-e_t)N_t)^{1-\alpha},$$

where $0 < e_t < 1$ is the enrolment rate at date t and where A_t is the total factor productivity (TFP), subject to exogenous random shocks.

We consider the enrolment process as the unique source of public intervention and e_t as exogenous. By doing so, we implicitly assume it is the state's privilege to enrol forces. This corresponds to the medieval duty known as "ost" in the (continental) European feodal times. A more contemporary view may be as follows. Assume the state has the power to tax wages in order to enrol soldiers. The state's budget balance implies,

$$w_t^m e_t N_t = \tau_t(w_t^m e_t N_t + w_t(1-e_t)N_t),$$

where w_t^m and w_t are respectively the wages paid to soldiers and civilians and τ_t is the tax rate. We then have,

$$e_t = \frac{\tau_t w_t}{(1-\tau_t)w_t^m + \tau_t w_t}.$$

In particular if soldiers and civilians are equally paid $e_t = \tau_t$. Accordingly e_t is less than τ_t if and only if soldiers are better paid than civilians.

If we set $A_t' = A_t(1-e_t)^{1-\alpha}$ we see that supplying time to the military sector amounts to divide the TFP (A_t). We then write,

$$Y_t = A_t' K_{t-1}^{\alpha} N_t^{1-\alpha}.$$

The law of motion for the capital stock is given by,

$$K_t = A_K K_{t-1}^{\delta_t} I_t^{1-\delta_t},$$

where $I_t = Y_t - C_t$ is the investment flow at date t and δ_t is the stochastic random rate of depreciation taking values in $]0,1[$. This law of motion is an adaptation of the model proposed by Hercowitz and Sampson (1991) (see also Collard (1999)), in which we incorporate a possible random depreciation rate, just as Ambler and Paquet (1994). This extra source of randomness captures the increased depreciation of capital induced by short–term and/or long–lasting conflicts. Notice that with the above formula we need $K_{t-1}/I_t > 1$ to insure that *ceteris paribus*, K_t decreases when δ_t drops. This condition is merely a matter of interpretation, and will cause no problem for the resolution.

The associated Lagrangian is,

$$\begin{aligned}
&\log(C_t - hN_t^{1+\mu}) + \lambda_t \left(A_K K_{t-1}^{\delta_t} (A_t K_{t-1}^{\alpha} ((1-e_t)N_t)^{1-\alpha} - C_t)^{1-\delta_t} - K_t \right) \\
&+\beta E_t \left[\log(C_{t+1} - hN_{t+1}^{1+\mu}) + \lambda_{t+1} \left(A_K K_t^{\delta_{t+1}} (A_{t+1} K_t^{\alpha} ((1-e_{t+1})N_{t+1})^{1-\alpha} - C_{t+1})^{1-\delta_{t+1}} - K_{t+1} \right) \right].
\end{aligned}$$

The first order conditions for optimality provide the following system:[1]

$$\frac{1}{C_t - hN_t^{1+\mu}} = \lambda_t K_t (1-\delta_t)/I_t, \tag{2}$$

$$\frac{h(1+\mu)N_t^{1+\mu}}{C_t - hN_t^{1+\mu}} = \lambda_t K_t (1-\delta_t)(1-\alpha) Y_t / I_t, \tag{3}$$

$$\lambda_t K_t = \beta E_t \left[\lambda_{t+1} K_{t+1} \left(\delta_{t+1} + (1-\delta_{t+1}) \alpha Y_{t+1}/I_{t+1} \right] \right), \tag{4}$$

$$K_t = A_K K_{t-1}^{\delta_t} I_t^{1-\delta_t}. \tag{5}$$

$$\tag{6}$$

If we divide (4) by (3) we get,

$$h(1+\mu)N_t^{1+\mu} = (1-\alpha)Y_t = A'_t (1-\alpha) K_{t-1}^{\alpha} N_t^{1-\alpha},$$

[1] At this point we assume that the FOC provide a (local) maximum to the problem. A full analysis of Second Order Conditions is beyond the scope of this paper. The SOC have been checked numerically after parameters have been calibrated (see section 4 below).

or,

$$N_t = \left(\frac{A'_t(1-\alpha)}{h(1+\mu)} K_{t-1}^{\alpha} \right)^{1/(\alpha+\mu)}.$$

Notice that employment is an increasing function of the capital if and only if $\alpha + \mu > 0$. We assume this holds true in the rest of the paper.

Plugging this into $Y_t = A'_t K_{t-1}^{\alpha} N_t^{1-\alpha}$ gives,

$$Y_t = \left(\frac{1-\alpha}{h(1+\mu)} \right)^{\frac{1-\alpha}{\alpha+\mu}} (A'_t)^{\frac{1+\mu}{\alpha+\mu}} K_{t-1}^{\alpha \frac{1+\mu+\alpha}{\alpha+\mu}}. \tag{7}$$

Denote $I_t/Y_t = S_t$, and $\lambda_t K_t = X_t$ for all t. The system formed by Equations (3) and (5) are now equivalent to the following system,

$$S_t = \frac{X_t(1-\delta_t)(\mu+\alpha)}{(1+\mu)(1+X_t(1-\delta_t))}, \tag{8}$$

$$\frac{X_t}{\beta} = \alpha \frac{1+\mu}{\mu+\alpha} + E_t[X_{t+1}(\delta_{t+1} + \alpha \frac{1+\mu}{\mu+\alpha}(1-\delta_{t+1}))]. \tag{9}$$

It is easily seen that $S_t \in [0, 1-(1-\alpha)/(1+\mu)] \subset [0,1]$ as soon as $X_t > 0$, $1 > \alpha > 0$, and $\mu \geq 0$.

We consider that δ_t evolves according to the following stochastic process,

$$\delta_t = \delta(\eta_t) + \sigma_\delta \epsilon_{\delta,t} + \phi_\delta \epsilon_{\delta,t-1},$$

where $(\eta_t)_{t\geq 0}$ is a Markov process of order 1 with a finite number of states J, and $(\epsilon_{\delta,t})_{t\geq 0}$ stands for a second–order stationary process. More precisely, we assume $E_t[\epsilon_{\delta,t+1}] = \epsilon_\delta$, and $Var_t[\epsilon_{\delta,t+1}] = 1$. We also assume that η_t is independent from $(\epsilon_{\delta,s})_{s\leq t}$.

The shock $\epsilon_{\delta,t}$ may thus be interpreted as the outcome of a short–term problem in the storage production function. This could be caused by natural disasters (massive storms, earthquakes...) or human actions (riots for instance). Hence, we shall assume $P(\sigma_\delta \epsilon_{\delta,t} + \phi_\delta \epsilon_{\delta,t-1} \leq 0) = 1$. As there is no reason for these shocks to last, the MA specification is well–suited.[2] Also notice that we need $P(\delta_t \in [0,1[) = 1$, an easily obtained condition for MA processes.

[2] Recall the AR specification for technological shocks is justified since upward or downward moves of the production functions will typically take time to spread and fully affect the aggregate production function.

Our model may be extended to more general MA processes, at least if $(\epsilon_{\delta,t})_{t\in\mathbf{Z}}$ is an *iid* process. However, the computation of the explicit solution soon becomes difficult.[3] Considering the interpretation given above, the order of the MA process is largely linked to the frequency of the data. Typically, a large order for the MA specification is needed if we use "high" frequency data (*e.g.* monthly). To keep computations and estimations within a feasible range, we choose annual data. In this case the MA(1) specification is supported by our dataset.[4]

The Markov process captures long–lasting conflicts where capital stocks are repeatedly destroyed. For instance, we could imagine $P(\eta_t \in \{1,2\}) = 1$ for all t. A possible interpretation in this case is that $\eta_t = 1$ (resp. $\eta_t = 2$) in peaceful (resp. bellicose) times, and $\delta(2) < \delta(1)$. This interpretation also allows ϕ_δ or σ_δ not to vary with η_t – although this may be done.

