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Ramsey Policies in a Small Open Economy with Sticky Prices and Capital*

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Abstract

In this paper we study jointly optimal fiscal and monetary policies in a small open economy framework with capital and sticky prices. We consider the case of distortionary taxes on labor and capital, and no public debt. As in a closed economy set–up, in the steady state, the optimal inflation rate is zero, as well as the optimal tax on capital. The dynamic properties of optimal monetary and fiscal policies in an open economy are qualitatively the same as those of a closed economy: the tax rate on capital income remains constant over the cycle, while both the nominal interest rate and the tax rate on labor income move although very smoothly, respectively to minimize the distortions implied by nominal rigidities and balance the budget.

Keywords: small open economy, sticky prices, optimal monetary and fiscal policies.

JEL Class.: E52, E62, E63, F41.

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1 Introduction

Previous literature in closed economies has shown that it was optimal to use unexpected variations in prices in a flexible prices setup to smooth taxes. However, when prices are sticky, a trade–off appears between using unexpected inflation to smooth taxes and the fact that adjusting the price level is costly (Schmitt-Grohe and Uribe (2004a)). When an open economy framework is considered, new issues come into play, such as whether to stabilize or not fluctuations on the exchange rate, and its implications on inflation and welfare (Benigno and de Paoli (2009)).

The literature of optimal taxation following Lucas and Stokey (1983) establishes that distorting taxes should be very smooth over time and states of nature, implying that capital taxes should be close to zero and that labor taxes should be roughly constant (e.g. Chari and Kehoe (1999)). Another result is that tax rates are an increasing function of the elasticity of input supply, i.e. the tax rate on labor increases when the elasticity of labor supply increases (see Chamley (1986)).

Schmitt-Grohe and Uribe (2004a) find that the major results about optimal fiscal policy are mostly unchanged in a closed economy when sticky prices are introduced. However, this is not the case in an open economy with nominal rigidities (Benigno and de Paoli (2009)). These authors argue that the open economy dimension introduces shocks to the terms–of–trade that directly affect labor supply decisions, although only for high levels on openness.

Regarding monetary policy, the literature underlines that the optimal policy in a competitive environment (with flexible prices) is to follow the Friedman rule, i.e. to set the nominal interest rate to zero at each period. However, the Friedman rule is found to be suboptimal when sticky prices are introduced (see for instance Woodford (2003)). In this case, the optimal monetary policy rule should aim at stabilizing the inflation rate to minimize the distortions associated to nominal rigidities.

However, to our knowledge little work has been done on the interaction between monetary and fiscal policy in an open economy environment. This may look surprising because the effectiveness and proper conduct of national macroeconomic policies should clearly depend on international linkages between national economies. Furthermore, previous literature omits capital as an input factor, when introducing this variable seems to be crucial for the results of optimal fiscal policy.

In this paper, we study the dynamic properties of setting taxes and monetary policy optimally in a small open economy with capital and Calvo sticky prices. The setup we use is a standard New Keynesian DSGE model in which the government levies distortionary taxes on inputs (capital and labor) to provide individuals with an exogenously determined amount of public good, and does not have access to subsidies to undo the distortions introduced by imperfect competition.

Our approach solves the Ramsey problem in the context of a small open economy with capital, extending the work of Schmitt-Grohe and Uribe (2004a). We study the optimal taxation system both in the Pareto–inefficient steady state and around this steady state (dynamics). There are several ways in which the open dimension, combined with the presence of nominal rigidities and monopolistic competition may change some of the traditional results about optimal taxation. First, monopolistic competition distorts steady state allocations and taxes on inputs (labor and capital), as well as the inflation tax, may help attaining the first–best. Second, as nominal rigidities imply departures from the Friedman rule, and as the dynamics of capital income are closely related to the path chosen for the nominal interest rate, the tax–smoothing result on capital income taxes may not apply in this environment. Finally, the open–economy dimension may clearly affect both the optimal level of tax rates and their dynamics over business cycles.

First, considering a closed economy version of our model, we show that it is able to reproduce most closed economy results:

- In accordance with Chamley (1986), the tax rate on capital income should be zero in the steady state and should be kept nearly constant over the cycle, even in a sticky price environment.

- Optimal monetary policy implies large departures from the Friedman rule and should be aimed at stabilizing the inflation of production prices.

- Fiscal and monetary policy instruments exhibit a large persistence, reflecting the need to smooth the implied distortions on labor supply and consumption.

Trade openness has no impact on the optimal steady state of the economy. Furthermore, we show that the volatility of policy instruments is affected only to the extent that the open–economy dimension implies an increase in the overall macroeconomic volatility, related to the presence of an additional shock (world demand shocks). As a consequence, closed–economy results are fully preserved, whatever the degree of trade openness or the nature of goods traded (consumption or capital goods).

The paper is structured as follows. The model and its general equilibrium conditions are presented in Section 2. Section 3 describes the parameterization used along the paper. Sections 4 and 5 report the results on optimal monetary and fiscal policy in the steady state and dynamics, respectively. In Section 6, we investigate an extension of the model with traded capital goods. The paper concludes with Section 7 and further research.

2 The model

2.1 Households

The model is composed of two areas: the domestic economy and the rest of the world, of size n and $1-n$, respectively. Both areas are symmetric, except for the size. In what follows below, we denote rest of the world variables by an asterisk.

Each country is populated by infinitely–living households. In each area, the representative household j maximizes a welfare index

$$\Omega(j) = E_0 \sum_{t=0}^{\infty} \beta^t u\left(c_t(j), \ell_t(j)\right) \tag{1}$$

subject to the budget constraint[1]

$$E_t\left\{q_{t,t+1} b_{t+1}(j)\right\} + p_{c,t} c_t(j) + p_t k_t(j) = b_t(j) + \Pi_t(j) + \left(1-\tau_{\ell,t}\right) w_t \ell_t(j) + p_t r_{k,t} k_{t-1}(j) \tag{2}$$

[1]The model features no money holdings, since money is endogenously supplied depending on the level of nominal interest rate.

and subject to the appropriate transversality conditions on claims and capital accumulation.

In Equation (1), the parameter β is the subjective discount factor; $c_t(j)$ is the consumption bundle chosen by the representative agent; $\ell_t(j)$ is the quantity of labor competitively supplied. In Equation (2), w_t and z_t are the nominal wage and return on physical capital rental; $r_{k,t} = 1 + (1 - \tau_{k,t}) \left(\frac{z_t}{p_t} - \delta\right)$ is the net (real) return on capital accumulation; $\Pi_t(j) = \int_0^n \Pi_t(i,j)di$ refers to profits paid by domestic firms (operating on monopolistic competition markets), indexed by i, to the representative domestic household. The variable $b_t(j)$ is a portfolio of state contingent assets held in period $t-1$, which pays in units of domestic aggregate consumption; $q_{t,t+1}$ denotes the stochastic discount factor for one–period ahead nominal payments attached to the portfolio. Finally, $p_{c,t}$ and p_t denote the consumer and producer price indices, respectively.

The representative household chooses $c_t(j)$, $\ell_t(j)$, $k_t(j)$, and $b_{t+1}(j)$ to maximize utility (1) subject to the budget constraint (2). First order conditions imply

$$-\frac{u_{\ell,t}}{u_{c,t}} - (1 - \tau_{\ell,t}) \frac{w_t}{p_{c,t}} = 0 \tag{3}$$

$$\beta \left(\frac{u_{c,t+1}}{u_{c,t}}\right) \left(\frac{p_{c,t}}{p_{c,t+1}}\right) = q_{t,t+1} \tag{4}$$

$$q_{t,t+1} \frac{p_{t+1}}{p_t} r_{k,t+1} = 1 \tag{5}$$

Equation (3) is the standard labor supply function, describing the intra–temporal trade off between consumption and leisure. Equation (4) is the Euler equation relating the intertemporal choice of consumption as a function of inflation and the return on the financial portfolio. Denoting $r_t = \frac{1}{E_t\{q_{t,t+1}\}}$ as the gross return on a riskless one–period bond, and taking conditional expectations on both sides of (4), the standard Euler equation writes

$$r_t \beta E_t \left\{ \left(\frac{u_{c,t+1}}{u_{c,t}}\right) \left(\frac{p_{c,t}}{p_{c,t+1}}\right) \right\} = 1$$

Finally, taking conditional expectations on both sides of Equation (5) shows that net returns on disposable assets should be equal in equilibrium

$$E_t \left\{ \frac{p_{t+1}}{p_t} r_{k,t+1} \right\} = r_t$$

We follow Benigno and de Paoli (2009) and assume that the aggregate consumption of consumer j is a composite of consumption of goods produced at home (h), and goods produced in the rest of the world (f) according to

$$c_t(j) = \left[\varphi^{\frac{1}{\mu}} c_{h,t}(j)^{\frac{\mu-1}{\mu}} + (1-\varphi)^{\frac{1}{\mu}} c_{f,t}(j)^{\frac{\mu-1}{\mu}}\right]^{\frac{\mu}{\mu-1}}$$

where $\varphi = 1-(1-n)\alpha$ refers to the relative weight of home and foreign goods, which is a function of the size of the domestic economy, n, and α, a measure of trade openness (see Corsetti (2006) and Goldberg and Tille (2008)). Symmetrically, the consumption of a representative household in the rest of the world is

$$c_t^*(j) = \left[\varphi^{*\frac{1}{\mu}} c_{h,t}^*(j)^{\frac{\mu-1}{\mu}} + (1-\varphi^*)^{\frac{1}{\mu}} c_{f,t}^*(j)^{\frac{\mu-1}{\mu}}\right]^{\frac{\mu}{\mu-1}}$$

where $\varphi^* = n\alpha$.

