GREDI

Groupe de Recherche en Économie et Développement International

Cahier de Recherche / Working Paper
11-05

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Stéphane Auray

Aurélien Eyquem

UNIVERSITÉ DE SHERBROOKE

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Stéphane Auray[†] Aurélien Eyquem[‡]

First version: December 2009
This version: January 2011

Abstract

This paper examines if taking into account changes in the number of producers, or equivalently changes in the product variety space over the business cycle, helps to understand and replicate international business cycle facts. To this end, we develop a two-country model in which the economy is driven by real and monetary policy shocks. It is characterized by an endogenous number of firms and varieties, sticky prices and financial markets incompleteness. We show that these features are crucial to reproduce international business cycle statistics. We also evaluate the welfare implications of various monetary policies and highlight the importance for monetary policymakers to respond moderately to output fluctuations.

Keywords: International Business Cycles, Endogenous Entry, Financial Markets Incompleteness, Sticky Prices, Monetary Policy, Welfare.

JEL Class.: E51, E58, F36, F41.

*We would like to thank the editors Fabio Ghironi as well as Benn Steil, and two anonymous referees for insightful comments. Aurélien Eyquem gratefully acknowledges the Fondation Banque de France for financial support.

†CNRS – THEMA (UMR 8184), EQUIPPE (EA 4018) – Universités Lille Nord de France (ULCO), Université de Sherbrooke (GREDI) and CIRPEE, Canada. Email: stephane.auray@gmail.com.

‡GATE LSE, Université de Lyon, and Ecole Normale Supérieure de Lyon, France, and GREDI, Université de Sherbrooke, Canada. Email: aurelien.eyquem@ens-lyon.fr.

1 Introduction

This paper describes a set of updated international business cycles facts and, observing that the number of producers varies over the business cycle[1], examines if taking into account changes in the number of producers as well as changes in the product variety space may provide a deeper understanding of international economic fluctuations. The paper presents a two-country model with endogenous entry, sticky prices and financial market incompleteness. We exploit this setup (*i*) to highlight the role of endogenous entry as well as financial markets incompleteness and sticky prices to reproduce international business cycle stylized facts and (*ii*) to deliver monetary policy implications in this context.

The analysis focuses on the properties of the international business cycles that the open-economy literature considers as key facts. Some of those facts are considered as puzzles since standard open-macroeconomic frameworks are unsuccessful in explaining and/or reproducing them. Using quarterly data on OECD countries between 1975 and 2004, we obtain similar results as other studies on international business cycles.[2] Empirical business cycles statistics are such that (*i*) imports and exports are positively correlated with GDP, (*ii*) the correlation is higher for imports than for exports, and (*iii*) net exports are counter-cyclical. Data also show that (*iv*) the real exchange rate is about 3 times more volatile than GDP and highly persistent, a fact most commonly referred to as the real exchange rate persistence puzzle, (*v*) cross-country correlations of outputs are higher than those of consumptions, which is commonly referred to as the output-consumption anomaly, and (*vi*) the real exchange rate is positively correlated with exports, and net exports, and negatively with imports. Finally, (*vii*) relative consumptions are weakly and negatively correlated with the real exchange rate, which contradicts the prediction of complete asset markets models of a positive and near unitary correlation. In the literature, this puzzling feature of the data is often called the Backus-Smith puzzle, from Backus and Smith's (1993) article that first documented it.

Not all these facts are puzzles. For instance, facts (*i*), (*ii*) and (*vi*) are conform both to common wisdom and the predictions of standard models. However, the puzzle arises because some facts relate to each other and reproducing one aspect of the data, such as the counter-cyclicality of net exports, is often achieved at the expense of not matching other facts. As

[1] See Bernard, Jensen, Redding and Schott (2009), Broda and Weinstein (2007) and Bilbiie, Ghironi and Melitz (2009).

[2] See for instance Backus and Kehoe (1992), Backus, Kehoe and Kydland (1992), Backus, Kehoe and Kydland (1993) or more recently Engel and Wang (2011).

most of these puzzles intuitively relate to each other, the literature seeks for explanations that may help matching jointly the highest number of international business cycle statistics. For example, sources of sluggishness in the dynamics of the real exchange rate may well explain the counter-cyclicality of net exports, as the external adjustment of the economy may be slower than the internal adjustment. Another example is the matching of the real exchange rate-consumption anomaly: theoretical explanations of this puzzle usually rely on stronger wealth effects than substitution effects after changes in the real exchange rate, which may at the same time explain why net exports are counter-cyclical.[3] A careful description of trade flows and their determinants is thus crucial to analyze the transmission of international business cycles and the properties of relative prices. For instance, Baxter (1995) and Engel and Wang (2011) respectively show that trade in capital goods, and trade in durable goods crucially affect the cross-country transmission of aggregate country-specific shocks.