Now consider $X_t = X(\eta_t) + \sigma_X(\eta_t)\epsilon_{\delta,t}$ as a possible solution to equation (9), and take into account the specification of the stochastic process $(\delta_t)_{t\geq 0}$. We must have for all state j,

$$\begin{aligned}
\tfrac{X(j)+\sigma_X(j)\epsilon_{\delta,t}}{\beta} &= \tfrac{\mu(1-\alpha)}{\mu+\alpha}\sum_{j'=1}^{J}\pi_{j,j'}((X(j')+\sigma'_X(j')\epsilon_\delta)(\delta(j')+(\sigma_\delta+\phi_\delta)\epsilon_\delta+\phi_\delta\epsilon_{\delta,t})+\sigma_X(j')\sigma_\delta) \\
&+\alpha\tfrac{1+\mu}{\mu+\alpha}\sum_{j'=1}^{J}\pi_{j,j'}(1+X(j')+\sigma_X(j')\epsilon_\delta),
\end{aligned}$$

where $\mathbf{P}(\eta_{t+1} = j'|\eta_t = j) = \pi_{j,j'}$.

This is equivalent to the following linear system:

$$\begin{aligned}
X(j) &= \tfrac{\beta\mu(1-\alpha)}{\mu+\alpha}\sum_{j'=1}^{J}\pi_{j,j'}((X(j')+\sigma'_X(j')\epsilon_\delta)(\delta(j')+(\sigma_\delta+\phi_\delta)\epsilon_\delta)+\sigma_X(j')\sigma_\delta) \\
&+\alpha\beta\tfrac{1+\mu}{\mu+\alpha}\sum_{j'=1}^{J}\pi_{j,j'}(1+X(j')+\sigma_X(j')\epsilon_\delta), \\
\sigma_X(j) &= \tfrac{\beta\mu(1-\alpha)}{\mu+\alpha}\phi_\delta\sum_{j'=1}^{J}\pi_{j,j'}(X(j')+\sigma_X(j')\epsilon_\delta).
\end{aligned}$$

We may now summarize our explicit solution.

[3] In case of a MA(8) process a linear system with more than 100 equations is involved.

[4] See Section 4.

- Exogenous processes:

$$
\begin{aligned}
&a'_t \in \mathbf{R},\\
&E_t[\epsilon_{\delta,t+1}] = \epsilon_\delta,\ Var_t[\epsilon_{\delta,t+1}] = 1,\ \mathbf{P}(\epsilon_{\delta,t} < 0) = 1,\\
&\mathbf{P}(\eta_{t+1} = j' | \eta_t = j) = \pi_{j,j'},\ \forall\, (j, j') \in \{1, \dots J\}^2,\\
&\delta_t = \delta(\eta_t) + \sigma_\delta \epsilon_{\delta,t} + \phi_\delta \epsilon_{\delta,t-1}.
\end{aligned}
$$

- Parameters and domains:

$$
\begin{aligned}
&\alpha, \beta, \in]0, 1[,\\
&\mu, h, \sigma_\delta, -\epsilon_\delta \in \mathbf{R}^{++},\\
&a_k, \phi_\delta \in \mathbf{R},\\
&0 \le \pi_{j,j'} \le 1\ \forall (j, j') \in \{1, \dots J\}^2,\ \textstyle\sum_{j'=1}^{J} \pi_{j,j'} = 1\ \forall\, j \in \{1, \dots, J\},\\
&\tfrac{\mu+\alpha}{\beta} X(j) = \mu(1-\alpha) \sum_{j'=1}^{J} \pi_{j,j'} ((X(j') + \sigma'_X(j')\epsilon_\delta)(\delta(j') + (\sigma_\delta + \phi_\delta)\epsilon_\delta) + \sigma_X(j')\sigma_\delta)\\
&\qquad\qquad + \alpha(1+\mu) \sum_{j'=1}^{J} \pi_{j,j'} (1 + X(j') + \sigma_X(j')\epsilon_\delta),\ \forall\, j \in,\\
&\sigma_X(j) = \tfrac{\beta\mu(1-\alpha)}{\mu+\alpha} \phi_\delta \sum_{j'=1}^{J} \pi_{j,j'} (X(j') + \sigma_X(j')\epsilon_\delta)\ \{1, \dots, J\} \delta_t \in [0, 1[,\\
&X_t > 0,\\
&K_{t-1}/I_t > 1.
\end{aligned}
$$

- Endogenous processes:

$$
\begin{aligned}
&X_t = X(\eta_t) + \sigma_X(\eta_t)\epsilon_{\delta,t},\\
&Y_t/I_t = S_t = \tfrac{X_t(1-\delta_t)(\mu+\alpha)}{(1+\mu)(1+X_t(1-\delta_t))},\\
&y_t = \tfrac{1-\alpha}{\alpha+\mu} \log\left(\tfrac{1-\alpha}{h(1+\mu)}\right) + \alpha \tfrac{1+\mu+\alpha}{\alpha+\mu} k_{t-1} + \tfrac{1+\mu}{\alpha+\mu} a'_t,\\
&k_t = a_k + \left(\delta_t + (1-\delta_t)\alpha \tfrac{1+\mu+\alpha}{\alpha+\mu}\right) k_{t-1} + (1-\delta_t)\left(s_t + \tfrac{1-\alpha}{\alpha+\mu} \log\left(\tfrac{1-\alpha}{h(1+\mu)}\right) + \tfrac{1+\mu}{\alpha+\mu} a'_t\right).
\end{aligned}
$$

The last issue to tackle is the existence of a stationary distribution for the endogenous process k_t. This is a key issue for (at least) two reasons. First, part of our inferential strategy relies on asymptotic approximations, that are valid only if our dynamic system accepts stationary solutions. Second, the identification of the welfare measure is not possible if no long–term distribution may be exhibited. Finally, notice that this question is not trivial since k_t evolves according to a random autoregressive coefficient with random volatility. This particular source of randomness is the consequence of the stochastic nature of the depreciation.[5] Ambler and Paquet (1994) obtain a similar result although they do not seem to discuss stationarity issues extensively.

We then derive the following result.

Proposition 1 *If,*

$$E\left[\log\left(\delta_t + (1-\delta_t)\alpha\frac{1+\mu+\alpha}{\alpha+\mu}\right)\right] < 0 \quad \textit{and} \quad \overline{lim}_{i\to+\infty}\frac{1}{i}\log(\mid a'_{t-i}\mid) < 0,$$

there exists a stationary solution to,

$$k_t = a_{k,t} + \rho_{k,t}k_{t-1} + \sigma_{k,t}a'_t, \tag{10}$$

where,

$$\begin{aligned}
a_{k,t} &= a_k + (1-\delta_t)\left(s_t + \tfrac{1-\alpha}{\alpha+\mu}\log\left(\tfrac{1-\alpha}{h(1+\mu)}\right)\right),\\
\rho_{k,t} &= \delta_t + (1-\delta_t)\alpha\tfrac{1+\mu+\alpha}{\alpha+\mu},\\
\sigma_{k,t} &= (1-\delta_t)\tfrac{1+\mu}{\alpha+\mu}.
\end{aligned}$$

Proof: We derive for all $t \geq 1$,

$$k_t = a_{k,t} + k_0\prod_{i=0}^{t-1}\rho_{k,t-i} + \sum_{i=1}^{t-1}(a_{k,t-i} + \sigma_{k,t-i}a'_{t-i})\prod_{j=1}^{i}\rho_{k,t+j-i}.$$

Now $a_{k,t}$ is a bounded stationary process (it is almost trivial considering $-1/exp(1) < (1-\delta_t)\log(1-\delta_t) < 0$). Let us denote $a_{k,t} \leq \overline{a}_k$. Since $\rho_{k,t} > 0$ we have,

$$k_t \leq \overline{a}_k + (k_0 + \overline{a}_k)\prod_{i=0}^{t-1}\rho_{k,t-i} + \sum_{i=1}^{t-1}\sigma_{k,t-i}a'_{t-i}\prod_{j=1}^{i}\rho_{k,t+j-i}. \tag{11}$$

A unique stationary solution to equation (10) exists whenever the right hand side of (11) a.s. converges to a single real valued random variable as t goes to infinity. Let us first study the a.s.

