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Abstract

We show in this paper that the growth rate of the Sen index is multi-decomposable, that is, decomposable simultaneously by groups and income sources. The multi-decomposition of the poverty growth yields respectively: the growth rate of the poverty incidence (poverty rate) decomposed by groups, the growth rate of the poverty depth (poverty gap ratios) decomposed by sources and groups, and the growth rate of inequality decomposed by sources and groups. We demonstrate that the multi-decomposition is not unique. It is mainly dependent on poverty lines defined on the space of income sources. An application to Scandinavian countries shows that poverty lines based on non-correlation between the sources of incomes imply serious underestimation of the contribution levels of the different components of the global poverty growth. The main contribution of our paper is to pay a particular attention to the poverty growth and its source components in order to avoid underestimation of poverty growth.

Key-words: Gini index, Sen index, Source decomposition, Subgroup decomposition.

JEL Classification: D31, D63, I32

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1 Introduction

Since the 80's, the literature on income inequality measurement has proposed two new fundamental ways of measuring income inequalities. The first one is the decomposition of income inequalities by subgroups. The second one is concerned with the decomposition of income inequalities by income sources. The first method was first axiomatized by Bourguignon (1979) and Shorrocks (1980, 1984). They highlight the possibility to decompose inequality into a within-group component that gauges income inequalities within each group of the population and a between-group component that captures income inequalities between the mean incomes of the groups. The second technique, first axiomatized by Shorrocks (1982), yields the ability to outline the contribution of each income source (wage, bonus, child support, ect.) to the overall amount of income inequalities. Since then, many authors have proposed solutions to merge the two methods, namely the multi-decomposition, aiming at obtaining a mixture of both decomposed components, see e.g., Shorrocks (1999), Yitzhaki (2002), Mussard (2004, 2006). Precisely, the resulting estimators provide 'source-within-group' and 'source-between-group' inequality components such as they account for the overall inequality exactly.

From our knowledge, the literature is quite silent about how this type of multi-decomposition can be adapted to poverty measures. Chakravarty, Mukherjee and Ranade (1998) propose a multidimensional poverty decomposition by introducing a class of poverty indices simultaneously decomposable by dimensions and groups of population.[1] This multi-decomposition rule is respected by the class of Foster-Greer-Thorbecke indices (FGT, 1984), and by poverty indices relying on fuzzy set theory (see Mussard and Pi Alperin, 2007a). Besides, the multi-decomposition is not available for all indices, and particularly, from our knowledge, not usable for the Sen index of poverty because of its multiplicative analytical structure. Indeed, being a non-additive poverty index, the Sen index multi-decomposition becomes an awkward task.

In the paper, we show in particular that the *growth rate of the Sen index* is "multi-decomposable". Although the multiplicative structure of the Sen index and its growth rate were studied by Xu and Osberg (2001) throughout a subgroup decomposition layout, we propose to adapt the multi-decomposition for simultaneous "source/subgroup" decompositions. As we will show with more precision in the paper, the Sen index is a mixture of three components: the poverty incidence (i.e. the head count ratio or proportion of poor), the poverty depth (i.e. the poverty gap ratio where gaps are defined to be the difference between incomes and the poverty line), and the inequality in poverty (inequality in poverty gap ratios). The analysis of the poverty change (poverty growth) between two periods is then a function of the incidence growth, the depth

[1] Dimensions is a larger concept compared with income source. More details about this technique are presented in Appendix 1.

change, and the inequality change. The structure of the Sen index growth brings out many information, therefore, adding the information of the multi-decomposition of the Sen index growth may be of interest to capture additional specific determinants of poverty.

The third component of the Sen index is the Gini index of the poverty gap ratios. A well-suited feature of this index is precisely the respect of the multi-decomposition property into "source/group" contributions. Therefore, the multi-decomposition of the growth rate of the Sen index will be connected with the Gini multi-decomposition studied in Mussard (2006, 2008) and Mussard and Pi Alperin (2007b). In particular, the multi-decomposition of the growth rate of the Sen index yields respectively:

• the growth rate of the poverty incidence decomposed in groups ;

• the growth rate of the poverty depth decomposed in sources and groups ;

• and the growth rate of the inequality of poverty gap ratios, decomposed in sources and groups. All these components sum up to the overall Sen index growth precisely.

The other challenge of the paper is the implementation of the multi-decomposition by choosing an adequate poverty line for each income source. Although there is some consensus about the poverty line of income (60% of the income median for European countries), there is no common practice to determine a poverty line for any given source of income. Subsequently, we propose three methods based on deprivation and fuzzy set theory. Our application to Scandinavian countries shows that poverty lines based on non-correlation between the sources of incomes imply serious underestimation of the contribution levels of the different components of the global poverty growth. The main contribution of our paper is to pay a particular attention to the poverty growth and its source components in order to avoid underestimation of poverty, which could lead to transfer cut-offs.

The remainder of the paper is organized as follows. In section 2, we introduce notations and discuss the feasibility of the multi-decomposition of the growth rate of the Sen index. In Section 3 we expose the result of the multi-decomposition especially when different perspectives of poverty lines are introduced. In section 4, the multidimensional decomposition of the Sen index growth is applied to Scandinavian countries. Finally, we close in Section 5.

2 The Multi-decomposition of the Sen Index

2.1 Notations and poverty identification

Let the number of income units, say individuals, in a population be n and the number of poor individuals with income below the poverty line z be q. In this population there are K distinct subgroups (sub-populations), $k \in \{1, 2, \dots, K\}$. In subgroup k, there are q_k poor individuals among n_k total individuals. The overall poverty rate (head count ratio) is $H = \frac{q}{n}$ and the poverty rate for subgroup k is $H_k = \frac{q_k}{n_k}$ with $H = \sum_{k=1}^{K} \frac{n_k}{n} H_k$. Let y_i be the income of the ith person and z the poverty line. The poverty gap ratio (sometime called relative poverty gap or poverty gap) is:

$$x_i = \begin{cases} \frac{z-y_i}{z}, & \forall z > y_i \\ 0, & \end{cases} \tag{1}$$

for all q poor individuals. Then, the vector of poverty gap ratios of the poor is given by: $\mathbf{x} = [x_1, \ldots, x_i, \ldots, x_q]$. Let y_i^m be the mth income source possessed by the ith person. Her total income is the sum of all income sources she received:

$$\sum_{m=1}^{M} y_i^m = y_i. \tag{2}$$

The identification of the poor is based on to whether or not the income of an individual y_i falls below the poverty line z. Obviously, this criterion is also applicable to any subgroup. However, when we attempt to analyse the contributions of shortfalls in income components to shortfalls in overall income, we have to consider different types of poverty lines. That is, the poverty line in overall income must be decomposable according to different sources:

$$\sum_{m=1}^{M} z^m = z. \tag{3}$$

We will explain in Section 3 three ways of dealing with this non-trivial problem of decomposing the poverty line in income sources. Accordingly, the poverty gap ratio in source m of individual i is expressed as:

$$x_i^m = \frac{z^m - y_i^m}{z}, \tag{4}$$

such as $\sum_{m=1}^{M} x_i^m = x_i$. While x_i is non-negative, its components x_i^m can be negative, showing that a person can be poor ($x_i^m > 0$) in one dimension (e.g. wages) but rich ($x_i^m \leq 0$) in another one (e.g. transfers). If the poverty line can be suitably decomposed, it is possible to define the poverty gap ratio in income source m of individual i in group k:

$$x_{ik}^m = \frac{z^m - y_{ik}^m}{z}. \tag{5}$$

The average poverty gap ratio of the population and that of group k are respectively given by:

$$\overline{x}_p = \sum_{i=1}^{q} \frac{x_i}{q} \; ; \; \overline{x}_p^{(k)} = \sum_{i=1}^{q_k} \frac{x_{ik}}{q_k}, \tag{6}$$

such as

$$\overline{x}_p = \sum_{k=1}^{K} \frac{q_k}{q} \overline{x}_p^{(k)}. \tag{7}$$

As the poverty gap ratio can be expressed in various ways such as by income source and subgroup, we can propose new decompositions for the Sen index. Before, let us review briefly the analytical structure of the Sen index and its third component, the Gini index.

2.2 Sen and Gini decompositions

Over the last two decades, one of the important developments in the literature on inequality and poverty measures was the new poverty measure introduced by Amartya Sen (1976), the so-called Sen index. This index is attractive – easy to understand and convenient for applied research and policy analysis – because of its decomposability into three measures of poverty: incidence (the poverty rate), depth (poverty gap ratio), and inequality (1 plus the Gini index of poverty gap ratios) (see Xu & Osberg (2001)). Naturally, economists and policy analysts would like to know whether it is possible to further decompose the Sen index components according to subgroups or income sources which allows researches to measure and, therefore, to appreciate how each of contributing components affects the overall inequality/poverty. Let us on the one hand remember the feasibility of subgroup decomposing the Gini index, which lead Xu and Osberg (2001) to propose a subgroup decomposition of the Sen index:[2]

$$\begin{aligned} S &= H\overline{x}_p(1+G) \\ &= \sum_{k=1}^{K} \frac{n_k}{n} H_k \sum_{k=1}^{K} \frac{q_k}{q} \overline{x}_p^{(k)} \left(1 + (G_w + G_{gb})\right), \end{aligned} \tag{8}$$

where G is the Gini index of poverty gap ratios of the poor, G_w is the contribution of inequalities within K subgroups, G_{gb} is the gross contribution of all inequalities between each and every pair of groups.[3]

On the other hand, the Gini index is also decomposable by income sources. Thus, it is possible to gauge inequalities in poverty due to source m, for all $m = 1, \ldots, M$:

$$G = \sum_{m=1}^{M} \mathcal{C}^m \tag{9}$$

[2] In this paper, we will not discuss the SST (S_{SST}) index but this does not diminish the usefulness of the SST index. The results provided in this paper are still relevant to the SST index as the Sen index and SST index are closely related and have a one-to-one mapping according to Xu and Osberg (2001):

$$S_{SST} = HS + 2H(1-H)\overline{x}_p.$$

That is, given H and $\overline{x}_p$, it is always possible to compute S_{SST} from S and vice versa.