There has been a recent and growing literature arguing that endogenous entry or changes in the variety space may help understand the propagation of economic fluctuations in closed economies (see Bénassy (1996), Bilbiie et al. (2009), Broda and Weinstein (2007), Etro and Colciago (2010), Wang and Wen (2007) and many others). Transposing this assumption in an open economy allows for a new description of the dynamics of trade flows and their relation with relative prices (see Cook (2002), Ghironi and Melitz (2005) and more recently Zlate (2010)), and may change traditional views on the international propagation of shocks. Taking into account the extensive margin of both Gross Domestic Product (GDP) and trade flows has thus been shown to improve business cycle properties of international macroeconomic models along various dimensions.[4] However, all these models with endogenous entry share the assumption of flexible prices. In addition, a wide strand of the literature underlines that nominal rigidities as well as monetary policy and assumptions related to how export decisions are made or constrained (local/producer currency pricing, distribution costs) are key elements to understand both the volatility and the persistence of international relative prices and more generally the properties of international business cycles (see Benigno (2004), Chari, Kehoe and McGrattan (2002), and Corsetti et al. (2008) among others).

[3] It can be done by breaking the risk-sharing condition implied by complete asset markets and assuming low values of the elasticity of substitution or high persistence of fundamental shocks, as shown by Corsetti, Dedola and Leduc (2008).

[4] For the reader to get used to the terms in this literature, we recall that the extensive margin is the part of output driven by firm entries that create new varieties while the intensive margin corresponds to the increase production of existing varieties.

Considering endogenous entry, price stickiness and monetary policy in an integrated framework is not new (see Bilbiie, Ghironi and Melitz (2007)) but little has been done with regard to international economic fluctuations. This is precisely the aim of this paper: do endogenous entry and price stickiness provide relevant mechanisms to explain international business cycle facts? To contrast the precise importance of endogenous entry and price stickiness in reproducing those facts, we also explore the role of other macroeconomic assumptions, and in particular the role of financial markets incompleteness. This is actually not trivial. Indeed, the way properties of goods markets affect the international transmission of shocks are crucially affected by the structure of financial markets. For instance, Baxter (1995), Ghironi (2006), Kehoe and Perri (2002) and Selaive and Tuesta (2003) among many others, show that incompleteness in financial markets affects both the volatility of relative prices and their correlation with relative consumptions.

Based on these important empirical and modeling aspects, we investigate the ability of a standard two-country new-Keynesian model with endogenous entry affecting both the internal components of GDP and exports to replicate international business cycle stylized facts. The model incorporates the following features: monopolistic competition, sticky prices, endogenous monetary policy reactions and incomplete financial markets. Notice that endogenous entry acts as investment in standard international real business cycle models, which allows for a direct comparison with these models. Since all versions of the model deliver satisfactory national business cycles statistics, the analysis focuses on the ability of our model to reproduce international dimensions of the data. The paper highlights that taking the extensive margin into account allows to go a significant step further in this task.[5]

First, we show that a flexible-price version of the model (with real shocks only) performs quite well in terms of persistence but fails to deliver counter-cyclical net exports.[6] In addition, although the introduction of endogenous entry/varieties lowers the correlation of relative consumptions with the real exchange rate, a model with flexible prices does not solve the real exchange rate-consumption anomaly. Considering sticky prices and real shocks only,

[5] The issue of comparability between the observed data (that do not take into account the extensive margin of variables) and the simulated data, is solved by deflating variables with an appropriate price index (see Bilbiie et al. (2009) for an extensive discussion of this issue).

[6] Incorporating an endogenous traded sector, as in Ghironi and Melitz (2005), and above all assuming inelastic labor supply may lead to counter-cyclical net exports. However, the mechanisms leading to counter-cyclical net exports in our model are very different, especially because labor supply is endogenous and elastic to changes in wages.