[5] See section 6 for further discussion on this point.

convergence of,

$$\sum_{i=1}^{t-1} \sigma_{k,t-i} a'_{t-i} \prod_{j=1}^{i} \rho_{k,t+j-i}.$$

A sufficient condition is the absolute convergence. We then study the following sum,

$$\sum_{i=1}^{t-1} \mid \sigma_{k,t-i} a'_{t-i} \mid \prod_{j=1}^{i} \mid \rho_{k,t+j-i} \mid .$$

Using the Cauchy principle, a sufficient condition for convergence is then given by,

$$\overline{lim}_{i \to +\infty} \sigma_{k,t-i}^{1/i} \mid a'_{t-i} \mid^{1/i} \prod_{j=1}^{i} \mid \rho_{k,t+j-i} \mid^{1/i} < 1.$$

Taking logarithms, a sufficient condition is then given by,

$$\overline{lim}_{i \to +\infty} \frac{1}{i} \log(\sigma_{k,t-i}) + \overline{lim}_{i \to +\infty} \frac{1}{i} \log(\mid a'_{t-i} \mid) + \overline{lim}_{i \to +\infty} \frac{1}{i} \sum_{j=1}^{i} \log(\mid \rho_{k,t+j-i} \mid) < 0.$$

As $\sigma_{k,t}$ is also bounded from above, we must show,

$$\overline{lim}_{i \to +\infty} \frac{1}{i} \log(\mid a'_{t-i} \mid) + \overline{lim}_{i \to +\infty} \frac{1}{i} \sum_{j=1}^{i} \log(\mid \rho_{k,t+j-i} \mid) < 0.$$

Now, if we have $E[\log(\delta_t + (1-\delta_t)\alpha \frac{1+\mu+\alpha}{\alpha+\mu})] < 0$ we get a.s.,

$$\overline{lim}_{i \to +\infty} \frac{1}{i} \sum_{j=1}^{i} \log(\mid \rho_{k,t+j-i} \mid) < 0,$$

and the Cauchy Principle also shows that in this case we have $\lim_{t \to +\infty} \prod_{i=0}^{t-1} \rho_{k,t-i} = 0.$ a.s. *Q.E.D.*

One may remark that the condition $\overline{lim}_{i \to +\infty} \frac{1}{i} \log(\mid a'_{t-i} \mid) < 0$ is very weak. For instance it does not rule out the possibility that a_t is not a stationary process. Second, the condition $E[\log(\delta_t + (1-\delta_t)\alpha \frac{1+\mu+\alpha}{\alpha+\mu})] < 0$ is also weaker than $E[\delta + (1-\delta)\alpha \frac{1+\mu+\alpha}{\alpha+\mu}] < 1$ (the stationarity condition in perpetual peaceful time with $\delta_t = \delta$).

4 Inferential strategy

We assume that the triplet (Y_t, I_t, N_t) is observed for $t = 1, \ldots, T$, and consider the case $J = 1$ in this section.[6] Moreover we specialize the processes $a_t = (1-\rho_a)a_\infty + \rho_a a_{t-1} + \sigma_a \epsilon_{a,t-1}$ and $e_t = e$. Notice that (e, a_∞) is not identifiable leading us to choose $e = 0$.

The unknown parameters and the unobserved values include,

$$
\begin{aligned}
&\text{production technological shocks} : a_\infty, \rho_a, \sigma_a, \\
&\text{storage technological shocks} : \delta(1), \phi_\delta, \sigma_\delta, \\
&\text{preference parameters} : \mu, \beta, h, \\
&\text{production technology} : \alpha, k_0, \\
&\text{storage technology} : A_k.
\end{aligned}
$$

We first fit some parameters using GMM and the easiest form of the model that is not rejected by the data, namely the case where the shocks $\epsilon_{\delta,t}$ are assumed to be uniformly distributed over $[-\sqrt{12}, 0]$ (recall we assume $Var[\epsilon_{\delta,t}] = 1$).

We used the equations,

$$
\begin{aligned}
h(1+\mu)N_t^{1+\mu} &= (1-\alpha)Y_t, \\
\tfrac{I_t}{C_t - hN_t^{1+\mu}} &= \tfrac{(1+\mu)I_t}{(\alpha+\mu)Y_t - (1+\mu)I_t},
\end{aligned}
$$

together with the two first conditional moments of $\frac{(1+\mu)I_t}{(\alpha+\mu)Y_t-(1+\mu)I_t}$, and the conditional moments of $\left(\frac{(1+\mu)I_t}{(\alpha+\mu)Y_t-(1+\mu)I_t}\right)\left(\frac{(1+\mu)I_{t-1}}{(\alpha+\mu)Y_t-(1+\mu)I_{t-1}}\right)$ and $\left(\frac{(1+\mu)I_t}{(\alpha+\mu)Y_t-(1+\mu)I_t}\right)^2\left(\frac{(1+\mu)I_{t-1}}{(\alpha+\mu)Y_t-(1+\mu)I_{t-1}}\right)$. These moments together with the distributional assumptions identify the parameters since,

$$
\begin{aligned}
X_t(1-\delta_t) &= \tfrac{(1+\mu)I_t}{(\alpha+\mu)Y_t-(1+\mu)I_t}, \\
X_t(1-\delta_t) &= X(1-\delta) + (\sigma_X(1-\delta) - \sigma_\delta)\epsilon_{\delta,t} - X\phi_\delta\epsilon_{\delta,t-1} - \sigma_\delta\sigma_X\epsilon_{\delta,t}^2 - \phi_\delta\sigma_X\epsilon_{\delta,t}\epsilon_{\delta,t-1},
\end{aligned}
$$

where we denote $(\delta, X, \sigma_X)' = (\delta(1), X(1), \sigma_X(1))'$.

Using $I_{t-3}, Y_{t-3}, W_{t-3}, I_{t-4}, Y_{t-4}, W_{t-4}, I_{t-5}, Y_{t-5}, W_{t-5}$ as instruments, we obtain the following

[6] This will cause no real problem since we use annual post–war US data. Over the period considered, this country did not experience conflicts inducing a sizeable impact on the capital stock. Moreover, collecting reliable macro–level data during major long–lasting conflicts is difficult. The main issue here is to obtain reliable values for the deep parameters governing the technology and the utility functions and some insights on the relative magnitudes of shocks.

estimates:

	GMM est.	std. err.	t-stat	p-value
μ	3.293	0.0372	88.47	$< .0001$
h	3.395E-6	5.458E-7	6.22	$< .0001$
β	0.960	0.033	28.99	$< .0001$
α	0.393	0.00960	41.00	$< .0001$
σ_δ	0.003777	0.000199	18.92	$< .0001$
δ	0.971	0.000283	3430.75	$< .0001$
ϕ_δ	0.00288	0.000236	12.18	$< .0001$

The estimates are quite in accordance with standard calibrations.[7] We also performed an over–identification test. The p–value for this test is almost one. The large value should not been taken too seriously for the sample size is small.