[3] The gross between group component is also decomposable into G_b – net between-group inequalities that excludes the overlap between the distributions of these groups – and G_t – the inequalities between the groups limited to the overlap between the conditional distributions or the intensity of transvariation (see Gini (1916), Dagum (1959, 1960, 1961, 1997)). It is precisely G_t that prevents the Gini index from being subgroup consistent (see Shorrocks (1984)). Now, we know that G_t measures stratification as well as distance between distributions, see Lerman and Yitzhaki (1991) and Dagum (1997) respectively.

where $\mathcal{C}^m$ stands for the contribution of source m to the Gini index of poverty gap ratios.

Splitting the Gini index in income sources contributions is not a unique exercise, as can be seen in Rao (1969), Pyatt (1976), Shorrocks (1982), Silber (1989), Lerman and Yitzhaki (1985), among others. However, there exists an income source decomposition in such a way that the Gini index, and in consequence the Sen index, may be decomposed simultaneously in all cells "subgroup k / source m". The Gini index multi-decomposition proposed by Mussard (2004, 2006, 2008) allows the Table 6 (in Appendix 1) to be determined (except we talk about inequality rather than poverty). This multi-decomposition is actually a subgroup Gini decomposition in which each element is further decomposed by income sources:

$$G = \sum_{m=1}^{M} (G_w^m + G_{gb}^m), \tag{10}$$

where G_w^m and G_{gb}^m are respectively the contributions of the mth source to G_w and G_{gb}.[4]

The advantage of the Gini multi-decomposition is quite similar to the poverty one. Prior to the proposal of this multi-decomposition approach, researchers used to compute the margins of Table 6 (in Appendix 1): either the contribution of source m or the contributions to subgroup k to the overall amount of inequality G. Instead of looking only at the margins of Table 6, the Gini multi-decomposition provides the contribution of the mth source of the within-group inequalities and of the between-group inequalities, respectively G_w^m and G_{gb}^m, that account for the global Gini index. If analysts were used exclusively the two traditional decomposition techniques, i.e., the "marginal" decompositions either by group or by income source, important mistakes could be done. Actually, the multidimensional decomposition brings to light the independence between these two types of decomposition. Suppose that marginal decompositions produce the two following results: source m and group k are the most important contributions. Then, it will be wrong to directly conclude that the couple "inequalities within group k / due to source m" is the most important contribution to G. Consequently, the multi-decomposition may yield a different couple than that of the marginal decompositions and compute precisely the greater value (or a lower one).

3 The results: multi-decomposition of poverty growth

3.1 Exogenous poverty lines

As mentioned in the previous Section, it is possible, and sometimes desirable, to consider decomposing poverty measures by income source and by subgroup simultaneously. Suppose that

[4] More details about this technique are presented in Appendix 2a.

the poverty line by sources z^m for all m is fixed exogenously. Then, Mussard and Xu (2006) present the following propositions.

Proposition 3.1 *If both income and poverty line can be decomposed by source, then the Sen index is multi-decomposable.*

Proof.

We know that:

$$\overline{x}_p = \sum_{k=1}^{K} \frac{q_k}{q} \overline{x}_p^{(k)} \text{ and } \overline{x}_p^{(k)} = \sum_{i=1}^{q_k} \frac{x_i}{q_k} = \sum_{i=1}^{q_k} \frac{\sum_{m=1}^{M} x_i^m}{q_k} = \sum_{m=1}^{M} \frac{\sum_{i=1}^{q_k} x_i^m}{q_k} = \sum_{m=1}^{M} \overline{x}_p^{m(k)}.$$

The average poverty gap ratio in source m for subgroup k is:

$$\overline{x}_p^{m(k)} = \frac{\sum_{i=1}^{q_k} x_i^m}{q_k}. \tag{11}$$

Then,

$$\overline{x}_p = \sum_{k=1}^{K} \frac{q_k}{q} \overline{x}_p^{(k)} = \sum_{k=1}^{K} \frac{q_k}{q} \sum_{m=1}^{M} \overline{x}_p^{m(k)} = \sum_{k=1}^{K} \sum_{m=1}^{M} \frac{q_k}{q} \overline{x}_p^{m(k)}. \tag{12}$$

Given Eq.(9), (10), the Sen index can be expressed as:

$$S = \sum_{k=1}^{K} \frac{n_k}{n} H_k \left(\sum_{m=1}^{M} \sum_{k=1}^{K} \frac{q_k}{q} \overline{x}_p^{m(k)} \right) \left(1 + \sum_{m=1}^{M} \left(G_w^m + G_{gb}^m \right) \right). \tag{13}$$

■

The result exhibits, under the condition that there exists a suitable decomposition of the poverty line by source of incomes (discussed in the next Section), a simultaneous source/group decomposition, namely the Sen index multi-decomposition, which is useful for analysts. For instance, it reveals whether inequalities in poverty gaps are due, e.g., to inequality in wages of the men group or to the inequality in rents and interest between men and women. Similarly, the multi-decomposition may exhibit inequalities in poverty gaps measured by wages within a region (say region A) and inequalities in poverty gaps measured by transfers between region A and B.

Proposition 3.2 *The growth rate of the Sen index is linearly decomposable into the growth rate of incidence, depth and inequality of poverty.*

Proof.

The growth rate of the Sen index can be expressed as:

$$\frac{dS}{S} = \frac{dH}{H} + \frac{d\overline{x}_p}{\overline{x}_p} + \frac{dG}{1+G}. \tag{14}$$

The change in the different components between periods $t-1$ and t can be approximated by $\Lambda\xi_t := d\xi \approx \xi_t - \xi_{t-1}$. The growth rate of poverty between periods $t-1$ and t is then:

$$\frac{\Lambda S_t}{S_{t-1}} \approx \frac{\Lambda\left(\sum_{k=1}^{K} \frac{n_k}{n} H_k\right)_t}{H_{t-1}} + \frac{\Lambda\left(\sum_{m=1}^{M} \sum_{k=1}^{K} \frac{q_k}{q} \overline{x}_p^{m(k)}\right)_t}{(\overline{x}_p)_{t-1}} + \frac{\Lambda\left(\sum_{m=1}^{M} \left(G_w^m + G_{gb}^m\right)\right)_t}{(1+G)_{t-1}}. \tag{15}$$

■

This result shows that the growth rate of the Sen index is an increasing function of the rates of incidence, depth, and inequality between $t-1$ and t. It can be shown that the Sen index depends on inequalities within groups, between groups, income sources and "groups/sources" inequality combinations.

This way of decomposing the Sen index is appealing from a practical point of view.[5] As depicted in Eq. (15), a change in the proportion of the poor and/or the average of income shortfalls below the poverty line is positively related to the change in the Sen index. The increase of inequality between periods t and $t-1$ is also positively related to the change in the Sen index. This observation confirms that the Sen index satisfies the principle of transfer. Indeed, a transfer of amount $\delta > 0$ from a higher-income individual to a lower-income one necessarily implies a decrease in overall inequality measured by the Gini index, which further reduces poverty intensity measured by the Sen index.

Based on the above framework the growth rate of the Sen index can be captured by changes in poverty incidence, depth, and inequality by group and across groups. The examples of this application include regional analysis and comparative studies. This framework also helps researchers/policy makers to identify changes in source components that are mainly responsible for changes in poverty intensity. On the other hand, the implementation perspective offered by the multi-decomposition of the Sen index growth is significantly related to the poverty line decomposition. In this respect, we propose hereafter three possibilities.

3.2 Poverty line decomposition

As said previously, when we attempt to analyse the contributions of shortfalls in income components to shortfalls in overall income, we have to consider different types of poverty lines. That is, the poverty line in overall income must be decomposable according to different income sources, and the poverty lines by source of incomes must sum up to the overall poverty line z.

A simple way to deal with a suitable decomposition of z is to compute direct fuzzy measures of income deprivation for each component and their contribution levels to global poverty. It is worth mentioning that the technique aiming at computing poverty lines by sources is based on

[5]This result can also be applied to measure the poverty difference between two distributions.

a postulate: the value of the overall poverty line z for the aggregate income is taken *a priori*. Indeed, we take z as 60% of the median income, and on this basis, we seek to achieve the determination of the poverty line by source of incomes.