the model fits much better with international business cycle statistics. It is able to reproduce the counter-cyclicality of net exports (together with pro-cyclical imports but counter-cyclical exports) and predicts a negative correlation between relative consumptions and the real exchange rate. The intuition behind the results relates to the overshooting of investment in the creation of new ventures under sticky prices. The latter is caused by the joint dynamics of monetary policy and inflation. For example, a positive domestic productivity shock enhances the international attractiveness of the domestic economy and new firms are massively created there. Because such a shock increases markups, the domestic economy experiences a disinflation, and the central bank lowers the nominal interest rate. Attracting international capital flows thus requires the nominal exchange rate to appreciate. Consequently, both net exports and the real exchange rate are counter-cyclical, which, since relative consumptions increase, yields a negative correlation between relative consumptions and the real exchange rate, together with counter-cyclical net exports. The joint presence of endogenous entry and price stickiness is thus crucial to match international business cycle facts. The introduction of monetary shocks allows to balance both the too negative correlation between relative consumptions and the real exchange rate, and the negative correlation of the real exchange rate with GDP, although at the cost of less overall persistence. Finally, a sticky prices version of the model incorporating both real and monetary shocks appears as a good compromise in terms of reproducing the observed properties of the international business cycles.

In a final section, we contrast the implications of the model for the conduct of monetary policy, with a special focus on the desirability of price stability. We use our model with real shocks only to evaluate the welfare losses of business cycles arising under various monetary policies. Welfare is evaluated using a second-order approximation of the life-time utility of both domestic and foreign households. First, when monetary policy reacts to output fluctuations, putting too much emphasis on price stability damages the welfare. The result builds on the joint counter-cyclicality of the real exchange rate and pro-cyclicality of hours worked. Because the real exchange is counter-cyclical, it appreciates when output and hours increase, which lowers the volatility of hours, and increases the welfare. Reacting too strongly to inflation thus reduces the intensity of the positive externality that a counter-cyclical real exchange rate exerts on hours. Second, when monetary policy does not respond to output, the real exchange rate becomes pro-cyclical due to the implied dynamics of real interest rates, and real exchange rate fluctuations are undesirable. In this case, the policy recommendation is to stabilize inflation fluctuations as much as possible. Notice however, that welfare is always higher when monetary policy responds to output than when it does not. Therefore, responding to output fluctuations is clearly welfare enhancing in our framework. Last but not least, the intensity of monetary policy responses to output fluctuations should not be too large. Large reactions of monetary policy to output fluctuations may indeed increase too much the volatility of investment, the volatility of varieties, and therefore the volatility of the extensive margin of welfare-based consumption, with negative consequences on welfare.

The remainder of the paper is organized as follows. Section 2 presents stylized facts about international business cycles in OECD countries. Section 3 builds a two-country model with monopolistic competition, sticky prices, incomplete financial markets and endogenous entry. Section 4 provides some insights about the dynamic properties of the model. Section 5 investigates the ability of our model to fit business cycle moments presented in Section 2. Section 6 investigates the welfare losses attached to monetary policies and provides some normative conclusions. Section 7 concludes.

2 Stylized facts

We start by reviewing national and international business cycles properties for the main OECD economies over the period 1975Q1–2007Q4. We use quarterly data from the OECD Economic Outlook 2008 for real aggregate consumption, real private gross fixed capital formation (investment), real GDP, hours worked, real exports and imports, net exports as a percentage of GDP, and the (trade weighted) real exchange rate. Quantities are taken in logs. The real exchange rate is converted to express the foreign price (or purchasing power) of one unit of foreign currency. Finally, series are detrended using a standard HP filter (with smoothing parameter $\lambda = 1600$).[7] Stylized facts are summarized in Tables 1 and 2.

As reported in other studies, all macroeconomic time series are persistent. Their autocorrelation is between 0.5 and 0.8. Most variables are positively correlated with their counterparts in the rest of the world.[8] Outputs are more strongly correlated with their counterparts in the rest of the world than consumptions. Consumption, investment and hours are pro-cyclical, but while the volatility of consumption is close to that of output, the volatility of investment is three times higher and the volatility of hours is 0.9 times that of output. Exports

[7] German series have also been smoothed to offset the consequences of the reunification.

[8] We build aggregate variables for the rest of the world by taking a weighted average of variables in OECD countries. In particular, relative consumption means consumption relative to the rest of the world.

Table 1: National facts (OECD averages).