We assume that the law of one price holds at the producer level, then the companion consumption price indexes are given by

$$p_{c,t} = \left[\varphi(p_t)^{1-\mu} + (1-\varphi)(\varepsilon_t p_t^*)^{1-\mu}\right]^{\frac{1}{1-\mu}}$$
$$p_{c,t}^* = \left[\varphi^*\left(\varepsilon_t^{-1} p_t\right)^{1-\mu} + (1-\varphi^*)(p_t^*)^{1-\mu}\right]^{\frac{1}{1-\mu}}$$

where ε_t denotes the nominal exchange rate.[2]

In these expressions, $\mu \geq 1$ is the elasticity of substitution between domestic and foreign goods. Standard Dixit and Stiglitz (1977) consumption subindexes are given by

$$c_{h,t}(j) = \left[\left(\frac{1}{n}\right)^{\frac{1}{\theta}} \int_0^n c_{h,t}(i,j)^{\frac{\theta-1}{\theta}} di\right]^{\frac{\theta}{\theta-1}}, \; c_{f,t}(j) = \left[\left(\frac{1}{1-n}\right)^{\frac{1}{\theta}} \int_n^1 c_{f,t}(i,j)^{\frac{\theta-1}{\theta}} di\right]^{\frac{\theta}{\theta-1}}$$
$$c_{h,t}^*(j) = \left[\left(\frac{1}{n}\right)^{\frac{1}{\theta}} \int_0^n c_{h,t}^*(i,j)^{\frac{\theta-1}{\theta}} di\right]^{\frac{\theta}{\theta-1}}, \; c_{f,t}^*(j) = \left[\left(\frac{1}{1-n}\right)^{\frac{1}{\theta}} \int_n^1 c_{f,t}^*(i,j)^{\frac{\theta-1}{\theta}} di\right]^{\frac{\theta}{\theta-1}}$$

where $c_{h,t}(j)$ ($c_{f,t}(j)$, respectively) is the consumption of final goods produced at home (in the rest of the world) by the representative consumer j, and $\theta > 1$ is the elasticity of substitution across domestic varieties of final goods.

[2]We measure the nominal exchange rate as the price of foreign currency in terms of the domestic currency.

Accordingly, optimal demands of domestic varieties can be expressed as

$$c_{h,t}(i,j)=\frac{\varphi}{n}\left[\frac{p_t(i)}{p_t}\right]^{-\theta}\left[\frac{p_t}{p_{c,t}}\right]^{-\mu}c_t\left(j\right),\ c^*_{h,t}(i,j)=\frac{\varphi^*}{n}\left[\frac{p_t(i)}{p_t}\right]^{-\theta}\left[\frac{p_t}{\varepsilon_t p^*_{c,t}}\right]^{-\mu}c^*_t\left(j\right)$$

In the baseline model, we assume that capital and public spending aggregates are composed of domestic varieties only

$$k_t\left(j\right)=\left[\left(\frac{1}{n}\right)^{\frac{1}{\theta}}\int_0^n k_t(i,j)^{\frac{\theta-1}{\theta}}di\right]^{\frac{\theta}{\theta-1}},\ \text{and}\ g_t=\left[\left(\frac{1}{n}\right)^{\frac{1}{\theta}}\int_0^n g_t(i)^{\frac{\theta-1}{\theta}}di\right]^{\frac{\theta}{\theta-1}}$$

where

$$g_{t+1}=\left(1-\rho_g\right)g+\rho_g g_t+\xi_{g,t+1}$$

with $\xi_{g,t+1}$ being an *i.i.d.* innovation.

Finally, let us define terms–of–trade as

$$s_t=\frac{\varepsilon_t p^*_t}{p_t} \tag{6}$$

This definition is meant to be consistent with the definition of the real exchange rate. Given this notational convention, an increase of s_t denotes a deterioration of terms–of–trade, implying an increase of the competitiveness of domestic goods.

2.2 Risk–sharing

Under the assumption of complete international markets of state–contingent assets, a relation similar to Equation (4) holds in the rest of the world

$$\beta\left(\frac{u_{c^*,t+1}}{u_{c^*,t}}\right)\left(\frac{p^*_{c,t}}{p^*_{c,t+1}}\right)\left(\frac{\varepsilon_t}{\varepsilon_{t+1}}\right)=q_{t,t+1}$$

which, combined with Equation (4) gives the following risk–sharing condition

$$\frac{u^*_{c,t}}{u_{c,t}}=\epsilon\frac{\varepsilon_t p^*_{c,t}}{p_{c,t}} \tag{7}$$

Equation (7) indicates that relative marginal utilities are related to the real exchange rate up to a constant ϵ that depends on initial conditions on relative net foreign asset position. Assuming symmetric initial conditions simply amounts to set $\epsilon=1$, which is consistent with the symmetric steady state around which we study the dynamic properties of the model.

2.3 Firms

Firms produce varieties $y_t(i)$ using domestic labor, $l_t(i)$, and physical capital, $k_{t-1}(i)$, according to the following production function:

$$y_t(i) = a_t k_{t-1}(i)^{\phi} l_t(i)^{1-\phi}$$

The total factor productivity, a_t, evolves according to:

$$a_{t+1} = (1-\rho_a)\,a + \rho_a a_t + \xi_{a,t+1}$$

where $\xi_{a,t+1}$ is an *i.i.d.* innovation. Input prices are related by the following efficiency condition:

$$\phi w_t l_t(i) = (1-\phi)\,z_t k_{t-1}(i)$$

with the nominal marginal cost being

$$mc_t(i) = mc_t = \frac{z_t^{\phi} w_t^{1-\phi}}{\phi^{\phi}(1-\phi)^{1-\phi}} a_t^{-1}$$

which is the same for all firms.

We assume that production prices are governed by Calvo (1983) pricing contracts. Only a fraction $1-\eta$ of randomly selected domestic firms is allowed to set new prices each period. The corresponding optimal price set by a firm allowed to reset is:

$$\overline{p}_t(i) = \frac{\theta}{\theta-1}\frac{\sum\limits_{v=0}^{\infty}(\eta\beta)^v E_t\{\lambda_{t+v}y_{t+v}(i)\,mc_{t+v}\}}{\sum\limits_{v=0}^{\infty}(\eta\beta)^v E_t\{\lambda_{t+v}y_{t+v}(i)\}}$$

where $y_t(i)$ is the aggregate demand addressed to firm i:

$$y_t(i) = c_{h,t}(i) + c^*_{h,t}(i) + k_t(i) - (1-\delta)\,k_{t-1}(i) + g_t(i)$$

In this expression, $c_{h,t}(i) = \int_0^n c_{h,t}(i,j)\,dj$, $c^*_{h,t}(i) = \int_n^1 c^*_{h,t}(i,j)\,dj$, and $k_t(i) = \int_0^n k_t(i,j)\,dj$. In addition, $\frac{\theta}{\theta-1}$ is the steady state mark–up signalling the distortion caused by monopolistic competition in final goods markets.

Aggregating among firms and assuming that Calvo producers set the same price when free to reset, the aggregate production price index is

$$p_t = \left[(1-\eta)\,\overline{p}_t(i)^{\ 1-\theta} + \eta p_{t-1}^{1-\theta}\right]^{\frac{1}{1-\theta}}$$

Production price inflation thus evolves according to[3]

$$\eta\pi_t^{\theta-1} + (1-\eta)\left(\frac{\theta}{\theta-1}\frac{v_{1,t}}{v_{2,t}}\right)^{1-\theta} = 1$$

where

$$v_{1,t} - \eta\beta E_t\left\{v_{1,t+1}\pi_{t+1}^{\theta}\left(\frac{s_t}{s_{t+1}}\right)^{\alpha}\right\} = u_{c,t}y_t\frac{mc_t}{p_t}$$

and

$$v_{2,t} - \eta\beta E_t\left\{v_{2,t+1}\pi_{t+1}^{\theta-1}\left(\frac{s_t}{s_{t+1}}\right)^{\alpha}\right\} = u_{c,t}y_t$$

Finally, the dispersion of production prices, $\Upsilon_t = \int_0^1 \left[\frac{p_t(i)}{p_t}\right]^{-\theta} di \geq 1$, is given by

$$\Upsilon_t = \eta\Upsilon_{t-1}\pi_t^{\theta} + (1-\eta)\left(\frac{\theta}{\theta-1}\frac{v_{1,t}}{v_{2,t}}\right)^{-\theta}$$

2.4 Government

The government has access to distorting taxes on inputs (labor and capital) to finance the exogenous stream of public spending. We only allow for balanced–budget policies, therefore the budget constraint of authorities in the domestic economy is

$$\tau_{\ell,t}\frac{w_t}{p_t}\ell_t + \tau_{k,t}\left(\frac{z_t}{p_t} - \delta\right)k_{t-1} = g_t$$

where $\ell_t = \int_0^n \ell_t(j)\,dj$, and $k_{t-1} = \int_0^n k_{t-1}(j)\,dj$.

2.5 Equilibrium

We focus on the case of a small open economy, i.e. $n \to 0$, in deriving the optimal monetary and fiscal policy. One implication of this assumption is that the aggregate consumer price index can be expressed as:

$$p_{c,t}^* = p_t^*, \quad \text{and} \quad p_{c,t} = \left[(1-\alpha)(p_t)^{1-\mu} + \alpha\left(\varepsilon_t p_t^*\right)^{1-\mu}\right]^{\frac{1}{1-\mu}} \tag{8}$$

[3]See appendix A for a detailed derivation of these conditions.