Let B^m be the fuzzy sub-set of individuals (or households) such that any individual $i \in B^m$ presents some degree of deprivation in the mth income source. Let y_i^m be the equivalent income level of the ith individual and for the mth component, and let y^{min} and y^{max} be respectively the maximum and minimum equivalent income thresholds which are exogenously determinated. Then, all individuals with an income value $y_i^m \geq y^{max}$ will not belong to the B^m sub-set. All individuals with an income value $y_i^m \leq y^{min}$ will completely belong to B^m. Finally, all individuals with an income value $y^{min} < y_i^m < y^{max}$ will belong to B^m with an intensity belonging to the open interval (0,1).[6]

The degree of membership of the ith individual ($i = 1, \ldots, n$) to B^m with respect to the mth component is defined as the quantity of the mth source ($m = 1, \ldots, M$) possessed by the ith individual. Formally:

$$s_i^m := B^m(m(i)) \ , \ 0 \leq s_i^m \leq 1 \ . \tag{16}$$

Particularly:

(i) $s_i^m = 1$, if the ith individual is fully deprived in the mth component;

(ii) $s_i^m = 0$, if the ith individual has the the mth income source level to be consider as totally non deprived;

(iii) $0 < s_i^m < 1$, if the ith individual possesses the mth component with an intensity belonging to the open interval (0,1).

Accordingly, it is possible to derive an unidimensional deprivation index (UDI) for each one of the mth income component as follows:

$$\phi^m = \frac{\sum_{i=1}^{n} s_i^m}{n} \tag{17}$$

where ϕ^m measures the degree of deprivation of the mth component for the entire population of n individuals.

Using the fuzzy sets technique allows one to calculate the contribution level of each component in two different ways. The first possibility is to consider the weight w_m proposed by Cerioli & Zani (1990) (CZ) in which w_m stands for the intensity of deprivation of X_m. It is an inverse function of the deprivation degree of the individuals on this component:

$$w_m = \log \left[\frac{n}{\sum_{i=1}^{n} s_i^m} \right] . \tag{18}$$

[6]Table 8 in Appendix 3 shows the y^{min} and y^{max} thresholds used in the empirical study.

Based on Dagum and Costa (2004), which introduced the decomposition by dimension (item) in the context of fuzzy sets, it is possible to gauge the contribution of the mth item to the overall amount of deprivation:

$$C^m = \frac{\phi^m w_m}{\sum_{m=1}^{M} w_m} \ , \tag{19}$$

where w_m is the weight attached to the mth component.

The second possibility is based on the system of weights proposed by Betti & Verma (1998) (BV). It takes into account the intensity of deprivation of m and it limits the influence of those components that are highly correlated. Betti & Verma (1998) defined the weight of any attribute as follows:

$$\widetilde{w}_m = w_m^a \cdot w_m^b \ , \tag{20}$$

where w_m^a only depends on the distribution of the mth attribute, whereas w_m^b depends on the correlation between m and the others attributes. In particular, w_m^a is determined by the coefficient of variation of s_i^m:

$$w_m^a = \frac{\sum_{i=1}^{n}(s_i^m - \bar{s}^m)^2}{\bar{s}^m n^{1/2}}. \tag{21}$$

The weights w_m^b are computed as follows:

$$w_m^b = \left[1 + \sum_{m'=1}^{M} \rho_{m,m'} F\left(\rho_{m,m'} < \rho_H\right)\right]^{-1} \times \left[\sum_{m'=1}^{M} \rho_{m,m'} F\left(\rho_{m,m'} \geq \rho_H\right)\right]^{-1} \tag{22}$$

where $\rho_{m,m'}$ is the correlation coefficient between items m and m' and $F(\cdot)$ is an indicator function valued to be 1 if the expression in brackets is true and 0 otherwise. ρ_H is a pre-determined cut-off correlation level between the two indicators.[7] w_m^b is the inverse of a measure of average correlation of item m with the other ones. The largest is the average correlation with item m, the lower is the resulting weight for item m.

Again, we deduce the contribution of the mth item to the overall amount of deprivation:

$$\widetilde{C}^m = \frac{\phi^m \widetilde{w}_m}{\sum_{m=1}^{M} \widetilde{w}_m} \ . \tag{23}$$

Since there is no common practice to determine a poverty line for each income component, we use the fuzzy set approach to compute in a more flexible way the direct measures of deprivation for each source (since it avoids using a poverty line which dichotomies the population). These measures are used to gauge the poverty lines by income sources. In addition, it allows to propose three different techniques to decompose the Sen index:

[7] ρ_H separates high and low correlations. Betti & Verma (1998) suggest setting ρ_H as to divide the ordered set of correlations at the point of the largest gap.

Proposition 3.3 *The growth rate of the Sen index is multi-decomposable according to* ($\imath$) *the unidimensional deprivation index,* ($\imath\imath$) *Cerioli & Zani's weight, and* ($\imath\imath\imath$) *Betti & Verma's weight.*

Proof.

($\imath$) From the unidimensional deprivation index (UDI) Eq.(17), we get:

$$z^m_{UDI} = z \cdot \frac{\phi^m}{\sum_{m=1}^{M} \phi^m} \ .$$

($\imath\imath$) From Cerioli & Zani's weight (CZ) Eq.(19):

$$z^m_{CZ} = z \cdot \frac{C^m}{\sum_{m=1}^{M} C^m} \ .$$

($\imath\imath\imath$) Finally, from Betti & Verma's weight (BV) Eq. (23) we have:

$$z^m_{BV} = z \cdot \frac{\widetilde{C}^m}{\sum_{m=1}^{M} \widetilde{C}^m} \ .$$

Substituting back one of these three expressions in Eq.(15) of Proposition 3.2 yields the desired result.[8] ■

This way of computing poverty lines by income sources allows a hierarchical order of the different sources to be addressed. Thereby, the total poverty line will be decomposed according to the importance of each income component among the population.

Those three techniques will be used in the empirical applications. Before, one has to clarify the implications of the decomposed poverty line on the multi-decomposition of the Sen index growth. Indeed, the analyst has to be aware about the implications of the poverty line decomposition.

3.3 Poverty line decomposition by income source: the implications

In Section 2, we introduced the definition of the poverty gap ratios computed for each income source poverty line. Indeed, the poverty gap ratio in source m of individual i is given by:

$$x^m_i = \frac{z^m - y^m_i}{z} \ ,$$

such as $\sum_{m=1}^{M} x^m_i = x_i$. As usual in poverty measurement, x_i is nonnegative, indicating that individual i is poor in income since her/his income level is below the poverty line. In our method, we allow for x^m_i to be negative in order to maintain the equality: $\sum_{m=1}^{M} x^m_i = x_i$. This means

[8] As the third term of the Sen index measures inequalities in poverty gaps, the readers must substituting the expression x^m_{ik} both in the second and third terms of the Sen index.

that a person can be poor in her aggregate income, but this does not imply systematically that the same person is also poor in all the income components such as transfers or child support benefits. If we suppose that $x_i^m < 0$ for certain $i \in \{1, \ldots, q\}$ and $m \in \{1, \ldots, M\}$, then we allow many income sources to contribute negatively to the poverty index. For instance, if many income transfers are seen to be higher than the poverty line of this source, then transfers may decrease the overall amount of poverty. For policy purposes, negative poverty gap ratios enable the analyst to implement simulations about income transfer policies in order to capture the contribution of this income component to the overall amount of poverty. Actually, it can be demonstrated that, for any given z, whatever the poverty line decomposition into income sources expressed by $z = z^1 + \ldots + z^m + \ldots + z^M$, the Sen index growth remains invariant.

Definition 3.4 Poverty Line Decomposition Invariance : *Let $S(\mathbf{x}; z)$ be a poverty index depending on a decomposed poverty line $z = z^1 + \ldots + z^m + \ldots + z^M$ that provides a vector of poverty gap ratios $\mathbf{x}$. Suppose three methods of poverty line decomposition denoted by $_{(\cdot)}$, respectively:*

$$\begin{aligned} z &= z^1_{(1)} + \ldots + z^m_{(1)} + \ldots + z^M_{(1)} =: z_{(1)} \implies \mathbf{x}_{(1)} \\ z &= z^1_{(2)} + \ldots + z^m_{(2)} + \ldots + z^M_{(2)} =: z_{(2)} \implies \mathbf{x}_{(2)} \\ z &= z^1_{(3)} + \ldots + z^m_{(3)} + \ldots + z^M_{(3)} =: z_{(3)} \implies \mathbf{x}_{(3)} \ . \end{aligned}$$

An index of poverty is invariant about Poverty Line Decompositions (PLD) *if, and only if,*

$$S(\mathbf{x}_{(1)}; z_{(1)}) = S(\mathbf{x}_{(2)}; z_{(2)}) = S(\mathbf{x}_{(3)}; z_{(3)}) \ . \tag{PLD}$$

The fact that the Sen index (or its growth rate) is invariant with respect to the poverty line decomposition method is quite obvious. Indeed, whatever the method, we get $\sum_m^M z^m_{(\cdot)} = z_{(\cdot)} = z$, that is, the sum of the components yields always the amount of the predefined poverty line. On the contrary, this does not imply systematically that all decomposed terms of the Sen index (or its growth rates) are invariant too. The first term $\Lambda\left(\sum_{k=1}^K \frac{n_k}{n} H_k\right)_t / H_{t-1} =: S_1$ of the growth rate Sen multi-decomposition and the third one $\Lambda\left(\sum_{m=1}^M \left(G_w^m + G_{gb}^m\right)\right)_t / (1+G)_{t-1} =: S_3$ respect the invariance principle of Poverty Line Decomposition (PLD). The second one is the sole term affected by the method of the poverty line decomposition $\Lambda\left(\sum_{m=1}^M \sum_{k=1}^K \frac{q_k}{q} \overline{x}_p^{m(k)}\right)_t / \left(\overline{x}_p\right)_{t-1} =:$ S_2. This result is summarised in the following proposition.

Proposition 3.5 *The terms S_1 and S_3 respect* (PLD)*, not S_2.*

Proof.