	$\sigma(u_t)$	$\sigma(u_t)/\sigma(y_t)$	$\rho(u_t, u_{t-1})$	$\rho(u_t, u_{t-2})$	$\rho(u_t, y_t)$	$\rho(u_t, u_t^*)$
Variables (u)						
Consumption	1.45	1.01	0.79	0.65	0.68	0.46
Investment	4.48	3.10	0.78	0.64	0.72	0.51
Output	1.44	1.00	0.78	0.64	1.00	0.55
Hours	1.27	0.88	0.87	0.73	0.62	0.50
Exports	3.43	2.38	0.63	0.44	0.40	0.55
Imports	4.27	2.96	0.74	0.55	0.61	0.59
Net exports	0.88	0.61	0.67	0.47	−0.26	0.35
Real exch. rate	4.02	2.79	0.81	0.58	0.08	0.35

Table 2: International facts

	Correlation with the real exchange rate			
	Exports	Imports	Net exp.	Rel. cons.
Austria	0.24	−0.52	0.52	−0.32
Canada	0.34	0.11	0.31	0.22
Germany	0.57	0.23	0.40	−0.05
Spain	0.31	−0.49	0.51	−0.16
Finland	0.23	−0.17	0.38	−0.52
France	0.48	0.14	0.27	0.13
U.K.	0.23	0.14	0.10	0.01
Ireland	0.38	0.33	−0.12	0.39
Italy	0.46	−0.19	0.59	−0.36
Japan	0.33	0.20	0.06	0.28
Netherlands	0.47	0.19	0.38	−0.12
Norway	0.12	−0.09	0.12	0.12
New-Zealand	0.22	−0.28	0.39	−0.36
Portugal	0.40	0.03	0.20	−0.37
Sweden	0.34	−0.09	0.44	−0.22
USA	0.35	0.02	0.17	−0.14
Average	0.34	−0.03	0.30	−0.09

and imports are twice as volatile as output, persistent and pro-cyclical. The correlation of imports with output is higher (0.6) than that of exports (0.4). Net exports are less volatile than output and countercyclical. Finally, real exchange rates are roughly three times more volatile than output, highly persistent and mainly a-cyclical.

International facts reported in Table 2 also match those usually reported in the literature. Exports are positively correlated with the exchange rate (expressed as the price of one unit of foreign currency in terms of domestic currency). This is reasonable since an increase in the real exchange rate implies a depreciation of the domestic currency that usually increases the competitiveness of domestic exporters. By the same logic, imports are negatively correlated with exchange rates, although evidence is less clear-cut given the large heterogeneity among OECD countries. As a consequence, net exports are positively correlated with exchange rates. Finally, relative consumptions are not correlated or negatively correlated with the real exchange rate, suggesting that cross-country risk-sharing is quite low.[9]

3 The model

We consider a two-country model with endogenous entry modeled as in Bilbiie et al. (2007), sticky prices and incomplete financial markets. Economies are buffeted by two types of shocks: productivity and monetary policy shocks. Financial markets are incomplete: domestic households have access to both foreign and domestic bonds, whereas foreign households have access to local bonds only (see Benigno (2009)). In both countries, households also have access to a mutual fund of local firms.

3.1 Households

In each country, the number of households with infinite life is normalized to one. In the home country, the representative household maximizes the welfare index

$$\Upsilon_0 = E_0 \left[\sum_{t=0}^{\infty} \beta^t u\left(c_t, \ell_t\right) \right],$$

subject to the budget constraint

[9] This is true under the assumption that preferences are additively separable, since the risk-sharing condition states that relative marginal utilities evolve in line with the real exchange rate.

$$\begin{aligned} b_t + e_t a_t + p_{c,t}\left(v_t\left(n_t + n_{e,t}\right)x_t + c_t\right) &= \left(1+i_{t-1}\right)b_{t-1} + e_t\left(1+i^*_{t-1}\right)a_{t-1} \\ &\quad + p_{c,t}\left(d_t + v_t\right)n_t x_{t-1} + \left(1-\zeta_t\right)w_t \ell_t - p_{c,t} ac_t - tax_t, \end{aligned}$$

and the appropriate transversality condition.

In these expressions, β is the subjective discount factor, c_t is the consumption bundle chosen by the representative household and ℓ_t is the quantity of labor competitively supplied. The welfare-based Consumption Prices Index (CPI hereafter) in the domestic country in period t is denoted by $p_{c,t}$, w_t is the nominal wage, and ζ_t is a tax on labor income intended to correct distortions from the presence of monopolistic competition with endogenous labor supply in the economy. Households have access to three different assets in the home economy: mutual fund shares of domestic firms (x_t) and two nominal bonds, in quantity b_t and a_t, that pay nominal interest rates i_{t-1} and i^*_{t-1} between periods $t-1$ and t. Accessing foreign bonds requires the payment of transaction costs, denoted ac_t, to financial service providers. These costs take the following quadratic functional form:

$$ac_t = \frac{\Phi}{2} e_t \left(a_t - a\right)^2,$$

where a is the steady-state holding of foreign bonds by domestic households. The cost is paid in units of consumption goods.