We have to check whether our assumption $P(\delta_t \in]0,1[) = 1$ is verified. Our estimates are clearly consistent with $\sigma > 0$ and $\phi > 0$. Now if this is true, the largest possible value for δ_t is δ which is clearly significantly smaller than 1. The smallest possible value is $\delta - (\sigma + \phi)/(\sqrt{12})$. The estimate for this quantity is 0.964622, with a standard error equal to 0.000136. We therefore conclude that our estimation is in accordance with the assumption $P(\delta_t \in]0,1[) = 1$. Similarly, it can be shown that $X_t > 0$ always – the estimate for the smallest possible value being 7.4889 with a standard error equal to 0.00932.

We also check that the MA(1) specification for δ_t is supported by the data. To this end we propose an exact test procedure as the sample size is small. Under the null hypothesis, $S_1, S_3, \ldots, S_{[T/2]}$ is an *iid* sequence. Using the technique proposed by Dufour and Roy (1985), we obtain a p–value equal to 0.243.

We extract the sequence of shocks $\epsilon_{\delta,t}$ by minimizing the sum of the squared differences between $X_t(1-\delta_t)$ and,

$$X(1-\delta) + (\sigma_X(1-\delta) - \sigma X)\epsilon_{\delta,t} - X\phi_\delta\epsilon_{\delta,t-1} - \sigma_\delta\sigma_X\epsilon^2_{\delta,t} - \phi\sigma_X\epsilon_{\delta,t}\epsilon_{\delta,t-1}.$$

[7] Notice that β cannot be estimated directly. It has been estimated using the link between β and X.

As an accuracy check to this approach, we perform a two–sided Kolmogorov–Smirnov test of the hypothesis that $\epsilon_{\delta,t}$ is uniformly distributed. The p–value of this test is 0.19.

This allows us to compute the sequence $\delta_t = \delta + \sigma\epsilon_t + \phi\epsilon_{t-1}$. The sequence k_t may be then be extracted (up to a normalizing constant) using,

$$k_t = a_k + \delta_t k_{t-1} + (1 - \delta_t)i_t.$$

Finally the production function gives $a_t = y_t - \alpha k_{t-1} - (1-\alpha)n_t$ from which we derive

$$\begin{aligned} \hat{\rho}_a &= 0.98189, \\ \hat{\sigma}_a &= 0.00869, \end{aligned}$$

where the standard error for $\hat{\rho}_a$ is 0.01727.

The AR(p) diagnostic checks for a_t are as follows,

	AR(1)		AR(2)		AR(3)		AR(4)	
Variable	F-Value	Pr > F	F	Pr > F	F	Pr > F	F	Pr > F
a_t	3.62	0.0646	1.04	0.3636	0.98	0.4134	0.60	0.6630

We then conclude that the AR(1) specification fits our data reasonably well. Finally we check whether $E\left[\log\left(\delta_t + (1-\delta_t)\alpha\frac{1+\mu+\alpha}{\alpha+\mu}\right)\right] < 0$ holds (notice that the AR(p) diagnostic check for a_t together with the fact that we reject $\rho_a = 1$ implies $\overline{lim}_{i\to+\infty}\frac{1}{i}\log(\mid a'_{t-i} \mid) < 0)$). We have,

$$E[\delta_t + (1-\delta_t)\alpha\frac{1+\mu+\alpha}{\alpha+\mu}] = (\delta - \sigma/\sqrt{48} - \phi/\sqrt{48})\left(1 - \alpha\frac{1+\mu+\alpha}{\alpha+\mu}\right) + \alpha\frac{1+\mu+\alpha}{\alpha+\mu}.$$

The estimate for the above quantity is 0.984 and the standard error is 0.000437. Hence, we conclude our model admits a stationary solution.

5 IRFs and welfare analysis

We now use the estimated model to analyze the short–term impact of shocks. Welfare computation is then used to question the cost/advantages of defense and peacekeeping policies.

5.1 Impulse Response Functions

As we derived an explicit dynamic system as a possible solution of our RBC model, Impulse Response Functions may be obtained without simulations.

First consider (generalized) productivity shocks affecting the process a'_t. Responses to temporary upward movement of the process a'_t display the usual hump–shaped pattern with the usual directions. More precisely, investment, consumption and working hours increase along with a'_t. Recall that increasing enrolment is equivalent to a negative productivity shocks. As a consequence, enrolment in perpetual peaceful times (that is when δ_t is fixed) destroys civilian working hours both directly (since $(1-e_t)N_t$ decreases) and indirectly since N_t also decreases.

We now consider responses after a negative shock to the depreciation rate, *i.e.* δ_t drops. Let us first show that K_t must then decrease. The condition $P(\sigma_\delta \epsilon_{\delta,t} + \phi_\delta \epsilon_{\delta,t-1} \leq 0) = 1$ implies $P(\delta_t \leq \overline{\delta}) = 1$ where $\overline{\delta} = \max\{\delta(1), \ldots, \delta(J)\}$. Now, it may be shown that,

$$(1-\delta)\log\left(\frac{X_t(1-\delta)(\mu+\alpha)}{(1+\mu)(1+X_t(1-\delta))}\right),$$

is a concave function of δ if $\delta \in [0,1[$. As a consequence, a necessary and sufficient condition for k_t to decrease when δ_t decreases is then,

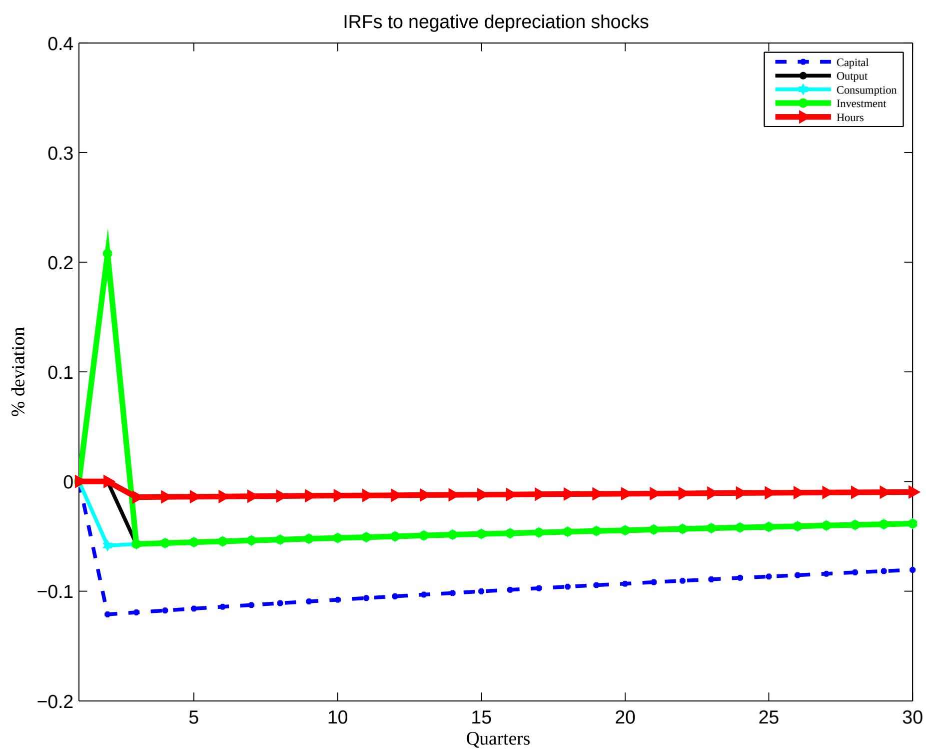
$$\frac{\mu(1-\alpha)-\alpha^2}{\mu+\alpha}k_{t-1} > \log\left(\frac{X_t(1-\overline{\delta})(\mu+\alpha)}{(1+\mu)(1+X_t(1-\overline{\delta}))}\right) + \frac{(1-\alpha)\log\left(\frac{1-\alpha}{h(1+\mu)}\right)(1+\mu)a'_t}{\alpha+\mu} - \frac{1-\overline{\delta}}{1+X_t(1-\overline{\delta})}.$$