See Appendix 2b. ■

In consequence, the poverty incidence (the head count ratio S_1) and the inequality in poverty gap ratios S_3 are invariant to the poverty line decomposition, in particular, they are invariant to our three poverty line decompositions z into: z^m_{UDI}, z^m_{CZ}, z^m_{BV}, for all $m = 1, \ldots, M$.

4 Empirical study: the Scandinavian countries

The empirical study is focused on Scandinavian countries. Their poverty level is rather a question of income than a question of non-monetary variables. The study of these countries gives the possibility of having positive income sources for all countries in order to apply the multi-decomposition of the Sen index growth.

4.1 The Database

EU-SILC (European Union Statistics on Income and Living Conditions) is an instrument aiming at collecting comparable cross sectional and longitudinal multidimensional micro data on income poverty and social exclusion. The EU-SILC survey was developed to be a flexible yet comparable instrument between the Community countries. It covers data and data sources of various types depending on the country: cross-sectional and longitudinal; household-level and person-level; economic and social; from new and existing national surveys; registers or other sources. Following pilot surveys in 2003, full-scale EU-SILC surveys were conducted in 15 countries in 2004, and in 25 countries in 2005. This cross-country data covers to date 27 countries (EU-27 minus Bulgaria and Romania, plus Norway and Iceland). The number is expected to reach around 30 countries, including all EU Member States.

Our analysis is based on cross-sectional data for the 5 Scandinavian Countries (Denmark (DK), Finland (FI), Iceland (IS), Norway (NO) and Sweden (SE)) and from EU-SILC 2006 and 2007[9]. The analysis will be conducted at the household level.[10] Table 1 shows the household-sample sizes (by country and by year) covered in this empirical study.

Table 1: Sample Sizes

Country	Year 2006			Year 2007		
	Men	Female	Total	Men	Female	Total
Denmark	4915	5140	10055	4968	5247	10215
Finland	9314	9605	18919	9133	9378	18511
Iceland	2505	2599	5104	2542	2637	5179
Norway	4846	4945	9791	5032	5095	10127
Sweden	5650	5862	11512	6068	6263	12331

Source: Eurostat, EU-SILC 2006 and 2007

[9] Versions 2006.1 from 01-03-08 and 2007.1 from 01-03-09 respectively for years 2006 and 2007.

[10] The interest of working at household and not at individual level is explained by the possibility of comparing the total income level perceived by each member of the household. Then, it is possible to consider the use of some income sources as family and housing allowances.

The 'total household income' is composed of gross personal income components (which will be the sum for all household members) and gross income components at household level. Three different income sources are considered: (i) income from wages (y_w); (ii) income from rents and interests (y_{ri}); and (iii) income from allowances and benefits (y_{ab}).[11]

To make possible comparisons between households, the income variables has been divided by the 'equivalent household size' in order to account for economies of scale.[12]

4.2 The poverty lines by income sources

As we notice before, the multi-decomposition of the growth rate of the Sen index is implementable if and only if it is possible to decompose the poverty line according to different sources. We identify in the previous sections three different ways of doing this[13]:

($\imath$) From the unidimensional deprivation index (UDI) Eq.(17), we get:

$$z^m_{UDI} = z \cdot \frac{\phi^m}{\sum_{m=1}^{M} \phi^m} \ .$$

($\imath\imath$) From Cerioli & Zani's weight (CZ) Eq.(19):

$$z^m_{CZ} = z \cdot \frac{C^m}{\sum_{m=1}^{M} C^m} \ .$$

($\imath\imath\imath$) Finally, from Betti & Verma's weight (BV) Eq. (23) we have:

$$z^m_{BV} = z \cdot \frac{\widetilde{C}^m}{\sum_{m=1}^{M} \widetilde{C}^m} \ .$$

We have computed, for each country and for each component, the unidimensional deprivation index by source (UDI) and their contribution values to the global deprivation using the CZ and BV systems of weights (these values are presented in Table 2).

The weight values enable the poverty lines to be computed by sources *via* those three systems of weights (see Tables 3 and 4).

[11] It is important to notice that for all Scandinavian countries data is collected from registers.

[12] Let $HM14_+$ be the number of households members aged 14 and over and $\mathbf{HM13}_-$ the number of households members aged 13 or less at the end of income reference period. The equivalent household size $= 1 + 0.5 \times (HM14_+ - 1) + 0.3 \times HM13_-$.

[13] In Appendix 2, the reader will find the construction of the s_i^m's.

Table 2: Weights used to calculate the income sources poverty lines

Country	Year 2006				Year 2007			
	Method	y_{ri}	y_{ab}	y_w	Method	y_{ri}	y_{ab}	y_w
Denmark	UDI	0.4367	0.4281	0.4137	UDI	0.4172	0.4254	0.4179
	CZ	0.3319	0.3332	0.3349	CZ	0.3337	0.3327	0.3336
	BV	0.1542	0.4936	0.3523	BV	0.1693	0.4330	0.3978
Finland	UDI	0.3923	0.4569	0.4479	UDI	0.3786	0.4591	0.4470
	CZ	0.3384	0.3299	0.3317	CZ	0.3389	0.3294	0.3317
	BV	0.1441	0.5210	0.3350	BV	0.1356	0.5333	0.3311
Iceland	UDI	0.4802	0.4603	0.4144	UDI	0.3683	0.4614	0.4102
	CZ	0.3278	0.3324	0.3398	CZ	0.3374	0.3273	0.3353
	BV	0.2537	0.4118	0.3345	BV	0.2231	0.4667	0.3102
Norway	UDI	0.4703	0.4523	0.4374	UDI	0.3823	0.4519	0.4348
	CZ	0.3299	0.3337	0.3364	CZ	0.3377	0.3297	0.3326
	BV	0.2013	0.4171	0.3816	BV	0.1678	0.4671	0.3651
Sweden	UDI	0.4687	0.4610	0.4412	UDI	0.3649	0.4605	0.4390
	CZ	0.3310	0.3326	0.3364	CZ	0.3386	0.3287	0.3327
	BV	0.3985	0.4352	0.1664	BV	0.1490	0.4542	0.3967

UDI: Unidimensional Deprivation Index by each income source.
CZ: Contribution values calculated using the system of weight proposed by Cerioli and Zani (1990).
BV: Contribution values calculated using the system of weight proposed by Betti and Verma (1998).

Table 3: Income sources poverty lines for 2006 (in €)

Countries	Method	z_{ri}	z_{ab}	z_w	z
Denmark	UDI	4644.70	4553.23	4400.07	
	CZ	4513.18	4530.85	4553.97	13598.00
	BV	2096.60	6711.30	4790.10	
Finland	UDI	3307.22	3851.82	3775.95	
	CZ	3700.40	3607.46	3627.14	10935.00
	BV	1575.58	5696.57	3662.86	
Iceland	UDI	6054.51	5803.61	5224.88	
	CZ	5599.81	5678.39	5804.80	17083.00
	BV	4333.96	7034.78	5714.26	
Norway	UDI	5763.94	5543.34	5360.72	
	CZ	5498.77	5562.11	5607.12	16668.00
	BV	3355.27	6952.22	6360.51	
Sweden	UDI	3637.05	3577.30	3423.65	
	CZ	3521.18	3538.20	3578.62	10638.00
	BV	4238.82	4629.19	1769.99	

UDI: Unidimensional Deprivation Index by each income source.
CZ: Contribution values calculated using the system of weight proposed by Cerioli and Zani (1990).
BV: Contribution values calculated using the system of weight proposed by Betti and Verma (1998).

Table 4: **Income sources poverty lines for 2007 (in €)**

Countries	Method	z_{ri}	z_{ab}	z_w	z
Denmark	UDI	4635.04	4726.14	4642.82	14004.00
	CZ	4673.13	4659.13	4671.73	
	BV	2370.64	6063.13	5570.23	
Finland	UDI	3272.34	3968.12	3863.54	11104.00
	CZ	3763.15	3657.66	3683.20	
	BV	1505.70	5921.76	3676.53	
Iceland	UDI	5116.52	6409.88	5698.60	17225.00
	CZ	5811.72	5637.74	5775.54	
	BV	3842.90	8038.91	5343.20	
Norway	UDI	5198.86	6145.34	5912.80	17257.00
	CZ	5827.69	5689.63	5739.68	
	BV	2895.72	8060.74	6300.53	
Sweden	UDI	3212.64	4054.32	3865.03	11132.00
	CZ	3769.30	3659.09	3703.62	
	BV	1658.83	5056.66	4416.51	

UDI: Unidimensional Deprivation Index by each income source.
CZ: Contribution values calculated using the system of weight proposed by Cerioli and Zani (1990).
BV: Contribution values calculated using the system of weight proposed by Betti and Verma (1998).

Each method verifies that $\sum_{m=1}^{M} z^m = z$, that is, the poverty lines by source of incomes z^m sum up to the overall poverty line z. Based on the three poverty lines, the multi-decomposition of the Sen index growth may be implemented.

4.3 Results

The poverty growth rates between 2006 and 2007 have been computed for all the Scandinavian countries. Table 5 indicates that the growth rate of the Sen index has decreased in all countries but with different intensities, going from -0.38457 to -0.06254 in Iceland and Finland, respectively.

Table 5: **Global growth rate of the Sen index**

Country	Growth rate
Denmark	-0.26365
Finland	-0.06254
Iceland	-0.38457
Norway	-0.10665
Sweden	-0.18336

The population is partitioned into male and female groups and the three types of poverty lines are decomposed into income sources. Accordingly, the growth rate of the Sen index can be explained by measuring the changes in poverty incidence, depth and inequality within male and female groups and across those two groups along the three different income sources.