Let us define aggregate output as $y_t = \left[\left(\frac{1}{n}\right)^{\frac{1}{\theta}} \int_0^n y_t(i)^{\frac{\theta-1}{\theta}} di\right]^{\frac{\theta}{\theta-1}}$. Then, using the definition of the consumer price index (8) and the terms–of–trade (6), the final goods market clears according to

$$y_t = (1-\alpha)\left((1-\alpha) + \alpha s_t^{1-\mu}\right)^{\frac{\mu}{1-\mu}} c_t + \alpha s_t^{\mu} c_t^* + k_t - (1-\delta) k_{t-1} + g_t$$

where c_t^* is assumed to be an exogenous process:

$$c_{t+1}^* = (1-\rho_{c^*}) c^* + \rho_{c^*} c_t^* + \xi_{c^*,t+1}$$

Using the risk–sharing condition

$$\frac{u_{c,t}^*}{u_{c,t}} = \left(\alpha + (1-\alpha) s_t^{\mu-1}\right)^{\frac{1}{\mu-1}}$$

the market clearing condition may be expressed as

$$y_t = g(c_t, s_t) + k_t - (1-\delta) k_{t-1} + g_t$$

Finally, the labor market clearing condition is

$$l_t = \int_0^n l_t(i)di = \int_0^n \ell_t(j)dj = \ell_t$$

and the aggregate production is given by

$$y_t = \Upsilon_t^{-1} a_t k_{t-1}^{\phi} \ell_t^{1-\phi} = y(a_t, \Upsilon_t, k_{t-1}, \ell_t)$$

Let $\{\mathcal{R}_t\}_{t=0}^{\infty} = \{r_t, \tau_{n,t}, \tau_{k,t}\}_{t=0}^{\infty}$ be a sequence of monetary and fiscal policies.

Let $\{\mathcal{S}_t\}_{t=0}^{\infty} = \{a_t, g_t, c_t^*\}_{t=0}^{\infty}$ be a sequence of exogenous shocks.

Let $\{\mathcal{Q}_t\}_{t=0}^{\infty} = \{y_t, n_t, c_t, k_t\}_{t=0}^{\infty}$ be a sequence of quantities.

Let $\{\mathcal{P}_t\}_{t=0}^{\infty} = \{\pi_t, p_t, w_t, z_t, mc_t, s_t, v_{1,t}, v_{2,t}, \Upsilon_t\}_{t=0}^{\infty}$ be a sequence of prices.

Definition *A competitive equilibrium is defined as an allocation $\{\mathcal{Q}_t\}_{t=0}^{\infty}$ and a sequence of prices $\{\mathcal{P}_t\}_{t=0}^{\infty}$ such that, (i) for a given sequence of prices $\{\mathcal{P}_t\}_{t=0}^{\infty}$, shocks $\{\mathcal{S}_t\}_{t=0}^{\infty}$ and a given policy $\{\mathcal{R}_t\}_{t=0}^{\infty}$, the sequence $\{\mathcal{Q}_t\}_{t=0}^{\infty}$ satisfies households first order conditions (including the transversality conditions) and maximizes firms profits; (ii) for a given sequence of quantities $\{\mathcal{P}_t\}_{t=0}^{\infty}$, shocks $\{\mathcal{S}_t\}_{t=0}^{\infty}$ and a given policy $\{\mathcal{R}_t\}_{t=0}^{\infty}$, the sequence $\{\mathcal{P}_t\}_{t=0}^{\infty}$ clears goods markets, financial markets and factors markets.*

3 Parametrization

Before turning to the analysis of the optimal monetary and fiscal policies, we assign numerical values to our deep parameters, in order to be able to proceed to numerical simulations.

Utility function. We adopt the following functional form of the utility function:

$$u\left(c_t\left(j\right), n_t\left(j\right)\right) = \frac{c_t\left(j\right)^{1-\sigma}}{1-\sigma} - \frac{\ell_t\left(j\right)^{1+\psi}}{1+\psi}$$

Preferences. The subjective discount factor, β, is equal to 0.988, consistent with an average annual real interest rate of 5% in the steady state. The inverse of the intertemporal elasticity of consumption is $\sigma = 2$ as in Corsetti et al. (2008). We set the inverse of the elasticity of labor supply with respect to wages, ψ, equal to 1. This value fits the range proposed by Canzoneri et al. (2007). The elasticity of substitution between domestic and foreign goods is $\mu = 1.5$ as in Backus and Kehoe (1994). The elasticity of substitution across domestic varieties of final goods determines the average mark–up and is set to $\theta = 7$, in accordance with Rotemberg and Woodford (1997).

Technology. The share of capital income in the GDP is $\phi = 0.36$. The depreciation rate, δ, is assumed to be 10% annually. The degree of price stickiness, i.e. the Calvo parameter, is $\eta = 0.75$, implying an average duration of 4 quarters for prices.

Shocks. The characteristics of productivity, public expenditure and world demand innovations are $\sigma\left(\xi_{a,t+1}\right) = \sigma\left(\xi_{c^*,t+1}\right) = 0.008$ and $\sigma\left(\xi_{g,t+1}\right) = 0.01$. The persistence of shocks is set to $\rho_a = \rho_g = \rho_{c^*} = 0.9$.

Openness. Finally, the degree of trade openness is $\alpha = 0$ in the case of a closed economy (our benchmark), and $\alpha = 0.3$ in the case of an open economy. This value is in the interval of parameters suggested by Pappa and Vassilatos (2007) and Galí and Monacelli (2005).

4 Optimal monetary and fiscal policy in the steady state

In this section, we study the optimal tax schedule in the steady state. Notice that monetary policy is not taken into consideration since the nominal interest rate is constant in the steady state, i.e. $r = \frac{1}{\beta}$. We focus our attention on the symmetric steady state where $u_c = u_c^*$, which implies $s = 1$. Furthermore, we assume that the technology shock, a, is normalized to get $\frac{mc}{p} = 1$ and that $\frac{g}{y} = \kappa = 0.25$.

Defining $\omega = \frac{w}{p}$, and $\zeta = \frac{z}{p}$, associated steady state conditions are

$$\ell^{\psi} c^{\sigma} = (1 - \tau_{\ell})\,\omega \tag{9}$$

$$\beta^{-1} = (1 + (1 - \tau_k)(\zeta - \delta)) \tag{10}$$

$$\frac{mc}{p} = \frac{\zeta^{\phi} \omega^{1-\phi}}{\phi^{\phi} (1 - \phi)^{1-\phi}} \tag{11}$$

$$\phi \omega \ell = (1 - \phi)\,\zeta k \tag{12}$$

$$\eta \pi^{\theta - 1} + (1 - \eta) \left(\frac{\theta}{\theta - 1} \frac{mc}{p} \frac{1 - \eta \beta \pi^{\theta - 1}}{1 - \eta \beta \pi^{\theta}} \right)^{1-\theta} = 1 \tag{13}$$

$$\Upsilon = \frac{(1 - \eta)}{(1 - \eta \pi^{\theta})} \left(\frac{\theta}{\theta - 1} \frac{mc}{p} \frac{1 - \eta \beta \pi^{\theta - 1}}{1 - \eta \beta \pi^{\theta}} \right)^{-\theta} \tag{14}$$

$$y (1 - \kappa) = c + \delta k \tag{15}$$

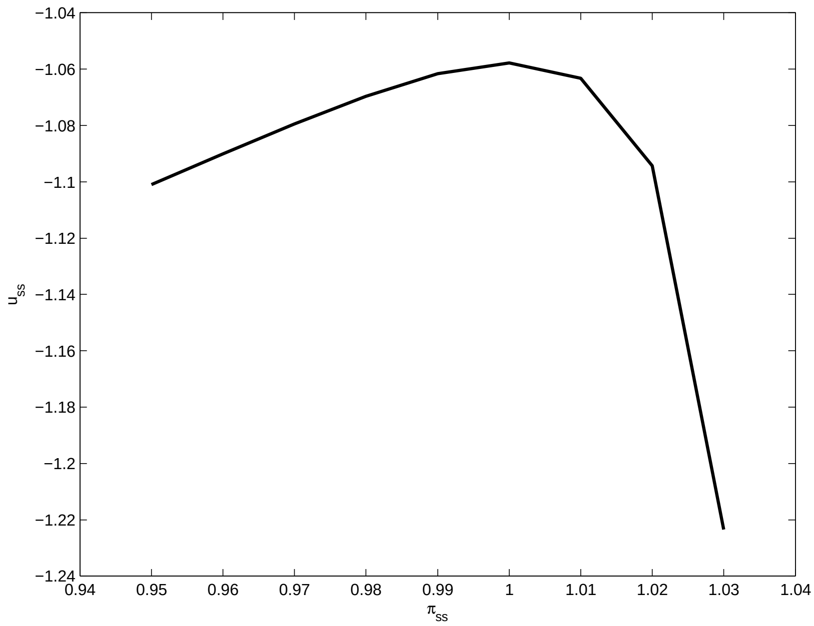
$$y \Upsilon = k^{\phi} \ell^{1-\phi} \tag{16}$$

$$\tau_{\ell} \omega \ell + \tau_k (\zeta - \delta)\, k = \kappa y \tag{17}$$

The Ramsey (optimal) steady state is the combination of steady state taxes (the inflation tax, the capital income tax and the labor income tax) that maximizes the steady state welfare.

The first result is that the Ramsey steady state implies a zero inflation tax and a zero tax rate on capital income. To show that, we first solve numerically the steady state for varying steady state levels of the inflation rate while letting tax rates being determined optimally. As depicted in Figure 1, the optimal level of steady state inflation appears to be $\pi = 1$.

Figure 1: Steady state level of utility for varying steady state levels of the inflation rate.



Then, fixing the steady state value of the inflation rate to unity, we solve numerically the steady state for varying levels of the capital income tax rate. Figure 2 plots the corresponding steady state level of utility and shows that the optimal steady state level of capital income tax rate is zero.