This condition is equivalent to $K_{t-1}/I_t > exp\left(-\frac{1-\overline{\delta}}{1+X_t(1-\overline{\delta})}\right)$ when I_t and X_t are computed for $\delta_t = \overline{\delta}$. Notice this condition is always verified if $K_{t-1}/I_t > 1$ (as we assume $X_t > 0$).

Now, as N_t is an increasing function of K_{t-1} only, we obtain that working hours also decrease with one–period delay, just as production. By the same argument, the marginal productivity of labor – hence real wage – also decreases one period after the shock. Of course, the effect is reinforced if the government chooses to enlarge enrolment. However, the marginal productivity of capital – hence real interest rates – grows if $\mu > \alpha^2/(1-\alpha)$. This condition is certainly verified since the hypothesis $\alpha < 1/2 < \mu$ is clearly not rejected.

The response of investment is positive during a few periods – as S_t increases. The result after two periods depends on ϕ. If this parameter is positive enough, then two periods after the initial shock, the investment may still be higher than its long–term level. After three periods, the investment is lower than its long–term level. Finally, as production decreases and investment increases, private consumption decreases. Consumption's decrease is immediate (since investment rises whereas production is constant in the first period). The fall in consumption will typically be the largest two periods after the depreciation shocks if ϕ is large enough (since production decreases while investment may still rise). Notice that the marginal propensity to consume $(1 - S_t)$ also decreases.

The chart below displays our IRFs after a depreciation rate shock using the estimates presented in the previous section.



The "myth" according to which "war may be good for the economy" is then highly partial. The only positive effect our model produces is an increase of gross private investment. Notice however that this effect is entirely due to the need to rebuilt destroyed capital. The net and mid–term effects are both clearly negative. Also, the claim according to which employment may be larger during war episodes does not show up. Finally, reversing the previous comparative exercises, our model is able to account for economic "booms" after long periods of conflicts, as this is may be analyzed as a positive shock on δ_t.

5.2 Welfare analysis

The conclusions to be drawn from our IRFs are limited, as they relate to the linkage between destructive episodes and business cycles. The fact that destruction by itself cannot be good for the economy goes back to the famous "broken window fallacy" controversy. It clearly cannot be used to justify massive disarmament. A more careful study of the benefits induced by enrolment must be derived from welfare analysis.

As mentioned in the introduction, the two kinds of risks we consider (temporary and long lasting shocks) are usually handled by two different peacekeeping forces: police and army. We first consider the optimal short–run policy, in which enrolment is entirely devoted to police forces, whereas enrolment in the army is consider in a second step.

5.2.1 Police forces and the short–run enrolment policy

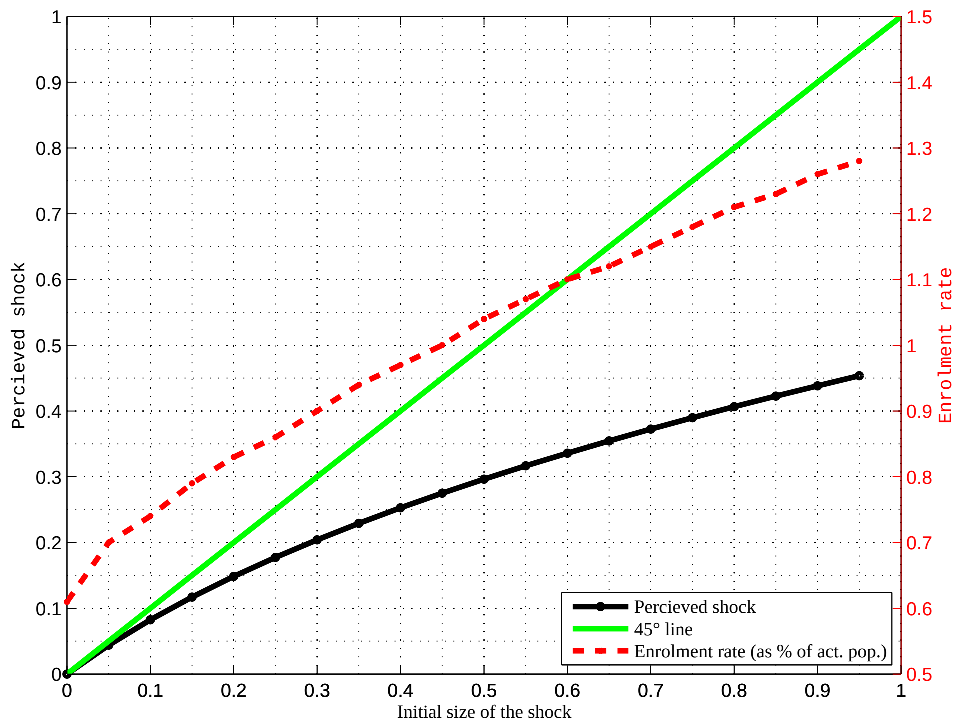
Since we consider only short–run policy, we consider here that $J = 1$. For the sake of illustration, assume that enrolment in the police force divides the size of the temporary shocks (that is $\epsilon_{\delta,t}/(\theta e_t)$). Enrolment at date t must then be chosen as to maximize,

$$\begin{aligned}&(1-\alpha)\tfrac{1+\mu}{\alpha+\mu}\log(1-e_t)+\beta\alpha\tfrac{1+\alpha+\mu}{\alpha+\mu}\left(1+\beta E_t\left[\delta_{t+1}+(1-\delta_{t+1})\alpha\tfrac{1+\alpha+\mu}{\alpha+\mu}\right]\right)k_t\\&-\log(1+X_t(1-\delta_t))-E_t\left[\log(1+X_{t+1}(1-\delta_{t+1}))\right].\end{aligned}$$

Calibrating θ such that the average amount of police forces in the US correspond to the optimal level for average shocks, we get the following chart:[8]

Optimal enrolment is an increasing function of the size of shocks with a moderate convexity effect. Notice also the evolution is not very large (keeping police forces within +/-15% of its actual value covers optimally more than 3/4 of the shocks). Also, the "perceived shocks" (that is $\epsilon_{\delta,t}/(\theta e_t)$ decrease more sharply for larger shocks. The optimal enrolment absorbs more than 50%

[8] The data are from the Bureau of Labor Statistics, see $http://www.bls.gov/oes/current/oes_n at.htm\#b33-0000$. Enrolment is calculated as the sum of occupations in the protective service sector plus active military forces inside the U.S., see $http://siadapp.dior.whs.mil/personnel/MILITARY/ms0.pdf$.



of the largest possible shock.