Figures 1 to 5 expose the contribution levels of each component of the Sen index to its total growth rate in all Scandinavian countries:

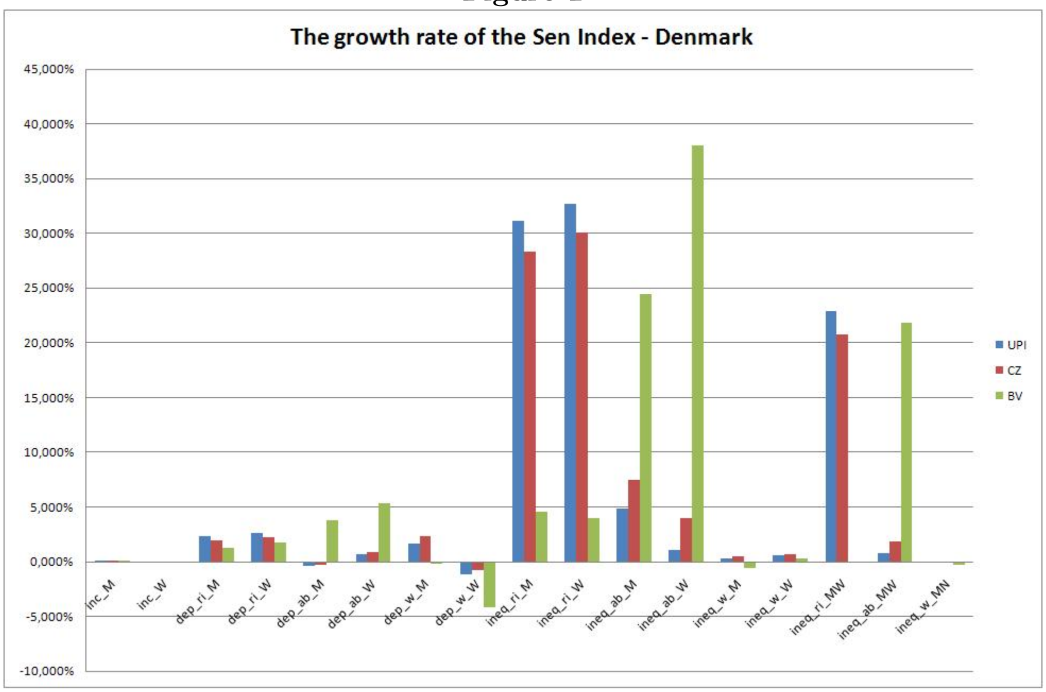
• 'inc', 'dep' and 'ineq' are the incidence change (between 2006 and 2007), depth change, and inequality change of the Sen index growth, respectively;

• 'ri', 'ab' and 'w' are respectively the incomes from rents and interests, from allowances and benefits, and from wages sources;

• men and women subgroups are indexed by the letters M and W.

For example, 'inq_w_M' corresponds to the 'change in the incomes from wage inequalities within the men's subgroup'.[14]

Figure 1



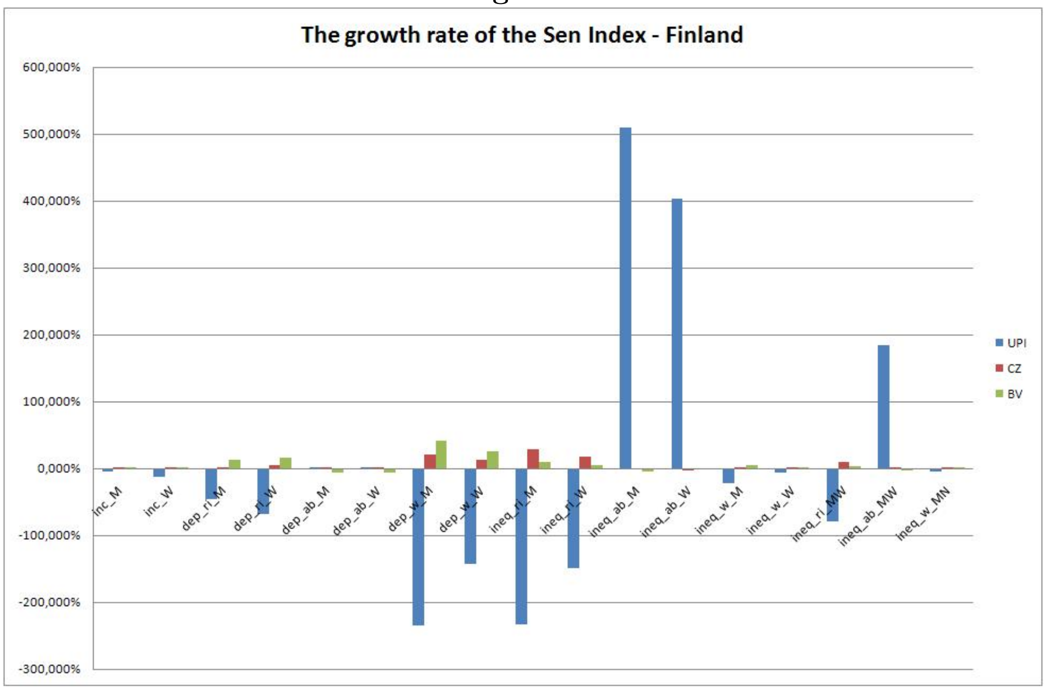
The poverty growth in Denmark (Figure 1) is principally explained by the inequality component, mainly by the changes in incomes from allowances and benefits within the men's group and between men and women, and by the changes in incomes from wages within the men's group. Concerning the depth term, only the *BV* method presents 'dep_ab_M', 'dep_ab_W' and 'dep_w_W' as the three most explicative components of the global poverty growth.

In Finland (Figure 2), both incidence and inequality terms have important contribution levels to explain the Sen index growth. The most explicative components are the incidence change of the women's group and the inequalities change from incomes, from allowances and benefits, and from wages in men's group. Even if the global contribution level of the second term is the same when using the three different methods for computing poverty lines by income sources, each

[14]These contribution values are depicted in Tables 10 to 29 in Appendix 3.

method proposes different contribution levels for each component. Then, the most explicative couples 'source/group' to changes in overall poverty are 'dep_ri_W' and 'dep_ab_M' when using the UDI method, 'dep_ri_W' when using the CZ method, and finally, 'dep_ri_W' and 'dep_ri_M' when using the BV method.

Figure 2



The inequality and incidence terms have the most important contribution levels to explain the poverty growth in Iceland and Norway (see Figures 3 and 4, respectively). The most explicative components are the inequalities in 'ab' and 'w' income sources in the men's group, and the incidence in men and women's groups.

Analysing the second term in Iceland, the depth in the rent and interest income source in the women's group appears as one of the most explicative component using either UDI, CZ or the BV method. On the other hand, Norway's growth rate is mainly explained by the 'dep_ri_W' and 'dep_w_W' components when using the UDI method; 'dep_ab_W' and 'dep_w_W' components when using the CZ method; and 'dep_ri_W' and 'dep_ab_W' components when using the BV method.