Our results thus nest the traditional results reported by Chamley (1986) and Judd (1985) for what concerns the optimal capital income tax rate in the steady state, and the result reported by Benigno and Woodford (2005) with respect to the optimal inflation tax. Using the set of Eqs. (9)–(17) and assuming $(\pi, \tau_k) = (1, 0)$, we get

$$\ell = \left[\left((1-\kappa)\, \varphi^{\phi} - \delta\varphi \right)^{-\sigma} \left(\omega - \kappa\varphi^{\phi} \right) \right]^{\frac{1}{\psi+\sigma}}$$

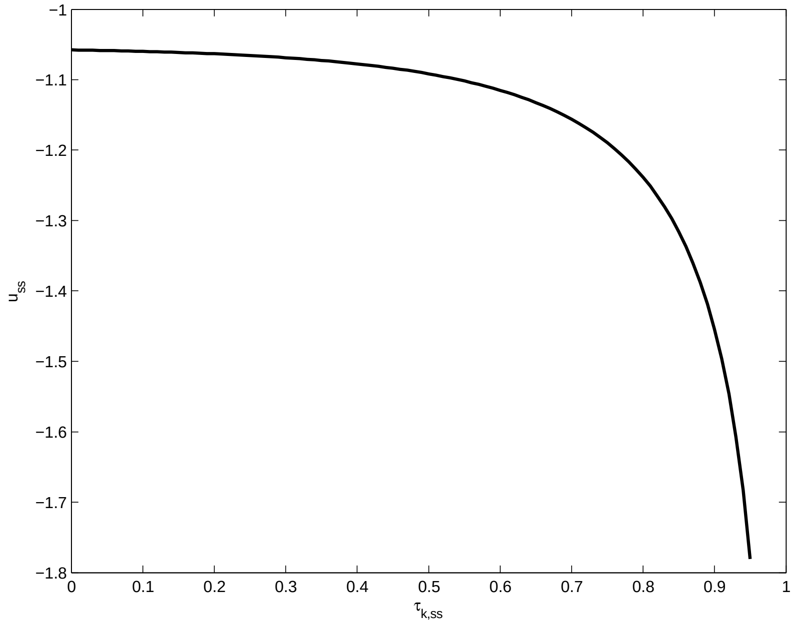
$$c = \left[\left((1-\kappa)\, \varphi^{\phi} - \delta\varphi \right)^{\psi} \left(\omega - \kappa\varphi^{\phi} \right) \right]^{\frac{1}{\psi+\sigma}}$$

$$k = \varphi\ell, \qquad y = \varphi^{\phi}\ell, \qquad \tau_{\ell} = \frac{\kappa\theta}{(\theta-1)(1-\phi)}$$

where $\zeta = \beta^{-1} - 1 + \delta$, $\omega = \left(\frac{\theta-1}{\theta} \phi^{\phi} (1-\phi)^{1-\phi} \zeta^{-\phi} \right)^{\frac{1}{1-\phi}}$, and $\varphi = \frac{\phi\omega}{(1-\phi)\zeta}$.

Interestingly, notice that the steady state optimal level of labor income tax does not depend on the intertemporal elasticity of substitution of consumption (σ) or labor (ψ). It

Figure 2: Steady state level of utility for varying steady state levels of the capital income tax rate.



depends positively on the steady state level of (exogenous) government expenditure (κ), negatively on the labor share (ϕ) (the higher the labor share, the higher the corresponding tax base, the lower the optimal tax rate on labor income) and negatively on the degree of monopolistic competition (θ) since

$$\frac{\partial \tau_n}{\partial \theta} = -\frac{\kappa}{(\theta - 1)^2 (1 - \phi)} < 0$$

The intuition is the following. In this steady state, the inflation rate is zero, implying that the distortions associated to monopolistic competition in the model affect the steady state through the level of the real marginal cost, impacting only on the real wage given that the real capital rental rate is unaffected since not taxed. A low level of competition generates important distortions by reducing the steady state value of the real wage inducing a higher tax rate on labor income to provision the exogenously given level of public spending in GDP, κ. A high level of competition, on the other hand, increases the real wage in the equilibrium and lowers the steady state labor supply and/or increases the steady state level of consumption.

5 Optimal monetary and fiscal policy in the non–steady state

We now turn to the characterization of the Ramsey policy in the non–steady state. We are more particularly interested in determining the impact of nominal rigidities and trade openness on standard results regarding optimal monetary and fiscal policy.

5.1 The dynamic Ramsey problem

The solution of the Ramsey problem is a sequence

$$\{\Xi_t\}_{t=0}^{\infty} = \left\{\tau_{\ell,t}, \tau_{k,t}, r_t, c_t, \ell_t, k_{t-1}, \omega_t, \zeta_{t+1}, \tfrac{mc_t}{p_t}, \pi_t, \upsilon_{1,t}, \upsilon_{2,t}, \Upsilon_t, s_t\right\}_{t=0}^{\infty},$$

that solves

$$\underset{\Xi_t}{Max} E_0 \sum_{t=0}^{\infty} \beta^t \int_0^n u\left(c_t\left(j\right), \ell_t\left(j\right)\right) dj,$$

subject to Eqs. (18)–(29) (detailed in Appendix B), where $\omega_t = \frac{w_t}{p_t}$, and $\zeta_t = \frac{z_t}{p_t}$.

The optimal fiscal and monetary policies are given by combinations of $\{\tau_{\ell,t}, \tau_{k,t}, r_t\}$ that maximize individuals' welfare. This problem is typically known to result in time–variant (time inconsistent) policies, as authorities may choose their policy instruments after agents have formed their expectations about forward variables, such as the inflation rate, and therefore may take advantage of this situation in period zero. As a consequence, from period one onwards, authorities have an incentive to change their optimal policy since agents now take into account prior commitments when forming their expectations. Therefore, to avoid such inconsistency issues, we adopt the timeless perspective (see Woodford (2000)) and assume that the optimization problem is constrained by some former prior commitment, that is consistent with the optimal commitment chosen for period one onwards. We thus solve our dynamic Ramsey program and analyze its dynamic properties when the economy is closed or open respectively.[4]

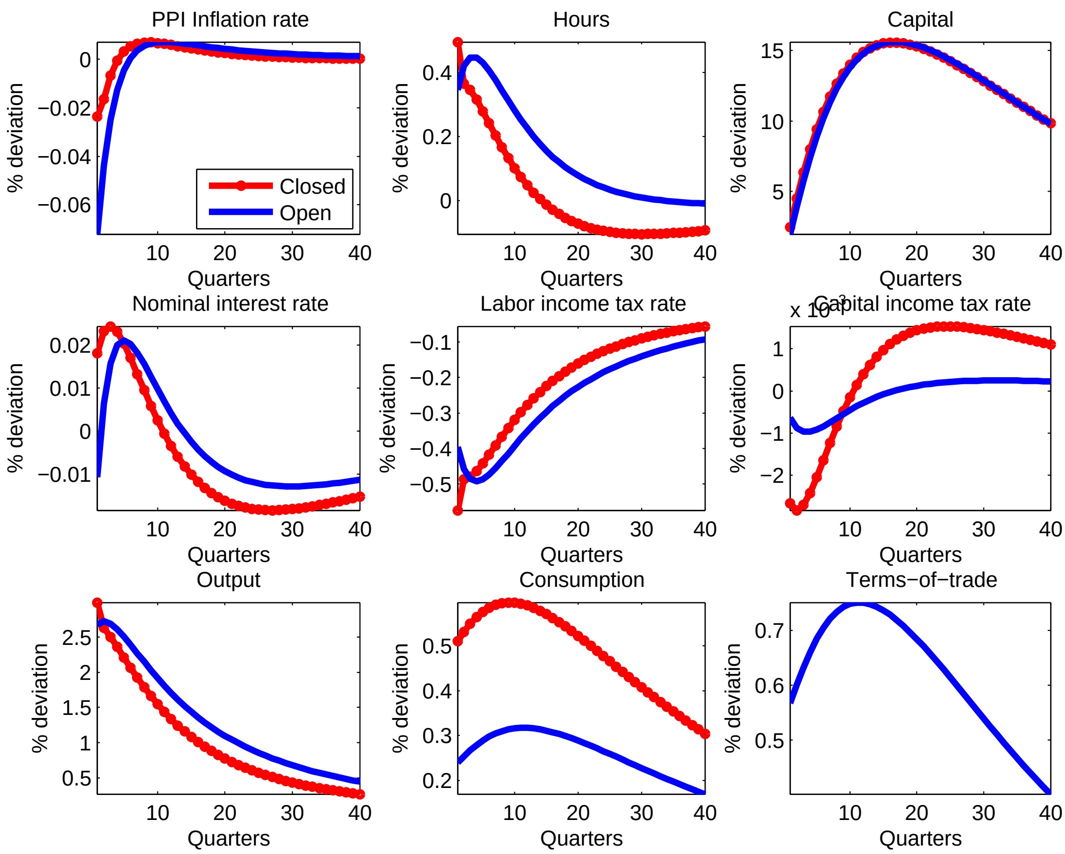
[4] To do this, we solve the Ramsey problem analytically (in level) using Matlab's symbolic toolbox and use Dynare's second order approximation algorithm (based on the method developed by Schmitt-Grohe and Uribe (2004b)) to simulate the model numerically.

5.2 Impulse response functions

In this section, we comment the optimal response of policy instruments, as well as other key macroeconomic variables, under the Ramsey policy after each type of shock.

Productivity shocks. Figure 3 plots the impulse response functions (henceforth IRFs) of the variables of interest after a unit productivity innovation.

Figure 3: IRFs after a unit productivity shock under Ramsey policies.