Finally, recall,

$$(1-\delta)\log\left(\frac{X(1-\delta)(\mu+\alpha)}{(1+\mu)(1+X(1-\delta))}\right),$$

is a concave function of δ. We conclude that the police technology, *ceteris paribus*, should aim at increasing the distribution of δ_t in the second–order sense. In other words, the welfare displays risk–adversity towards $\tilde{\delta}$, and there is a positive propensity to pay for lower–magnitude risks of riots and natural disasters.

5.2.2 Military forces and the long–run enrolment policy

We now consider long–run policy. Consequently, the government's objective is to maximize the whole welfare stream. If a stationary distribution exists, the welfare may be explicitly derived. Indeed, we have,

$$E_0\left[\sum_{t=0}^{+\infty}\beta^t\log(C_t-hN_t^{1+\mu})\right]=E_0\left[\sum_{t=0}^{+\infty}\beta^t y_t\right]+E_0\left[\sum_{t=0}^{+\infty}\beta^t\log\left(1-S_t-\frac{1-\alpha}{1+\mu}\right)\right].$$

The last sum of the above equation is,

$$\frac{\log(\mu+\alpha) - \log(1+\mu) - E[\log(1+\tilde{X}(1-\tilde{\delta}))]}{1-\beta},$$

where for any asymptotically stationary process x_t, we denote, $\tilde{x} \overset{d}{=} \lim_{t\to+\infty} x_t$.

Now recall we have,

$$y_t = \frac{1-\alpha}{\alpha+\mu}\log\left(\frac{1-\alpha}{h(1+\mu)}\right) + \alpha\frac{1+\mu+\alpha}{\alpha+\mu}k_{t-1} + \frac{1+\mu}{\alpha+\mu}a'_t.$$

Hence we derive,

$$\begin{aligned} E_0\left[\sum_{t=0}^{+\infty}\beta^t y_t\right] &= y_0 + \frac{\beta(1-\alpha)\log\left(\frac{1-\alpha}{h(1+\mu)}\right)}{(\alpha+\mu)(1-\beta)} + \alpha\frac{1+\mu+\alpha}{\alpha+\mu}E_0\left[\sum_{t=1}^{+\infty}\beta^t k_{t-1}\right] + \frac{\beta(1+\mu)a_\infty}{(1-\beta)(\alpha+\mu)}, \\ &= y_0 + \frac{\beta\left((1-\alpha)\log\left(\frac{1-\alpha}{h(1+\mu)}\right) + \alpha(1+\mu+\alpha)E[\tilde{k}] + (1+\mu)(a_\infty + (1-\alpha)E[\log(1-\tilde{e})])\right)}{(1-\beta)(\alpha+\mu)}. \end{aligned}$$

The computation of $E[\tilde{k}]$ in the general case is cumbersome. To avoid complications, we consider $\epsilon_{\delta,t} = 0$. Again, we consider that enrolment is entirely devoted to lower destructive effects of possible long–term conflicts.

We may then restrict the welfare analysis to,

$$\frac{\beta\left(\alpha(1+\mu+\alpha)E[\tilde{k}] + (1+\mu)(1-\alpha)E[\log(1-\tilde{e})]\right)}{\alpha+\mu} - E[\log(1+\tilde{X}(1-\tilde{\delta}))].$$

Let $P_j = P(\eta_t = j)$, we have (following, for instance, Francq and Zakoian (2001), section 4):

$$\begin{aligned} P_j E[k_t|\eta_t = j] &= a_k + (1-\delta(j))\left(\frac{1-\alpha}{\alpha+\mu}\log\left(\frac{1-\alpha}{h(1+\mu)}\right) + s_\infty(j) + \frac{1+\mu}{\alpha+\mu}(a_\infty + (1-\alpha)log(1-e(j)))\right), \\ &+ \left(\alpha\frac{1+\mu+\alpha}{\alpha+\mu} + \frac{\mu(1-\alpha)-\alpha^2}{\alpha+\mu}\delta(j)\right)\sum_{j'=1}^{J}\pi_{jj'}P_{j'}E[k_{t-1}|\eta_{t-1} = j'], \end{aligned}$$

where $s_\infty(j) = \log\left(\frac{\mu+\alpha}{1+\mu}\right) + x(j) + E[\log(1-\delta(j))] - E[\log(1+X(j)(1-\delta(j))]$.

We therefore conclude that the vector $(P_1E[k_t|\eta_t = 1], \ldots, P_JE[k_t|\eta_t = J])'$ is the solution of a linear system. Provided the matrix $Id_J - [\left(\alpha\frac{1+\mu+\alpha}{\alpha+\mu} + \frac{\mu(1-\alpha)-\alpha^2}{\alpha+\mu}(\delta(j)+\sigma\epsilon)\right)\pi_{jj'}]$ is invertible, there is only one solution and $E[\tilde{k}] = \sum_{j=1}^{J} P_j E[k_t|\eta_t = j]$.

It is easy to show that any positive shift in the distribution of δ increases the welfare. In other words, any defense technology that changes the initial distribution of δ to another one which stochastically dominates the first one at the first order, induces welfare gains.

Again, for concreteness, assume $\delta_t = \overline{\delta} + (\overline{\delta} - \delta(\eta_t))/(\theta e_t)$. As in the previous "police case", enrolment increases at the beginning of a war, grows up or down as war becomes more or less destructive and returns to its lowest level if we go back to peaceful time. Also remark that because of the term $(1-\delta(j))(1-\alpha)\frac{1+\mu}{\alpha+\mu}log(1-e(j))$, a more efficient defense technology is also more costly.

We finally use our model to address questions about the goal a long–run defense policy should aim at – beside trying to reduce the consequences of negative shocks on δ. Consider the case $J = 2$ (recall by convention we assume $\eta_1 = 1$ if and only if peace prevails at date t). We would like to know how the welfare depends on the sequence of war and peace. For instance, assume we must choose among two military technologies leading to the same average value of δ at the same enrolment cost. The first one aims at preventing long wars, at the cost of more frequent war episodes. The second tries to keep peace as long as possible, at the cost of more destructive war episodes. What is the best one?

If $J = 2$ the welfare may be written as,

$$\begin{aligned}
&\tfrac{\beta\alpha(1+\mu+\alpha)}{(1-\beta)(\alpha+\mu)}E[k_t|\eta_t = 2] - \tfrac{\log(1+X(1-\delta(2)))}{1-\beta},\\
&+P_1\left(E[k_t|\eta_t = 1] - E[k_t|\eta_t = 2] - \tfrac{\log(1+X(1-\delta(1)))-\log(1+X(1-\delta(2)))}{1-\beta}\right),
\end{aligned}$$

where $P_1 = \frac{1-\pi_{22}}{2-\pi_{11}-\pi_{22}}$ is the (marginal, stationary) probability of peace. Moreover, we have,

$$\begin{aligned}
&P_1E[k_t|\eta_t = 1] = a_1 + b_1(\pi_{11}P_1E[k_t|\eta_t = 1] + (1-\pi_{11})(1-P_1)E[k_t|\eta_t = 2]),\\
&(1-P_1)E[k_t|\eta_t = 2] = a_2 + b_2((1-\pi_{22})P_1E[k_t|\eta_t = 1] + \pi_{22}(1-P_1)E[k_t|\eta_t = 2]),\\
&E[\tilde{k}] = P_1E[k_t|\eta_t = 1] + (1-P_1)E[k_t|\eta_t = 2],\\
&a_j = a_k + (1-\delta(j))\left(\tfrac{1-\alpha}{\alpha+\mu}\log\left(\tfrac{1-\alpha}{h(1+\mu)}\right) + s_\infty(j) + \tfrac{1+\mu}{\alpha+\mu}(a_\infty - (1-\alpha)\log(1-e(j))\right),\\
&b_j = \alpha\tfrac{1+\mu+\alpha}{\alpha+\mu} + \tfrac{\mu(1-\alpha)-\alpha^2}{\alpha+\mu}\delta(j).
\end{aligned}$$

Let us denote $\Omega = \begin{pmatrix} b_1\pi_{11} & b_1(1-\pi_{11}) \\ b_2(1-\pi_{22}) & b_2\pi_{22} \end{pmatrix}$ and $w = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$.