Figure 3

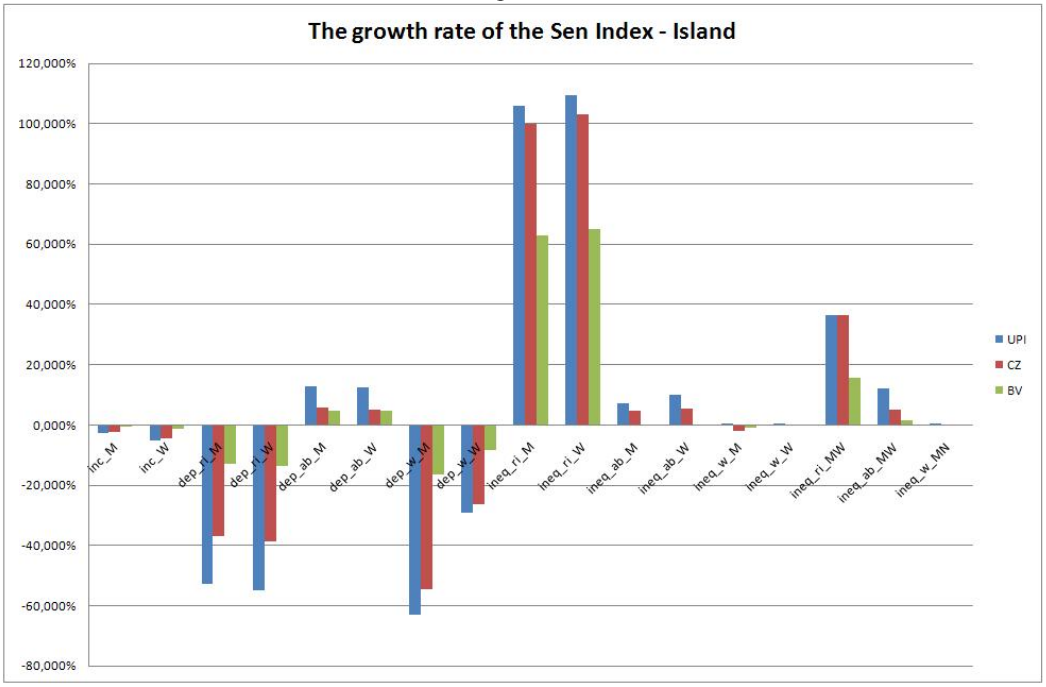


Figure 4

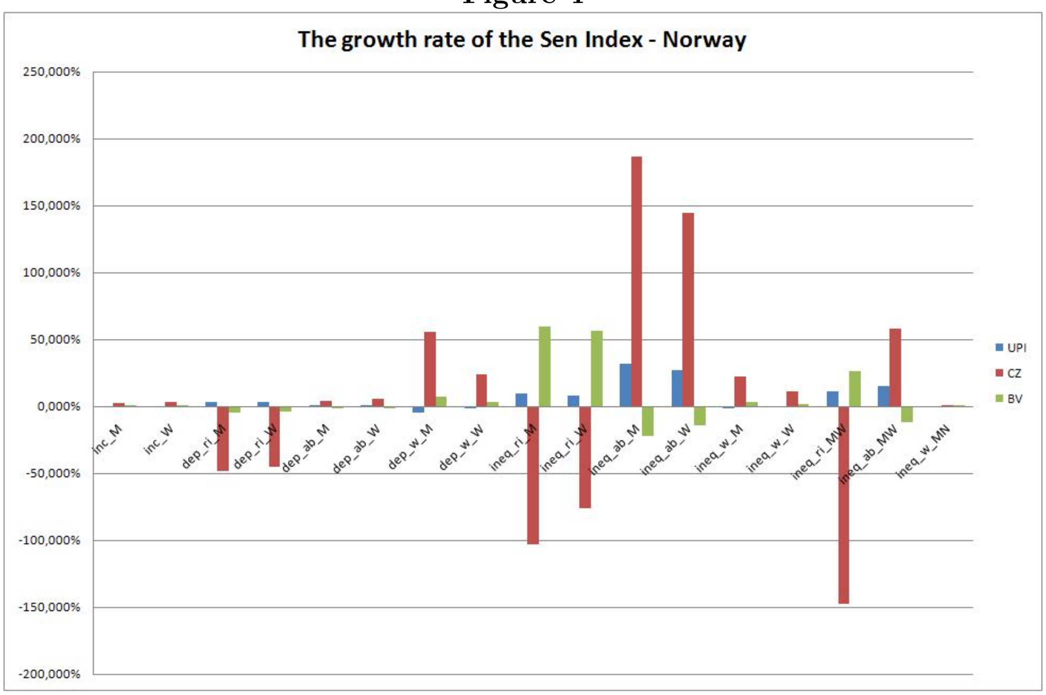


Figure 5

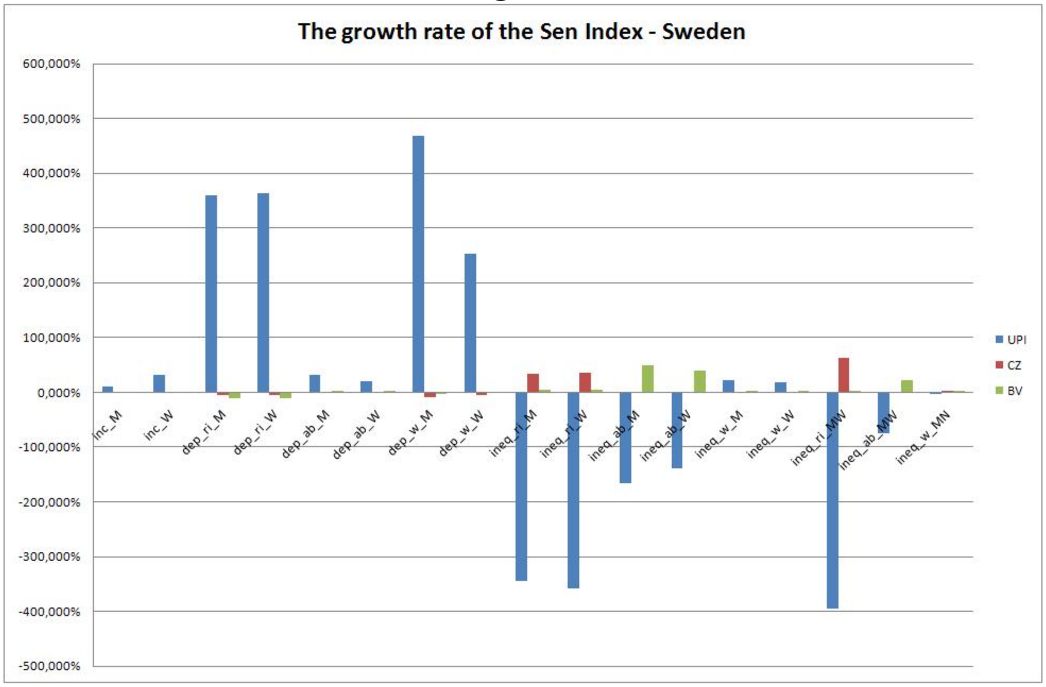


Figure 5 shows that the poverty growth in Sweden is explained by the incidence growth in the men's group component. The depth growth indicates that even if the most explicative components are rents, interests and wages (in both groups) using either UDI, CZ or BV, their contribution levels significantly vary along the three methods.

Finally, changes in poverty between 2006 and 2007 in all Scandinavian countries are principally explained by changes in the inequality term of the Sen index, in particular for rents and interest income sources, and for the women group.

5 Conclusion

In this paper, we show that the growth rate of the Sen index is multi-decomposable. The change in poverty intensity provides a quantitative evaluation of the contributions of the poverty growth of incidence, depth, and inequality components. Precisely, the method yields the contribution of each component, the contribution of each group, and the contribution of the inequalities in poverty within- and between groups, each term being decomposed in turn by the influence of the different income sources. We measure the contribution to the overall Gini index of the following combinations: "inequalities within group K / due to source m" and "inequalities between group $K-1$ and K / due to source m". This multi-decomposition can also be extended into the case where a parametric model can be used to describe the data generating process of poverty gap ratios. For that purpose, it is possible to gauge the impact of significant explanatory variables and specific social groups on overall poverty.

This methodology may contribute to open the way on new issues such as exploring the intimate interrelation between inequalities and poverty in a more general framework using fuzzy set theory (see e.g. Cheli and Lemmi (1995) or Dagum and Costa (2004)) by means of a decomposition technique relying on fuzzy set theory.

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6 Appendix 1: The multidimensional decomposition of poverty

The principle of the multidimensional poverty decomposition proposed by Chakravarty, Mukherjee and Ranade (1998) is depicted in Table 6 below.

Table 6: Structure of the poverty multi-decomposition

Dimensions → Groups ↓	Dimension 1	...	Dimension m	...	Dimension M	Total Groups
Poverty in group 1	$\mathcal{P}_1^1$	...	$\mathcal{P}_1^m$	...	$\mathcal{P}_1^M$	$\mathcal{P}_1$
⋮	⋮	...	...	...	⋮	⋮
Poverty in group k	$\mathcal{P}_k^1$	...	$\mathcal{P}_k^m$	...	$\mathcal{P}_k^M$	$\mathcal{P}_k$
⋮	⋮	...	...	...	⋮	⋮
Poverty in group K	$\mathcal{P}_K^1$	...	$\mathcal{P}_K^m$	...	$\mathcal{P}_K^M$	$\mathcal{P}_K$
Total Dimensions	$\mathcal{P}^1$	...	$\mathcal{P}^m$	...	$\mathcal{P}^M$	S

We observe a population of households with which we compute the global poverty, denoted by S, based on the observation of many socio-economic characteristics (dimensions). The global poverty is computed over several dimensions being income, health, education, etc., – dimensions are indexed by $m = 1, \ldots, M$. The poverty multi-decomposition technique depicted in Table 1 yields the amount of global poverty S as well as the contribution of all dimensions to S. These contributions are given in line "Total Dimensions" of Table 1: $\mathcal{P}^m$ for all $m = 1, \ldots, M$. For instance, based on the computation of these contributions, one can argue that health contributes with a 45% to the total amount of poverty S. It is worth mentioning that this property is not available for most indices we find in the literature, e.g., the Sen index. On the other hand, if the analyst is able to partition the population into k sub-populations ($k = 1, \ldots, K$) such as gender, races, or other social characteristics, then the multi-decomposition method aims at providing the contribution of the kth group ($k = 1, \ldots, K$) to the overall amount of poverty S. These contributions are given in column "Total Groups" of Table 1: $\mathcal{P}^k$ for all $k = 1, \ldots, K$. One can argue, for example, that men contribute with a 70% to the global poverty S. Again, this decomposition is not available for all existing indices. Importantly, the feature relying on the multi-decomposition concept consists in capturing all the cells of Table 1, i.e., to bring out the contribution of all the following couples: "Poverty in subgroup k and dimension m" to the global poverty S. These contributions are given in Table 1: $\mathcal{P}_k^m$ for all $k = 1, \ldots, K$ and $m = 1, \ldots, M$. This provides, for instance, a result such as "men's health contribute with a 60% to the global poverty".

The poverty multi-decomposition yields three results. The total poverty S is the aggregation of three types of contribution indices:

- S is the sum over all subgroup contributions $\mathcal{P}_k$ for all $k = 1, \ldots, K$;
- S is the sum over all dimension contributions $\mathcal{P}^m$ for all $m = 1, \ldots, M$,
- S is the sum over all couples "subgroups and dimensions" contributions $\mathcal{P}_k^m$ for all $k = 1, \ldots, K$ and for all $m = 1, \ldots, M$.