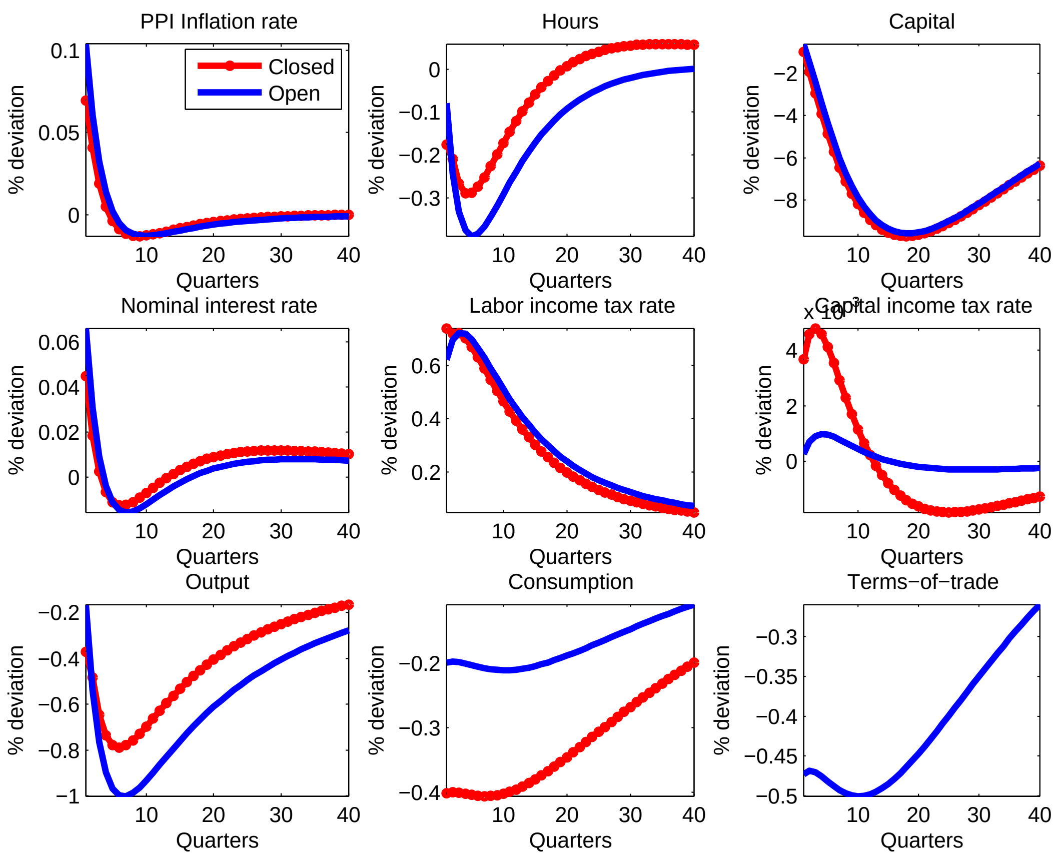
An increase in productivity produces the standard effects on most macroeconomic variables. The marginal cost of production drops, driving firms to increase their inputs demands and inflation to fall. Both output and consumption increase as well as hours and the stock of capital, just as in the standard RBC set–up, driven by the increase of real wages and real capital revenues (not shown in the figure).

In the event of an increase in productivity, the Ramsey planner faces a tension between smoothing taxes to reduce the volatility of labor and reducing them because individuals may want to work more due to higher productivity. Which effect is stronger will depend on preferences, mainly on the elasticity of labor supply. Given the parameterization employed in this model, the second effect dominates, and labor income taxes fall considerably for more than 20 quarters. The tax rate on capital remains nearly constant, as it reaches a minimum of 0.0025% for a 1% productivity shock. Since there is no public debt in the model, taxes on labor income serve as a mechanism to balance the budget each period. Notice that the optimal monetary policy, reflected by movements in the nominal interest rate, is contractionary during the first 10 quarters and exhibits a moderate expansionary stance for the next 30 periods. This increase of the nominal interest rate shows the non–optimality of the Friedman rule in this case, also depicted in recent literature (see e.g. Faia (2008)), and is intended to moderate the increase of hours worked, that affect negatively households welfare.

When we consider the open economy, the increase in domestic productivity drops the terms–of–trade (i.e. there is a real depreciation) and domestic goods become more competitive. The deterioration in the terms–of–trade for domestic consumers reduces the response of consumption now that foreign goods become relatively more expensive. Home prices must fall by more in the open economy case to clear the market. The nominal interest rate falls on impact to increase afterwards. This affects labor supply which rises as in the closed economy case, but becomes more persistent, and translates into a more persistent labor income tax rate too. Interestingly, trade openness seems to exert a moderating effect on the volatility of the tax rate on capital income. Notice that the open dimension of the economy does not significantly alter the dynamics of the optimal policy, except for the reaction of the nominal interest rate which becomes more persistent and volatile. In contrast to the closed economy case, inflation becomes more volatile under the optimal policy when the economy is open to smooth the path for consumption, despite the cost of adjusting prices.

Public spending shocks. Figure 4 plots the same IRFs after a unit public spending innovation.

Figure 4: IRFs after a unit public spending shock under Ramsey policies.



The shock produces an increase of public demand for final goods, however not met by an increase in output. With lump–sum taxes, private consumption would fall but hours would increase, which would make it possible for firms to increase their production and would lead to a smoother reaction of inflation. Since lump–sum taxes have been ruled out and since the Chamley (1986) result on capital income taxation applies, i.e. taxes on capital income remain flat over the cycle, the government finances the increase of spending by substantially increasing the tax rate on labor income. As a consequence, hours drop, as well as output, which increases the response of the inflation rate. Note that as shown in Schmitt-Grohe and Uribe (2004a), in the presence of price stickiness the Ramsey planner will resort to the use of taxes on labor income spread out over 40 quarters to minimize its negative effects. This is also shown in the second–order moments below, which report high persistence of policy instruments. The optimal monetary policy consists of rising the nominal interest rate but moderately to limit the depressing effect of the public spending shock on output.

The increase in domestic demand drives inflation up, which in the open economy translates into an improvement in the terms–of–trade, which damages the competitiveness of domestic firms. As a consequence, the traditional "real exchange rate crowding out" effect after a public spending shock applies in this model, since output drops more sharply when the economy is open than in the case of a closed economy.

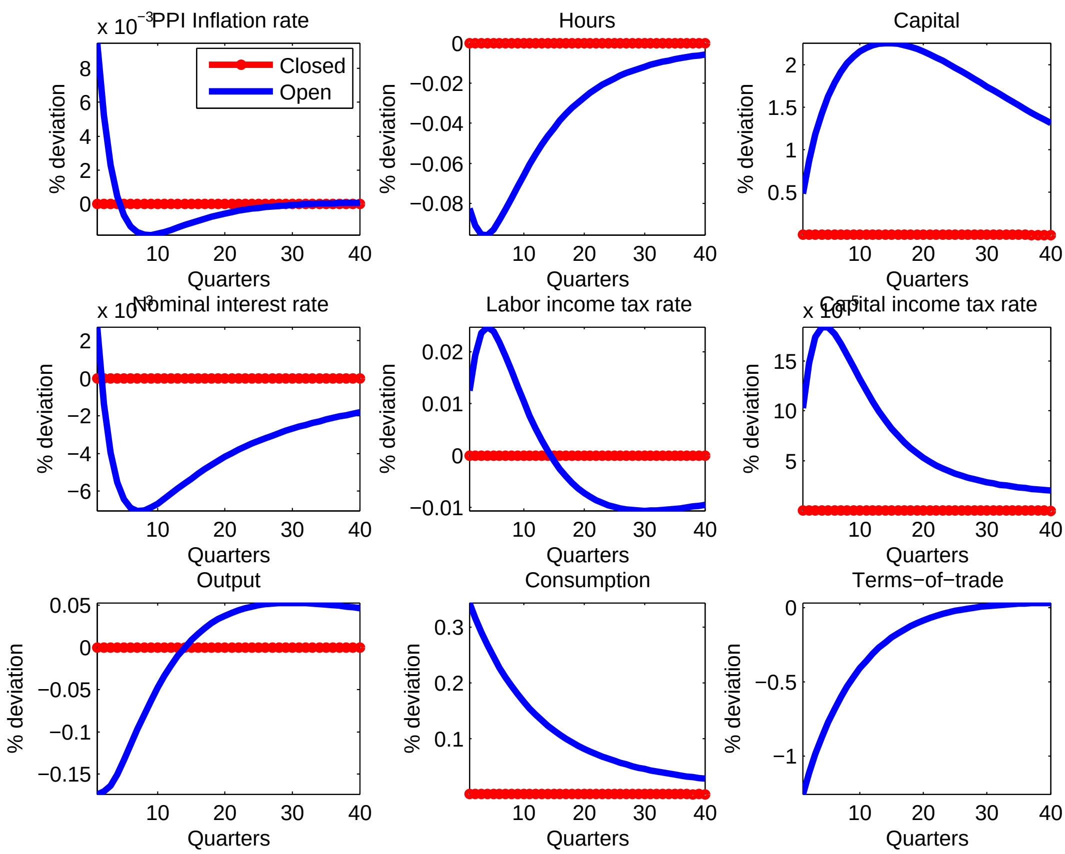
World demand shocks. Finally, Figure 5 plots the IRFs after a unit world demand innovation.

Within our framework, the macroeconomic effects of a world demand shock are the same as a terms–of–trade shock. As a consequence, when the economy is closed, all macroeconomic variables remain flat.

In an open economy, the risk–sharing condition leads the terms–of–trade to rise (the real exchange rate to fall), triggering an expenditure switching effect and boosting the level of domestic private consumption. Notice however that the increase of private consumption occurs through the increase of imported final goods, at the expense of domestically produced goods, implying a depressing effect on domestic global demand.

The shock also modifies households' labor supply decisions, both through the channel of private consumption and through what Benigno and de Paoli (2009) call the terms–of–trade spillover. On the one hand, the real depreciation makes its possible to buy less expensive goods abroad, with a negative impact on labor supply. On the other hand, the real depreciation undermines the domestic purchasing power of households, pushing them to increase their labor supply. In our setting, the first effect dominates and hours drop, driving the real wage up, as well as the marginal production cost of firms. A moderate inflationary stance thus arises and the optimal response of the nominal interest

Figure 5: IRFs after a unit world demand shock under Ramsey policies.



rate consists in a restrictive policy during 5 to 7 quarters, followed by a long lasting expansionary policy over the next 25–30 quarters. Here again, it is noticeable that the path of the monetary policy instrument is very smooth. The combined effect of the world demand shock on the labor income is positive for 20 quarters (the increase of wages is dominated by the drop of hours), and then negative for the next 20 quarters (the increase of real wages more than compensates the drop of hours). Since in this case, the optimal tax rate on capital income is almost zero all the time, the tax rate on labor income adjusts to balance government's budget. Consequently the tax rate on labor income increases during 20 quarters and then falls under its steady state level for another 20 quarters.