We have $E[\tilde{k}] = (1,1)\Omega^{-1}w$, and then, $dE[\tilde{k}] = -(1,1)\Omega^{-1}d\Omega\Omega^{-1}w$, or,

$$
\begin{aligned}
sgn(dE[\tilde{k}]) &= -sgn\left((1,1)\begin{pmatrix} b_2\pi_{22} & -b_2(1-\pi_{22}) \\ -b_1(1-\pi_{11}) & b_1\pi_{11} \end{pmatrix} d\Omega \begin{pmatrix} b_2\pi_{22} & -b_2(1-\pi_{22}) \\ -b_1(1-\pi_{11}) & b_1\pi_{11} \end{pmatrix} w\right), \\
&= -sgn\left(\begin{pmatrix} b_2\pi_{22} - b_1(1-\pi_{11}) \\ b_1\pi_{11} - b_2(1-\pi_{22}) \end{pmatrix} d\Omega \begin{pmatrix} b_2\pi_{22}w_1 - b_2(1-\pi_{22})w_2 \\ -b_1(1-\pi_{11})w_1 + b_1\pi_{11}w_2 \end{pmatrix}\right), \\
&= -sgn\left(\begin{pmatrix} b_2\pi_{22} - b_1(1-\pi_{11}) \\ b_1\pi_{11} - b_2(1-\pi_{22}) \end{pmatrix} \begin{pmatrix} b_1 d\pi_{11} & -b_1 d\pi_{11} \\ -b_2 d\pi_{22} & b_2 d\pi_{22} \end{pmatrix} \begin{pmatrix} b_2\pi_{22}w_1 - b_2(1-\pi_{22})w_2 \\ -b_1(1-\pi_{11})w_1 + b_1\pi_{11}w_2 \end{pmatrix}\right).
\end{aligned}
$$

If we consider the case $\delta(1) \simeq \delta(2)$, that is to say war episodes are not too destructive. We get the following approximation:

$$
\begin{aligned}
sgn(dE[\tilde{k}]/E[\tilde{k}]) &\simeq -sgn\left(\begin{pmatrix} \pi_{22} + \pi_{11} - 1 \\ \pi_{11} + \pi_{22} - 1 \end{pmatrix} \begin{pmatrix} d\pi_{11} & -d\pi_{11} \\ -d\pi_{22} & d\pi_{22} \end{pmatrix} \begin{pmatrix} 2\pi_{22} - 1 \\ 2\pi_{11} - 1 \end{pmatrix}\right), \\
&= sgn\left(\begin{pmatrix} \pi_{11} + \pi_{22} - 1 \\ \pi_{11} + \pi_{22} - 1 \end{pmatrix} \begin{pmatrix} (\pi_{11} - \pi_{22})d\pi_{11} \\ (\pi_{22} - \pi_{11})d\pi_{22} \end{pmatrix}\right), \\
&= sgn(\pi_{11} + \pi_{22} - 1)sgn(\pi_{11} - \pi_{22})sgn(d\pi_{11} - d\pi_{22}).
\end{aligned}
$$

Let us be optimistic and assume peace is a more stable state than war $\pi_{11} > \pi_{22}$. Consider a reasonably peaceful place in which $\pi_{11} + \pi_{22} > 1$. Then, clearly increasing π_{11} and/or reducing π_{22} increases the welfare (since P_1 increases in this case, the second term of the welfare is also increasing). Now, suppose it is no longer possible to increase P_1. Then increasing $d\pi_{11}$ also leads to increase $d\pi_{22}$. It may then be checked that the welfare increases with π_{11}. The opposite is true if the transition matrix that drives the Markov process is "anti-diagonal dominant", *i.e* both π_{11} and π_{22} are small.

This characteristic of the model is directly related to the randomness of the autoregressive coefficient that moves across time. It may be used to illustrate some aspects of the strategic debate that emerged during the last U.S. presidential campaign. In a nutshell, the alternative doctrines were the following. The "pro-Bush" view aims at engaging in more frequent and hopefully limited conflicts. Recall indeed that the war against Iraq was supposed to end up quickly with a "pro–Occidental" democratic regime, leading to a positive breakthrough over the entire Middle–East.

The opposite view – exposed by Carter (see his 2002 Nobel Prize reception speech) and recently highlighted by Obama – urges to avoid war episodes, even if it invloves "discussion with our enemies". Our model suggests that the right choice depends on the stability of peace and war states. The case for the "pro-Bush" view is the unstable one – π_{11} and π_{22} are both close to zero – whereas the "Carter" view is more relevant in a stable world – if π_{11} and π_{22} are large enough. Notice that considering the Correlate of War dataset, we obtain $\hat{\pi}_{11} = 0.75$ and $\hat{\pi}_{22} = 0.151$ for the U.S. over the entire period (1821-2003). These data then seems to back the "Carter" view.

This welfare analysis may also justify the choice of dissuasive weapons (in so far dissuasion aims at preserving peace) when the situation is stable enough. Recall however, that a more efficient defense technology is always more costly. Hence, if the objective is to reach a given level of welfare, a bellicose technology is the cheapest way to achieve this goal.

Finally consider a limiting case where $\pi_{11} = \pi_{22} = 1/2$ (that is war and peace are equally frequent and stable states), then if P_1 (also equal to 1/2) cannot be changed, either longer peace and war episodes or both shorter ones can be chosen. Any of these policy increases the welfare. Interestingly both policies display a self–fulfilling feature: if we choose to increase (resp. decrease) π_{11} and π_{22} with $P_1 = 1/2$ then it is beneficial to increase (resp. decrease) this quantities again, if such a technology is available.

6 Some extensions of the model

Our model admits several extensions, some of which are now presented and discussed. The first one generalizes the shocks that may disturb the accumulation of capital. The representative household now faces the following problem,

$$\begin{aligned}
\max \quad & E_0 \left[\sum_{t \geq 0} \beta^t \log(C_t + h(N_t)^{1+\mu} \right], \\
s.t. \quad & Y_t = A_t' K_{t-1}^{\alpha} N_t^{1-\alpha}, \\
& K_t = A_{k,t} K_{t-1}^{\delta_t} I_t^{1-\delta_t}, \\
& Y_t = C_t + I_t,
\end{aligned}$$

where $A_{k,t}$ is a positive exogenous shock. As Equations (3) to (5) remain unchanged, we easily see that the exact solution found in section 3 holds except we now have,

$$k_t = a_{k,t} + \left(\delta_t + (1-\delta_t)\alpha\frac{1+\mu+\alpha}{\alpha+\mu}\right)k_{t-1} + (1-\delta_t)\left(s_t + \frac{1-\alpha}{\alpha+\mu}\log\left(\frac{1-\alpha}{h(1+\mu)}\right) + \frac{1+\mu}{\alpha+\mu}a'_t\right).$$

We may then remark the differences between the two sources of shocks in the accumulation of capital. Assume for instance δ_t is constant. Then decreasing $A_{k,t}$ is equivalent to a negative productivity shock (only the magnitude differs). As a consequence, after such a shock the investment decreases. The "rebuilding effect" appears only if δ changes.