7 Appendix 2a: The multi-decomposition of the Gini index

Let us divide the total economic surface into K groups, K_k, of size n_k $(k = 1, ..., h, ..., K)$. The multi-decomposition of the Gini index of the poverty gaps ratios x_{ik}^m for all $m = 1, \ldots, M$ is computed as follows:

$$G = \sum_{m=1}^{M} \frac{\sum_{k=1}^{K} \sum_{i=1}^{n_k} \sum_{r=1}^{n_k} \left(x_{im}^k + x_{rm}^k - 2x_{ir,m}^{*k} \right)}{2n^2 \overline{x}_p}$$
$$+ \sum_{m=1}^{M} \frac{2 \sum_{k=2}^{K} \sum_{h=1}^{k-1} \sum_{i=1}^{n_k} \sum_{r=1}^{n_h} \left(x_{im}^k + x_{rm}^h - 2x_{ir,m}^{*kh} \right)}{2n^2 \overline{x}_p} ,$$

where $x_{ir,m}^{*k}$ is an operator that takes the mth contribution of the minimum between x_i^k and x_r^k; and $x_{ir,m}^{*kh}$ is an operator that takes the mth contribution of the minimum between x_i^k and x_r^h.

Consequently, it is possible to implement the couples "within-group k/component m" (contributions $G_{k,k}^m$ in Table 7 below) and "between-group k and h/component$m$" (contributions $G_{k,h}^m$ in Table 7 below). In other words, the differences in income inequalities within and between groups are determined by the M explanatory income sources.

The intuition of this technique is presented in the following table.

Table 7: Structure of the multi-decomposition of the Gini index

Sources → Groups ↓	Source 1	...	Source m	...	Source M	Total Groups
Inequality in group 1	$G_{1,1}^1$	...	$G_{1,1}^m$	...	$G_{1,1}^M$	$\mathcal{C}_{1,1}$
⋮	⋮	...	...	...	⋮	⋮
Inequality in group K	$G_{K,K}^1$	...	$G_{K,K}^m$	...	$G_{K,K}^M$	$\mathcal{C}_{K,K}$
Inequality between groups 1 & 2	$G_{1,2}^1$	...	$G_{1,2}^m$	...	$G_{1,2}^M$	$\mathcal{C}_{1,2}$
⋮	⋮	...	...	...	⋮	⋮
Inequality between groups $K-1$ & K	$G_{K-1,K}^1$	...	$G_{K-1,K}^m$	...	$G_{K-1,K}^M$	$\mathcal{C}_{K-1,K}$
Total Sources	$\mathcal{C}^1$	...	$\mathcal{C}^m$	...	$\mathcal{C}^M$	G

Prior to the proposal of this multi-decomposition approach, researchers often analyse the margins of Table 7: either the contributions of source m to the overall inequality (see $\mathcal{C}^m$ in line "Total Sources") or the contributions to G from inequalities in groups $1, \ldots, K$ (e.g., $\mathcal{C}_{1,1}, \ldots, \mathcal{C}_{K,K}$) and inequalities between any pair in all K groups (see $\mathcal{C}_{1,2}, \ldots, \mathcal{C}_{K-1,K}$) respectively in column "Total Groups"). But instead of looking only at the margins of Table 7, the multi-decomposition approach yields $\left(K + \frac{K(K-1)}{2}\right) M$ sub-indices. For example:

• the contribution to G issued from the inequality in source M (e.g. wages) of group K is:

$$G_{K,K}^M = \frac{\sum_{i=1}^{n_K} \sum_{r=1}^{n_K} \left(x_{iM}^K + x_{rM}^k - 2x_{ir,M}^{*K} \right)}{2n^2 \overline{x}_p} ,$$

• the contribution to G issued from the inequality in source 1 (e.g. transfers) between groups $K-1$ and K is:

$$G^1_{K-1,K} = \frac{\sum_{i=1}^{n_K}\sum_{r=1}^{n_{K-1}}\left(x^K_{iM} + x^{K-1}_{rM} - 2x^{*K,K-1}_{ir,M}\right)}{2n^2\overline{x}_p}\ .$$

There are many reasons that motivate the use of the multi-decomposition technique. The sub-indices are not Gini indices but contribution indices, which satisfy the following properties. Remember that $\mathcal{C}^m$ denote the contribution of the mth source to the Gini index. Let $\mathcal{C}^m_.$ be the same contribution computed either on within-group or between-group inequalities.

($\imath$) If the distributions of the q income sources are q replications (say q identical variables), then: $\mathcal{C}^1_. = \ldots = \mathcal{C}^m_. = \ldots = \mathcal{C}^M_. \Longrightarrow G = q \cdot \mathcal{C}^m_.$.

($\imath\imath$) If the q source distributions are equally distributed, then: $\mathcal{C}^1_. = \ldots = \mathcal{C}^m_. = \ldots = \mathcal{C}^M_. = 0 \Longrightarrow G = 0$.

($\imath\imath\imath$) $\mathcal{C}^m_. \in \mathbb{R}$. This means that some sources (e.g. transfers) may diminish the overall inequality, i.e. when $\mathcal{C}^m_. < 0$.

($\imath v$) Let x^1 and x^2 be the distributions of source 1 and 2, respectively. If $x^2 = \lambda x^1$, $\lambda > 0$, then $\mathcal{C}^2_. = \lambda \mathcal{C}^1_.$.

8 Appendix 2b: Proof of Proposition 3.5

As can be seen in S_1, the variables do not depend on $m \in \{1, \ldots, M\}$, thus S_1 respect (PLD). The term S_2 depends on $\overline{x}_p^{m(k)}$, which is the average poverty gap ratio of source m in group k. It is clear that changing the poverty line z^m imply a modification of the poverty gap ratios $(z^m - y^m_{ik})/z$, which in turns imply a modification of $\overline{x}_p^{m(k)}$. About S_3, following the multi-decomposition of the Gini index of the poverty gap ratios (Appendix 1a), we get:

$$\begin{aligned} G = \sum_{m=1}^{M} & \frac{\sum_{k=1}^{K}\sum_{i=1}^{n_k}\sum_{r=1}^{n_k}\left(\frac{z^m - y^k_{im}}{z} + \frac{z^m - y^k_{rm}}{z} - 2\frac{z^m - y^{*k}_{ir,m}}{z}\right)}{2n^2\overline{x}_p} \\ & + \sum_{m=1}^{M}\frac{2\sum_{k=2}^{K}\sum_{h=1}^{k-1}\sum_{i=1}^{n_k}\sum_{r=1}^{n_h}\left(\frac{z^m - y^k_{im}}{z} + \frac{z^m - y^h_{rm}}{z} - 2\frac{z^m - y^{*kh}_{ir,m}}{z}\right)}{2n^2\overline{x}_p}. \end{aligned}$$

As can be seen in the previous equation, it is possible to neutralize the z^m's, that is:

$$\begin{aligned} G = \sum_{m=1}^{M} & \frac{\sum_{k=1}^{K}\sum_{i=1}^{n_k}\sum_{r=1}^{n_k}\left(-y^k_{im} - y^k_{rm} + 2y^{*k}_{ir,m}\right)}{z \cdot 2n^2\overline{x}_p} \\ & + \sum_{m=1}^{M}\frac{2\sum_{k=2}^{K}\sum_{h=1}^{k-1}\sum_{i=1}^{n_k}\sum_{r=1}^{n_h}\left(-y^k_{im} - y^h_{rm} + 2y^{*kh}_{ir,m}\right)}{z \cdot 2n^2\overline{x}_p}\ . \end{aligned}$$

In consequence, the within-group contributions $G^m_{k,k}$ (for all $k = 1, \dots, K$ and $m = 1, \dots, M$) and the between-group ones $G^M_{k,h}$ (for all $k, h = 1, \dots, K$ and $m = 1, \dots, M$) are independent from the values of the z^m's (and dependent on z). S_3 respects (PLD).

9 Appendix 3: The degree of membership

In order to compute the direct measures of income deprivation for each component we define in Table 8 the degree of membership of each household to the mth component, that is, s_i^m:

Table 8: Degree of membership for each income component

Income component	Degree of membership
If $y_i^{me} \leq y_{O.6M}^{me}$	1
If $y_{O.6M}^{me} < y_i^{me} \leq y_M^{me}$	$(y_M^{me} - y_i^{me})/(y_M^{me} - y_{O.6M}^{me})$
If $y_i^{me} > y_M^{me}$	0

y_M^{me} is median equivalent income for the mth component.
$y_{O.6M}^{me}$ is 60% of the income median for the mth component.

Table 9 below shows the income median and the 60% of the income median values for all countries in 2006 and 2007.These values provide the degree of membership of each income source.

Table 9: Income Median (in €)

Country	Year 2006				Year 2007			
	Value	y_{ri}	y_{ab}	y_w	Value	y_{ri}	y_{ab}	y_w
Denmark	y_M^{me}	2462.81	1595.31	16915.86	y_M^{me}	2617.47	1597.86	17572.21
	$y_{O.6M}^{me}$	1477.69	957.18	10149.52	$y_{O.6M}^{me}$	1570.48	958.71	10543.33
Finland	y_M^{me}	2735.33	2057.50	9034.67	y_M^{me}	3133.33	1988.00	9820.67
	$y_{O.6M}^{me}$	1641.20	1234.50	5420.80	$y_{O.6M}^{me}$	1880.00	1192.80	5892.40
Iceland	y_M^{me}	156.57	1372.56	17262.94	y_M^{me}	5771.06	1179.285	17319.22
	$y_{O.6M}^{me}$	93.94	823.54	10357.76	$y_{O.6M}^{me}$	3462.64	707.57	10391.53
Norway	y_M^{me}	81.99	2219.67	15061.09	y_M^{me}	669.72	2240.41	16389.84
	$y_{O.6M}^{me}$	49.19	1331.80	9036.65	$y_{O.6M}^{me}$	401.83	1344.24	9833.90
Sweden	y_M^{me}	61.38	2254.49	10069.07	y_M^{me}	3346.45	2449.89	10294.61
	$y_{O.6M}^{me}$	36.83	1352.70	6041.44	$y_{O.6M}^{me}$	2007.87	1469.93	6176.77

Source: Eurostat, EU-SILC 2006 and 2007.