5.3 Second–order moments

In this section, we report second–order moments generated by the model under the Ramsey policy for the benchmark parametrization and compare the results to those in the literature on optimal monetary and fiscal policy. Table 1 reports the means, standard deviations, autocorrelations and cross–correlations of key macroeconomic variables and policy instruments both in the closed and the open economy set–up.

As observed in the dynamic analysis above, the zero capital tax result is preserved which is consistent with the findings in Chamley (1986) and Chari and Kehoe (1999), among others. Notice that this is the case in both the closed and open economy case, as well as the fact that optimal inflation tax is zero, as in Benigno and Woodford (2005). All policy instruments are very smooth, they show high autocorrelation and little volatility. As for optimal monetary policy, the nominal interest rate is acyclical but shows a positive correlation with inflation which suggests that inflation could be a nice monetary policy target (as in all studies studying the optimal monetary policy alone when prices are sticky). Finally, notice that the open dimension does not affect optimal policies, but tends to smooth the paths of consumption and capital while increasing the volatility of inflation and the tax on labor income. This is due to the fact that allowing for terms–of–trade variations which affect individuals' welfare gives way to a more active optimal policy, mainly instrumented in labor and inflation taxes (Corsetti and Pesenti (2001);

Benigno and de Paoli (2009)).

Table 1: Second order moments – closed vs open economy set–up.

	Mean	*Std. dev.*	*Auto.corr*	$Corr(x,y)$	$Corr(x,\pi)$	$Corr(x,s)$
			Closed economy ($\alpha = 0$)			
y	2.223	0.0719	0.9405	–	0.0444	–
n	0.676	0.0123	0.9279	0.5350	-0.1571	–
k	18.46	0.9129	0.9985	0.7105	0.2193	–
r	1.012	0.0013	0.8979	-0.0357	0.2634	–
τ_n	0.456	0.0263	0.9431	-0.7878	0.1387	–
τ_k	0.000	0.0002	0.9584	-0.1452	0.3763	–
π	1.000	0.0010	0.6219	0.0444	–	–
c	1.206	0.0350	0.9864	0.8542	0.0818	–
s	–	–	–	–	–	–
			Open economy ($\alpha = 0.3$)			
y	2.223	0.0875	0.9675	–	-0.0792	0.8283
n	0.676	0.0161	0.9726	0.8441	-0.1351	0.6832
k	18.46	0.9075	0.9985	0.7915	0.1628	0.8384
r	1.012	0.0012	0.7665	0.0070	0.5049	-0.2187
τ_n	0.456	0.0284	0.9678	-0.8320	0.2154	-0.7654
τ_k	0.000	0.0000	0.9716	-0.4147	0.2995	-0.1611
π	1.000	0.0015	0.5998	-0.0792	–	-0.0263
c	1.206	0.0201	0.9805	0.8325	-0.0107	0.6924
s	1.000	0.0493	0.9688	0.8283	-0.0263	–

Notice however that a higher volatility of policy instrument is achieved jointly with a higher persistence: as movements of policy instrument tend to create additional distortions in the economy, the central planner needs to make policy changes smoother over the cycle.

5.4 Sensitivity analysis

In this section, we proceed to a sensitivity analysis by varying key parameters.

High trade openness. For instance, Table 2 displays the mean, standard deviation and autocorrelation of the same key variables in the case of high levels of trade openness, typically $\alpha \geq 0.5$, implying an average 100% degree of trade openness at least. As in the previous subsection, the volatility of consumption and capital is dramatically reduced with trade openness while the volatility of hours, inflation, the nominal interest rate and the labor income tax rate increases.

Table 2: Second order moments – high trade openness

	Mean	*Std. dev.*	*Auto.corr*	*Mean*	*Std. dev.*	*Auto.corr*
	$\alpha = 0.5$			$\alpha = 0.6$		
y	2.223	0.0900	0.9771	2.223	0.0878	0.9627
n	0.676	0.0177	0.9674	0.676	0.0188	0.8408
k	18.46	0.8849	0.9985	18.46	0.8427	0.9985
r	1.012	0.0013	0.6720	1.012	0.0015	0.6740
τ_n	0.456	0.0284	0.9765	0.456	0.0281	0.9572
τ_k	0.000	0.0000	0.8055	0.000	0.0000	0.2296
π	1.000	0.0020	0.5980	1.000	0.0025	0.6090
c	1.206	0.0156	0.9580	1.206	0.0143	0.9405
s	1.000	0.0439	0.9663	1.000	0.0414	0.9631

High risk–aversion. Similarly, Table 3 reports the results for higher risk–aversion ($\sigma = 6$). As in the previous case, hours, inflation and labor income taxes are more volatile in the open–economy set–up. In contrast with the previous case however, consumption and capital are more volatile, mostly because domestic consumption is now more deeply affected by real exchange rate fluctuations caused by terms–of–trade shocks through the risk–sharing condition.

Table 3: Second order moments – high risk–aversion ($\sigma = 6$)

	Mean	*Std. dev.*	*Auto.corr*	*Mean*	*Std. dev.*	*Auto.corr*
	$\alpha = 0$			$\alpha = 0.3$		
y	1.998	0.0497	0.9177	1.998	0.0714	0.9659
n	0.607	0.0133	0.9693	0.607	0.0159	0.9514
k	16.59	0.8514	0.9985	16.59	0.8786	0.9985
r	1.012	0.0016	0.9472	1.012	0.0017	0.8216
τ_n	0.456	0.0257	0.9293	0.456	0.0284	0.9703
τ_k	0.000	0.0001	0.9782	0.000	0.0000	0.9388
π	1.000	0.0008	0.6304	1.000	0.0018	0.5917
c	0.8571	0.0036	0.6937	0.8571	0.0080	0.6178
s	–	–	–	1.000	0.0843	0.9175

Varying the elasticity of labor supply. Our last sensitivity analysis investigates the impact of different labor supply elasticities for a given level of trade openness ($\alpha = 0.3$). In particular, this elasticity is a key parameter in determining the size of the terms–of–trade spillover (Benigno and de Paoli (2009)), as well as the volatility of hours, and thereby the volatility of the labor income tax rate. Table 4 therefore reports our results for values of ψ ranging from 0.5 to 5.

Table 4 shows that the elasticity of labor supply plays a key role in the overall macroeconomic volatility. The most striking feature of these results is the positive relation between volatility and persistence under Ramsey plans: as the elasticity of labor supply increases (ψ goes down), macroeconomic volatility increases alongside with persistence.

Table 4: Second order moments – varying the elasticity of labor supply

	Mean	*Std. dev.*	*Auto.corr*	*Mean*	*Std. dev.*	*Auto.corr*
		$\psi = 0.5$			$\psi = 0.7$	
y	2.056	0.1017	0.9765	2.128	0.0948	0.9726
n	0.625	0.0228	0.9808	0.647	0.0196	0.9775
k	17.07	1.0526	0.9987	17.68	0.9820	0.9986
r	1.012	0.0021	0.7522	1.012	0.0016	0.7425
τ_n	0.456	0.0328	0.9773	0.456	0.0308	0.9731
τ_k	0.000	0.0001	0.9793	0.000	0.0000	0.9761
π	1.000	0.0026	0.7070	1.000	0.0020	0.6592
c	1.115	0.0228	0.9832	1.154	0.0215	0.9819
s	1.000	0.0610	0.9734	1.000	0.0553	0.9713
		$\psi = 3$			$\psi = 5$	
y	2.601	0.0737	0.9526	2.783	0.0716	0.9487
n	0.790	0.0074	0.9477	0.845	0.0049	0.9312
k	21.60	0.7576	0.9984	23.13	0.7298	0.9984
r	1.012	0.0007	0.9252	1.012	0.0007	0.9407
τ_n	0.456	0.0220	0.9493	0.456	0.0202	0.9406
τ_k	0.000	0.0000	0.9460	0.000	0.0000	0.9308
π	1.000	0.0005	0.4020	1.000	0.0003	0.3285
c	1.410	0.0175	0.9772	1.509	0.0170	0.9765
s	1.000	0.0361	0.9626	1.000	0.0326	0.9612

6 An extension with tradable capital goods

In this section, we modify the model to take trade in capital goods into account. This extension is proposed to check if modifying some key international linkages, for instance opening the capital goods markets, may alter our results.

6.1 Set–up

Physical capital accumulated by households is now a bundle of domestic and foreign capital goods. The bundle has a symmetric structure with respect to consumption goods

$$k_t(j) = \left[\chi^{\frac{1}{\mu}} k_{h,t}(j)^{\frac{\mu-1}{\mu}} + (1-\chi)^{\frac{1}{\mu}} k_{f,t}(j)^{\frac{\mu-1}{\mu}} \right]^{\frac{\mu}{\mu-1}}$$

where $\chi = 1 - (1-n)\gamma$ is a function of the size of the small open economy, n, and γ, the degree of openness on capital goods markets. Symmetrically, the bundle of a representative household in the rest of the world is:

$$k_t^*(j) = \left[\chi^{*\frac{1}{\mu}} k_{h,t}^*(j)^{\frac{\mu-1}{\mu}} + (1-\chi^*)^{\frac{1}{\mu}} k_{f,t}^*(j)^{\frac{\mu-1}{\mu}} \right]^{\frac{\mu}{\mu-1}}$$

where $\chi^* = n\gamma$.