This first extension is also useful for a second one. Assume now that part of the domestic consumption is produced abroad. This may be roughly handled in the following specification,

$$\begin{aligned}
\max \quad & E_0 \sum_t \beta^t \log(C_t + h(N_t)^{1+\mu}), \\
s.t. \quad & Y_t = A'_t K_{t-1}^{\alpha} N_t^{1-\alpha}, \\
& K_t = A_k K_{t-1}^{\delta_t} I_t^{1-\delta_t}, \\
& Y_t = (1-\psi_t)C_t + I_t,
\end{aligned}$$

where ψ_t is an exogenous process interpreted as the proportion of domestic consumption produced abroad. The problem is equivalent to the following one,

$$\begin{aligned}
\max \quad & E_0 \left[\sum_t \beta^t \log(C_t + h(N_t)^{1+\mu})\right] \\
s.t. \quad & Y'_t = A''_t K_{t-1}^{\alpha} N_t^{1-\alpha} \\
& K_t = A_{k,t} K_{t-1}^{\delta_t} (I'_t)^{1-\delta_t} \\
& Y'_t = C_t + I'_t
\end{aligned}$$

where $Y'_t = Y_t/(1-\psi_t), A''_t = A'_t/(1-\psi_t), A_{k,t} = A_k(1-\psi_t)^{1-\delta_t}, I'_t = I_t/(1-\psi_t)$. Now the capital accumulates according to,

$$\begin{aligned}
k_t \quad & = a_k + \left(\delta_t + (1-\delta_t)\alpha\tfrac{1+\mu+\alpha}{\alpha+\mu}\right)k_{t-1} + (1-\delta_t)\left(s_t + \tfrac{1-\alpha}{\alpha+\mu}\log\left(\tfrac{1-\alpha}{h(1+\mu)}\right) + \tfrac{1+\mu}{\alpha+\mu}a'_t\right) \\
& -(1-\delta_t)\log(1-\psi_t)\tfrac{1-\alpha}{\alpha+\mu}.
\end{aligned}$$

As $\log(1-\psi_t)\frac{1-\alpha}{\alpha+\mu} \leq 0$, trade openness lowers the impact of negative shocks on δ_t and mitigates the effects of positive shocks. In other words, debuts of conflicts are less destructive but ends of conflicts induces more moderate recovery.

This extension may also be used to question the impacts of embargo/blockades/sieges together with or in lieu of conflicts. A blockade (or complete embargo) amounts to fix $\psi_t = 0$. When used instead of open conflicts, blockades have the same negative impact as a negative productivity shock. When used together with a destructive attack, blockades reinforce the destructive impact. This last effect is easy to understand. Recall indeed that destructions induce larger responses of investment and sharper drops of private consumption, since production drops. Now, the sudden blockade destroys parts of the previously "free" consumption. Hence, the investment cannot be too large, and rebuilding is less effective. Blockades/sieges sharply reinforce the negative welfare effects of conflicts. This is especially true if labor is inefficient and painful (α is small and/or μ is large).

7 Conclusion

In this paper, we assess the impact of destructions caused by various conflicts or natural disasters on business cycles. To do this, we build on a standard Real Business Cycle model that allows for explicit solutions in order to account for the impact of large shocks. Conflicts may be either temporary and small–scale or much more intense and long–lasting. In the first case, the depreciation rate of physical capital is affected by random shocks. In the second case, it may switch from a low level in peaceful times to a high level in bellicose times. The model is estimated on US annual data and used to investigate the welfare consequences of alternative defense technologies. These technologies involve enrolment, which is modelled as a conscription policy. We show that, in the short–run, increasing enrolment is an optimal response to destructions induced by riots or natural disaster. In the longer run, our model allows to question the goal a defense policy should aim at. These include lowering the magnitude, the variability and the frequency of conflicts. In particular, *ceteris paribus*, a country is better–off avoiding war – even at the cost of longer conflicts – than promoting frequent but small–scaled conflicts.

In further research, we intend to use the above model to investigate many open questions related to military policies during conflict and peaceful periods. Indeed, the actual role of the State is not limited to conscription. Fiscal policies, price and production controls, massive production of money have all been used during war times (one also recall from Sun-Tzu's Art of War "when the army engages in protracted campaigns the resources of the state will not suffice"). Also the possible productivity impacts induced by the defense technology may not be negligible. In our model, these would act as substitutes to enrolment, which is clearly not a sufficient account for this effect. International issues are of course a major issue and the treatment of the last section only pays lip service to it. For instance, the results put forth by Martin et al. (2008b) tell us that the probability of conflicts cannot be treated as exogenous. Finally demographic impacts may also play a large role in the recovery process after major conflicts.

References

Ambler, S. and Paquet, A. (1994), Stochastic Depreciation and the Business Cycle, *International Economic Review*, **35**, 1, 101-116.

Azam, J., Collier, P. and Hoeffler, A. (2001), International Policies on Civil Conflict: An Economic Perspective, University of Oxford.

Arias, A. and Ardila, L. (2003), Military Expenditures and the Economic Activity: The Colombian Case, documento CEDE 2003-20.

Baxter, M. and King, R. (1993), Fiscal Policy in General Equilibrium, *American Economic Review*, **83**, 315-334.

Collard, F. (1999), Spectral and Persistence Properties of Cyclical Growth, *Journal of Economic Dynamics and Control*, **23**, 3, 463-488.

Collier, P. (1999), On the Economic Consequences of Civil War, *Oxford Economic Papers*, **51**, 168-183.

Francq, C. and Zakoian, J.M. (2001), Stationarity of Multivariate Markov-switching ARMA Models, *Journal of Econometrics*, **102**, 339-364.

Heo, U. (1999), Defense Spending and Economic Growth in South Korea: The Indirect Link, *Journal of Peace Research*, **36** , 6, 699-708.

Hercowitz, Z. and Sampson, M. (1991), Output Growth, the Real Wage, and Employment Fluctuations, *American Economic Review*, **81**, 5, 1215-1237.

Hess, G. (2003), Economic Welfare Cost of a Conflict: An Empirical Assessment, CESifo working paper 852.

McGrattan, E. and Ohanian, L. (2008), Does Neoclassical Theory Account for the Effect of Big Fiscal Shocks : Evidence from WWII, Federal reserve bank of Minneapolis, Staff Report 315.

McGrattan, E. and Braun, R. (1993), The Macroeconomics of War and Peace, *NBER Macroeconomics Annual 1993,* MIT Press.

Martin P., Mayer T. and Thoenig, M. (2008a), Civil Wars and International Trade, *Journal of the European Economic Association, Papers and Proceedings*, **6**, 3, 541-550.

Martin P., Mayer T. and Thoenig, M. (2008b), Make Trade not War?, *Review of Economic Studies*, **75**, 3, 865-900.

Ohanian, L. (1997), The Macroeconomic Effects of War Finance in the United States: WWII and the Korean War, *American Economic Review*, **87**, 1, 23-40.

Ramey, V. (2008), Identifying Government Spending Shocks : It's All in the Timing, mimeo.

Ramey, V. and Shapiro, M. (1997), Costly Capital Reallocation and the Effects of Government Spending, *Carnegie-Rochester Conference Series on Public Policy*, **48**, 145-194.

Bilmes, L. and Stiglitz, J. (2008), The Three Trillion Dollar War: The True Cost of the Iraq Conflict, The New York Times Bestseller, New York.

Stroup, M. and Heckelman, J. (2001), Size of the Military Sector and Economic Growth: A Panel Data Analysis of Africa and Latin America, *Journal of Applied Economics*, **IV**, 2, 329-360.