10 Appendix 4: The growth rate of the Sen index - First, Second and Third term

10.1 Denmark

Table 10: The growth rate of Sen's Index - Denmark - First Term

Value	Inc_M	Inc_W
Absolute value (AV)	-0.00084	0.00215
Relative contribution (RC)	(0.32)	(-0.81)

Table 11: The growth rate of Sen's Index - Denmark - Second Term

Method	Value	Dep_ri_M	Dep_ri_W	Dep_ab_M	Dep_ab_W	Dep_w_M	Dep_w_W
UDI	**AV**	-0.01137	-0.01478	-0.00152	0.00145	0.00395	0.00678
	RC	(4.29)	(5.58)	(0.57)	(-0.55)	(-1.49)	(-2.56)
CZ	**AV**	-0.00658	-0.00718	-0.00269	-0.00051	0.00034	0.00114
	RC	(2.50)	(2.72)	(1.02)	(0.20)	(-0.13)	(-0.43)
BV	**AV**	0.00002	-0.00048	-0.02692	-0.03638	0.01797	0.03031
	RC	(-0.01)	(0.18)	(10.21)	(13.80)	(-6.82)	(-11.50)

Table 12: The growth rate of Sen's Index - Denmark - Third Term

Value	Ineq_ri_M	Ineq_ri_W	Ineq_ab_M	Ineq_ab_W	Ineq_w_M	Ineq_w_W
AV	-0.02504	-0.00004	-0.07824	0.00005	-0.07758	0.00003
RC	(9.45)	(0.01)	(29.53)	(-0.02)	(9.28)	(-0.01)

Table 13: The growth rate of Sen's Index - Denmark - Third Term - *continuation*

Value	Ineq_ri_MW	Ineq_ab_MW	Ineq_w_MW
AV	-0.01148	-0.03146	-0.02571
RC	(4.33)	(11.87)	(9.70)

10.2 Finland

Table 14: The growth rate of Sen's Index - Finland - First Term

Value	Inc_M	Inc_W
AV	-0.00803	-0.02087
RC	(12.84)	(33.37)

Table 15: The growth rate of Sen's Index - Finland - Second Term

Methods	Value	Dep_ri_M	Dep_ri_W	Dep_ab_M	Dep_ab_W	Dep_w_M	Dep_w_W
UDI	**AV**	-0.00830	-0.01888	0.00784	0.00298	0.00406	0.00864
	RC	(13.26)	(0.18)	(-12.54)	(-4.76)	(-6.50)	(-13.81)
CZ	**AV**	-0.00499	-0.01394	0.00561	-0.00041	0.00299	0.00708
	RC	(7.97)	(22.28)	(-8.97)	(0.65)	(-4.77)	(-11.33)
BV	**AV**	-0.01007	-0.01785	0.01205	0.00601	0.00163	0.00458
	RC	(16.11)	(28.53)	(-19.27)	(-9.61)	(-2.60)	(-7.32)

Table 16: The growth rate of Sen's Index - Finland - Third Term

Value	Ineq_ri_M	Ineq_ri_W	Ineq_ab_M	Ineq_ab_W	Ineq_w_M	Ineq_w_W
AV	-0.00301	0.00000	-0.00940	0.00001	-0.00933	0.00000
RC	(4.81)	(0.01)	(15.04)	(-0.01)	(14.91)	(-0.01)

Table 17: The growth rate of Sen's Index - Finland - Third Term - *continuation*

Value	Ineq_ri_MW	Ineq_ab_MW	Ineq_w_MW
AV	-0.00138	-0.00378	-0.00309
RC	(2.21)	(6.05)	(4.94)

10.3 Iceland

Table 18: The growth rate of Sen's Index - Iceland - First Term

Value	Inc_M	Inc_W
AV	-0.04411	-0.08473
RC	(11.47)	(22.03)

Table 19: The growth rate of Sen's Index - Iceland - Second Term

Methods	Value	Dep_ri_M	Dep_ri_W	Dep_ab_M	Dep_ab_W	Dep_w_M	Dep_w_W
UDI	**AV**	-0.06346	-0.17368	0.02896	0.04115	0.01488	0.08222
	RC	(16.50)	(45.16)	(-7.53)	(-10.70)	(-3.87)	(-21.38)
CZ	**AV**	-0.04484	-0.12532	0.01918	0.01343	0.00604	0.06158
	RC	(11.66)	(32.59)	(-4.99)	(-3.49)	(-1.57)	(-16.01)
BV	**AV**	-0.05340	-0.15696	0.03286	0.05975	0.00092	0.04690
	RC	(13.89)	(40.81)	(-8.54)	(-15.54)	(-0.24)	(-12.20)

Table 20: The growth rate of Sen's Index - Iceland - Third Term

Value	Ineq_ri_M	Ineq_ri_W	Ineq_ab_M	Ineq_ab_W	Ineq_w_M	Ineq_w_W
AV	-0.01858	-0.00003	-0.05824	0.00003	-0.05780	0.00002
RC	(4.83)	(0.01)	(15.14)	(-0.01)	(15.03)	(-0.01)

Table 21: The growth rate of Sen's Index - Iceland - Third Term - *continuation*

Value	Ineq_ri_MW	Ineq_ab_MW	Ineq_w_MW
AV	-0.00856	-0.02347	-0.01919
RC	(2.23)	(6.10)	(4.99)

10.4 Norway

Table 22: The growth rate of Sen's Index - Norway - First Term

Value	Inc_M	Inc_W
AV	-0.01861	-0.02135
RC	(17.45)	(20.02)

Table 23: The growth rate of Sen's Index - Norway - Second Term

Methods	Value	Dep_ri_M	Dep_ri_W	Dep_ab_M	Dep_ab_W	Dep_w_M	Dep_w_W
UDI	**AV**	-0.02184	-0.04582	0.00582	0.00798	0.00631	0.03088
	RC	(20.48)	(42.97)	(-5.46)	(-7.48)	(-5.91)	(-28.96)
CZ	**AV**	-0.00721	-0.00817	-0.00170	-0.01200	-0.00080	0.01321
	RC	(6.76)	(7.6)	(1.60)	(11.25)	(0.75)	(-12.39)
BV	**AV**	-0.01544	-0.04126	0.01105	0.02924	-0.00532	0.00507
	RC	(14.48)	(38.69)	(-10.36)	(-27.42)	(4.99)	(-4.75)

Table 24: The growth rate of Sen's Index - Norway - Third Term

Value	Ineq_ri_M	Ineq_ri_W	Ineq_ab_M	Ineq_ab_W	Ineq_w_M	Ineq_w_W
AV	-0.00502	-0.00001	-0.01569	0.00001	-0.01555	0.00001
RC	(4.71)	(0.01)	(14.71)	(-0.01)	(14.59)	(-0.01)

Table 25: The growth rate of Sen's Index - Norway - Third Term - *continuation*

Value	Ineq_ri_MW	Ineq_ab_MW	Ineq_w_MW
AV	-0.00230	-0.00631	-0.00516
RC	(2.16)	(5.92)	(4.83)

10.5 Sweden

Table 26: The growth rate of Sen's Index - Sweden - First Term

Value	Inc_M	Inc_W
AV	-0.02302	-0.06504
RC	(12.55)	(35.47)

Table 27: The growth rate of Sen's Index - Sweden - Second Term

Methods	Value	Dep_ri_M	Dep_ri_W	Dep_ab_M	Dep_ab_W	Dep_w_M	Dep_w_W
UDI	**AV**	-0.04636	-0.12766	0.02100	0.04585	0.02650	0.05653
	RC	(25.29)	(69.62)	(-11.45)	(-25.01)	(-14.45)	(-30.83)
CZ	**AV**	-0.02497	-0.08816	0.00949	0.02554	0.01661	0.03735
	RC	(13.62)	(48.08)	(-5.18)	(-13.93)	(-9.06)	(-20.37)
BV	**AV**	-0.11453	-0.25565	0.02131	0.03671	0.09435	0.19367
	RC	(62.46)	(139.42)	(-11.62)	(-20.02)	(-51.46)	(-105.62)

Table 28: The growth rate of Sen's Index - Sweden - Third Term

Value	Ineq_ri_M	Ineq_ri_W	Ineq_ab_M	Ineq_ab_W	Ineq_w_M	Ineq_w_W
AV	-0.00713	-0.00001	-0.02231	0.00001	-0.02213	0.00001
RC	(3.89)	(0.01)	(12.17)	(-0.01)	(12.07)	(0.00)

Table 29: The growth rate of Sen's Index - Sweden - Third Term - *continuation*

Value	Ineq_ri_MW	Ineq_ab_MW	Ineq_w_MW
AV	-0.00328	-0.00898	-0.00734
RC	(1.79)	(4.90)	(4.00)