Given that the law of one price holds, companion price indexes are:

$$\begin{aligned} p_{k,t} &= \left[\chi (p_t)^{1-\mu} + (1-\chi)(\varepsilon_t p_t^*)^{1-\mu} \right]^{\frac{1}{1-\mu}} \\ p_{k,t}^* &= \left[\chi^* \left(\varepsilon_t^{-1} p_t\right)^{1-\mu} + (1-\chi^*)(p_t^*)^{1-\mu} \right]^{\frac{1}{1-\mu}} \end{aligned}$$

In these expressions, $\mu \geq 1$ is the elasticity of substitution between domestic and foreign capital goods. This elasticity is the same as that of consumption goods. Standard Dixit and Stiglitz (1977) subindexes are:

$$\begin{aligned} k_{h,t}(j) &= \left[\left(\frac{1}{n}\right)^{\frac{1}{\theta}} \int_0^n k_{h,t}(i,j)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}, \; k_{f,t}(j) = \left[\left(\frac{1}{1-n}\right)^{\frac{1}{\theta}} \int_n^1 k_{f,t}(i,j)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}} \\ k_{h,t}^*(j) &= \left[\left(\frac{1}{n}\right)^{\frac{1}{\theta}} \int_0^n k_{h,t}^*(i,j)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}, \; k_{f,t}^*(j) = \left[\left(\frac{1}{1-n}\right)^{\frac{1}{\theta}} \int_n^1 k_{f,t}^*(i,j)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}} \end{aligned}$$

Accordingly, optimal demands of domestic capital goods varieties are

$$k_{h,t}(i,j) = \frac{\chi}{n} \left[\frac{p_t(i)}{p_t}\right]^{-\theta} \left[\frac{p_t}{p_{k,t}}\right]^{-\mu} k_t(j), \; k_{h,t}^*(i,j) = \frac{\chi^*}{n} \left[\frac{p_t(i)}{p_t}\right]^{-\theta} \left[\frac{p_t}{\varepsilon_t p_{k,t}^*}\right]^{-\mu} k_t^*(j)$$

Letting $p_{k,t}$ denote the price of the bundle of capital goods build by households, their budget constraint is now:

$$E_t\{q_{t,t+1} b_{t+1}(j)\} + p_{c,t} c_t(j) + p_{k,t} k_t(j) = b_t(j) + \Pi_t(j) + (1-\tau_{\ell,t}) w_t \ell_t(j) + p_{k,t} r_{k,t} k_{t-1}(j)$$

First order conditions remain unchanged, except for what concerns the return on capital accumulation:

$$E_t \left\{ \frac{p_{k,t+1}}{p_{k,t}} r_{k,t+1} \right\} = r_t$$

The optimization program of firms is unchanged, except that $y_t(i)$, the aggregate demand addressed to firm j, now features some additional external demand

$$y_t(i) = c_{h,t}(i) + c^*_{h,t}(i) + k_{h,t}(i) - (1-\delta) k_{h,t-1}(i) + k^*_{h,t}(i) - (1-\delta) k^*_{h,t-1}(i) + g_t(i)$$

Keeping in mind that the domestic economy is a small open economy implying, $n \to 0$, capital goods price indexes become

$$p^*_{k,t} = p^*_t \text{ and } p_{k,t} = \left[(1-\gamma)(p_t)^{1-\mu} + \gamma (\varepsilon_t p^*_t)^{1-\mu} \right]^{\frac{1}{1-\mu}}$$

and the corresponding inflation rate of domestic capital goods prices is

$$\pi_{k,t} = \left[(1-\gamma) \pi_t^{1-\mu} + \gamma \pi_t^{1-\mu} \left(\frac{s_t}{s_{t-1}} \right)^{1-\mu} \right]^{\frac{1}{1-\mu}}$$

The equilibrium of goods markets is now

$$\begin{aligned} y_t = {} & (1-\alpha) \left((1-\alpha) + \alpha s_t^{1-\mu} \right)^{\frac{\mu}{1-\mu}} c_t + \alpha s_t^{\mu} c^*_t \\ & + (1-\gamma) \left((1-\gamma) + \gamma s_t^{1-\mu} \right)^{\frac{\mu}{1-\mu}} (k_t - (1-\delta) k_{t-1}) + \gamma s_t^{\mu} inv^*_t + g_t \end{aligned}$$

where inv^*_t is exogenous and affected by the same world demand shocks that consumption

$$inv^*_{t+1} = (1-\rho_{c^*}) inv + \rho_{c^*} inv^*_t + \xi_{c^*,t+1}$$

6.2 Parametrization and dynamics

The steady state remains symmetric, implying that steady state results still apply in this extended framework. The parametrization is meant to keep the overall trade openness constant, i.e. $\alpha + \gamma = 0.3$. Consequently, when abstracting from trade in capital goods ($\gamma = 0$), $\alpha = 0.3$, and when taking trade in capital goods into account, $\gamma = \alpha = 0.15$. The optimal dynamics after exogenous shocks is modified, as depicted in Figures 6 to 8 (in Appendix C).

Not much is changed in the case of internal shocks, except that terms–of–trade are much more volatile, as in Corsetti, Dedola and Leduc (2008). This high variation is required since the volatility if the trade balance has increased. Relative prices must therefore be more volatile to guarantee risk–sharing across countries. Our main results concerning the response of tax rates under Ramsey policies are clearly unchanged. In the case of an external demand shock, the depressive effects of the drop of terms–of–trade, triggered by the risk–sharing condition are amplified since terms–of–trade are more volatile. In particular, private consumption is much more responsive, magnifying the expenditure switching effect. As a consequence, hours, capital and output are clearly depressed as compared to a situation without trade in capital goods. In both cases, our results about optimal taxation remain qualitatively the same for reasonable parameter values.[5]

Summing up, all macroeconomic effects of trade in capital goods occur through the channel of the trade balance, requiring an additional volatility of terms–of–trade to guarantee risk–sharing across countries, which keeps our previous results about optimal monetary and fiscal policy mostly unchanged.

7 Conclusion

In this paper, we analyze the joint determination of optimal fiscal and monetary policies in a small open economy with sticky prices and capital. Solving for the optimal Ramsey policy, we show that our model nests the standard results about optimal fiscal policy (both in the steady state and over the cycle) and optimal monetary policy in a closed–economy set–up.

More precisely, we find that the optimal tax rate on capital income and the optimal inflation tax are zero in the steady state. The tax rate on capital income remains constant over the cycle, while both the nominal interest rate and the tax rate on labor income move, respectively to minimize the distortions implied by nominal rigidities and balance

[5]We also analyze the second order moments implied by the assumption of tradable capital goods (see Table 5 in Appendix C)

the budget. At the same time, movements of policy instruments are quite persistent to smooth the response of the economy to shocks and minimize the implied distortions (for example on labor supply decisions).

Opening the economy, the overall macroeconomic volatility increases due to the presence of an additional (world demand) shock. However, it does not fundamentally alter the results about optimal monetary and fiscal policy. Finally, additional international linkages, e.g. trade in capital goods, are not able to challenge standard closed–economy results, such as the zero tax rate on capital income.

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A Recursive formulation of pricing conditions

Putting price setting behavior in a recursive problem, we start from

$$\frac{\overline{p}_t(i)}{p_t} = \frac{\theta}{\theta-1}\frac{p_{1,t}}{p_t p_{2,t}}$$

$$p_{1,t} = \sum_{v=0}^{\infty} (\eta\beta)^v E_t \left\{ \lambda_{t+v} y_{t+v}(i)\, mc_{t+v} \right\}$$

$$p_{2,t} = \sum_{v=0}^{\infty} (\eta\beta)^v E_t \left\{ \lambda_{t+v} y_{t+v}(i) \right\}$$

Using $y_t(i) = \left[\frac{p_t(i)}{p_t}\right]^{-\theta} y_t$ and $\lambda_t = \frac{u_{c,t}}{p_{c,t}}$:

$$p_{1,t} = \sum_{v=0}^{\infty} (\eta\beta)^v E_t \left\{ \frac{u_{c,t+v}}{p_{c,t+v}} p_{t+v}^{\theta} y_{t+v} mc_{t+v} \right\}$$

$$p_{2,t} = \sum_{v=0}^{\infty} (\eta\beta)^v E_t \left\{ \frac{u_{c,t+v}}{p_{c,t+v}} p_{t+v}^{\theta} y_{t+v} \right\}$$

and using a recursive transformation:

$$p_{1,t} - \eta\beta E_t \{p_{1,t+1}\} = u_{c,t}\frac{p_t^{\theta}}{p_{c,t}} y_t mc_t$$

$$p_{2,t} - \eta\beta E_t \{p_{2,t+1}\} = u_{c,t}\frac{p_t^{\theta}}{p_{c,t}} y_t$$

Finally, defining

$$v_{1,t} = \frac{p_{1,t} p_{c,t}}{p_t^{\theta+1}}, \text{ and } v_{2,t} = \frac{p_{2,t} p_{c,t}}{p_t^{\theta}}$$

we get:

$$\frac{\overline{p}_t}{p_t} = \frac{\theta}{\theta-1}\frac{v_{1,t}}{v_{2,t}}$$

where

$$v_{1,t} - \eta\beta E_t \left\{ v_{1,t+1} \pi_{t+1}^{\theta} \left(\frac{s_t}{s_{t+1}} \right)^{\alpha} \right\} = u_{c,t} y_t \frac{mc_t}{p_t}$$

$$v_{2,t} - \eta\beta E_t \left\{ v_{2,t+1} \pi_{t+1}^{\theta-1} \left(\frac{s_t}{s_{t+1}} \right)^{\alpha} \right\} = u_{c,t} y_t$$

B Summary of equilibrium conditions

The model is expressed in real terms. We therefore define $\omega_t = \frac{w_t}{p_t}$, and $\zeta_t = \frac{z_t}{p_t}$. Equilibrium conditions are given by

$$-\frac{u_{n,t}}{u_{c,t}} = (1-\tau_{n,t})\,\omega_t s_t^{-\alpha} \tag{18}$$

$$1 = r_t \beta E_t \left\{ \frac{u_{c,t+1}}{\pi_{t+1} u_{c,t}} \left(\frac{s_t}{s_{t+1}} \right)^{\alpha} \right\} \tag{19}$$

$$r_t = E_t \left\{ \pi_{t+1} \left(1 + (1-\tau_{k,t+1})(\zeta_{t+1} - \delta)\right) \right\} \tag{20}$$

$$\tfrac{mc_t}{p_t} = \frac{\zeta_t^{\phi} \omega_t^{1-\phi}}{\phi^{\phi}(1-\phi)^{1-\phi}} a_t^{-1} \tag{21}$$

$$\phi \omega_t n_t = (1-\phi)\,\zeta_t k_{t-1} \tag{22}$$

$$\eta \pi_t^{\theta-1} + (1-\eta) \left(\frac{\theta}{\theta-1} \frac{v_{1,t}}{v_{2,t}} \right)^{1-\theta} = 1 \tag{23}$$

$$v_{1,t} = \eta \beta E_t \left\{ v_{1,t+1} \pi_{t+1}^{\theta} \left(\frac{s_t}{s_{t+1}} \right)^{\alpha} \right\} + u_{c,t} \Upsilon_t^{-1} a_t k_{t-1}^{\phi} n_t^{1-\phi} \tfrac{mc_t}{p_t} \tag{24}$$

$$v_{2,t} = \eta \beta E_t \left\{ v_{2,t+1} \pi_{t+1}^{\theta-1} \left(\frac{s_t}{s_{t+1}} \right)^{\alpha} \right\} + u_{c,t} \Upsilon_t^{-1} a_t k_{t-1}^{\phi} n_t^{1-\phi} \tag{25}$$

$$\Upsilon_t = \eta \Upsilon_{t-1} \pi_t^{\theta} + (1-\eta) \left(\frac{\theta}{\theta-1} \frac{v_{1,t}}{v_{2,t}} \right)^{-\theta} \tag{26}$$

$$\Upsilon_t^{-1} a_t k_{t-1}^{\phi} n_t^{1-\phi} = g(c_t, s_t) + k_t - (1-\delta)\,k_{t-1} + g_t \tag{27}$$

$$c_t^* = c_t s_t^{(1-\alpha)/\sigma} \tag{28}$$

and the budget constraint of the government:

$$\tau_{n,t} \omega_t n_t + \tau_{k,t} (\zeta_t - \delta)\, k_{t-1} = g_t \tag{29}$$

C IRFs and second–order moments: $\gamma = 0$ vs $\gamma = 0.15$

Figure 6: IRFs after a unit productivity shock under Ramsey. Closed vs open capital goods markets.

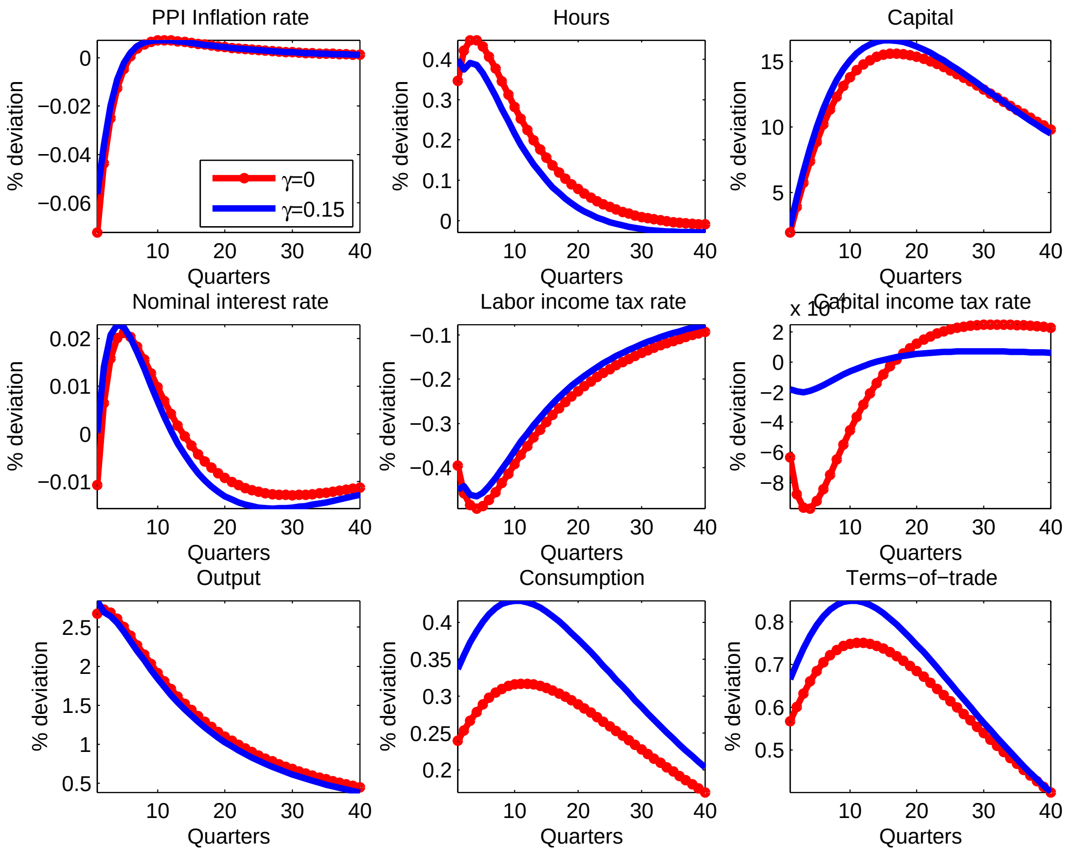

Table 5: Second order moments – the impact of trade in capital goods

Variable	*Mean*	*St. dev.*	*Auto.corr*	*Mean*	*St. dev.*	*Auto.corr*
	$\alpha = 0.3$ *and* $\gamma = 0$			$\alpha = \gamma = 0.15$		
y	2.223	0.0875	0.9675	2.223	0.0829	0.9600
n	0.676	0.0161	0.9726	0.676	0.0135	0.9600
k	18.47	0.9075	0.9985	18.47	0.9242	0.9983
r	1.012	0.0012	0.7665	1.012	0.0012	0.8110
τ_n	0.456	0.0284	0.9678	0.456	0.0267	0.9598
τ_k	0.000	0.0000	0.9716	0.000	0.0000	0.9638
π	1.000	0.0015	0.5998	1.000	0.0013	0.6081
c	1.206	0.0201	0.9805	1.206	0.0246	0.9865
s	1.000	0.0493	0.9688	1.000	0.0532	0.9640

Figure 7: IRFs after a unit public spending shock under Ramsey policies. Closed vs open capital goods markets.

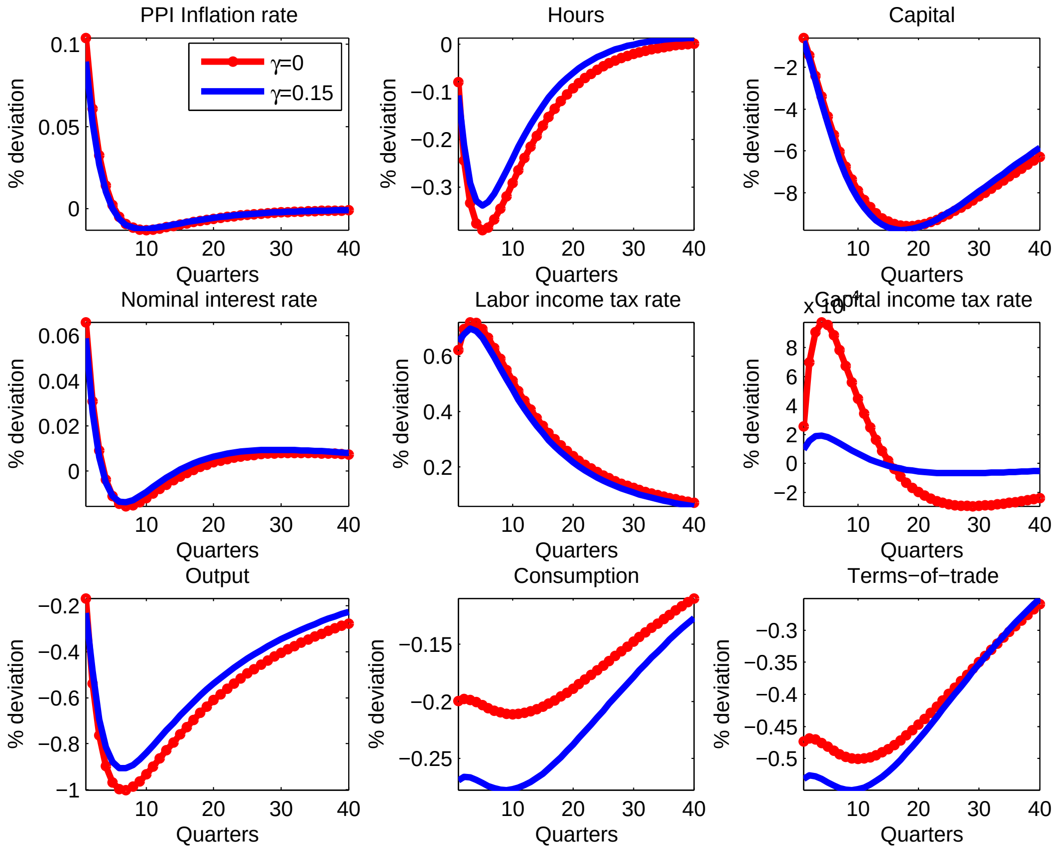

Figure 8: IRFs after a unit world demand shock under Ramsey policies. Closed vs open capital goods markets.