In period t, the household determines the optimal fraction x_t of the mutual fund of firms to be held. v_t denotes the average value of the firms in period t and d_t the real amount of dividends. Finally, tax_t is a lump–sum transfer from the government.

First-order conditions for the domestic households are:

$$\begin{aligned} \beta E_t\left[\frac{u_{c,t+1}}{u_{c,t}}\frac{\left(1+i_t\right)}{\left(1+\pi_{c,t+1}\right)}\right] - 1 &= 0, \\ \beta E_t\left[\frac{e_{t+1}}{e_t \Psi_t}\frac{u_{c,t+1}}{u_{c,t}}\frac{\left(1+i^*_t\right)}{\left(1+\pi_{c,t+1}\right)}\right] - 1 &= 0, \\ v_t - \left(1-\delta\right)\beta E_t\left[\frac{u_{c,t+1}}{u_{c,t}}\left(d_{t+1} + v_{t+1}\right)\right] &= 0, \\ -\frac{u_{\ell,t}}{u_{c,t}} - \left(1-\zeta_t\right)\varpi_t \frac{p_{d,t}}{p_{c,t}} &= 0, \end{aligned}$$

where $\pi_{c,t} = \frac{p_{c,t}}{p_{c,t-1}} - 1$ is the aggregate CPI inflation rate, $p_{d,t}$ is the Producer Prices Index (PPI hereafter), $\varpi_t = \frac{w_t}{p_{d,t}}$ is the PPI-deflated real wage, and $\Psi_t \equiv 1+\Phi e_t (a_t - a)$. Combining Euler equations on domestic and foreign bonds, we get:

$$E_t \left[\frac{u_{c,t+1}}{(1+\pi_{c,t+1})} \left((1+i_t) - \frac{e_{t+1}}{e_t \Psi_t} (1+i_t^*) \right) \right] = 0,$$

which indicates that the expected uncovered interest rate parity does not hold, because of Ψ_t.[10]

Foreign households face the exact same program except that they are allowed to allocate wealth only on shares and local bonds:

$$\begin{aligned} a_t^* + p_{c,t}^* \left(v_t^* \left(n_t^* + n_{e,t}^* \right) x_t^* + c_t^* \right) &= \left(1 + i_{t-1}^*\right) a_{t-1}^* \\ &+ p_{c,t}^* \left(d_t^* + v_t^* \right) n_t^* x_{t-1}^* + \left(1 - \zeta_t^*\right) w_t^* \ell_t^* - tax_t^*. \end{aligned}$$

First-order conditions thus imply:

$$\begin{aligned} \beta E_t \left[\frac{u_{c^*,t+1}}{u_{c^*,t}} \frac{(1+i_t^*)}{\left(1+\pi_{c,t+1}^*\right)} \right] - 1 &= 0, \\ v_t^* - (1-\delta)\beta E_t \left[\frac{u_{c^*,t+1}}{u_{c^*,t}} \left(d_{t+1}^* + v_{t+1}^* \right) \right] &= 0, \\ -\frac{u_{\ell^*,t}}{u_{c^*,t}} - (1-\zeta_t^*)\varpi_t^* \frac{p_{d,t}^*}{p_{c,t}^*} &= 0. \end{aligned}$$

The aggregate consumption of the representative domestic household at time t is defined over a continuum of goods Ω, with a preference for local goods, modeled as a home bias coefficient $(1-\alpha)$, where $\alpha < \frac{1}{2}$.[11] National varieties are imperfectly substitutable with elasticity $\theta > 1$ and national goods (variety bundles) are imperfectly substitutable with elasticity $\phi > 0$. Aggregate consumption is thus:

$$c_t = \left[(1-\alpha)^{\frac{1}{\phi}} \left(\int_{\omega \in \Omega} c_{d,t}(\omega)^{\frac{\theta-1}{\theta}} d\omega \right)^{\frac{\theta(\phi-1)}{(\theta-1)\phi}} + \alpha^{\frac{1}{\phi}} \left(\int_{\omega \in \Omega} c_{m,t}(\omega)^{\frac{\theta-1}{\theta}} d\omega \right)^{\frac{\theta(\phi-1)}{(\theta-1)\phi}} \right]^{\frac{\phi}{\phi-1}}.$$

Price aggregates are defined over sets of available goods Ω_t with (time–varying) mass n_t, corresponding to the number of domestic firms and Ω_t^* with (time–varying) mass n_t^*,

[10] Although the assumption of transaction costs is not required to get financial markets incompleteness, this assumption helps pin down a unique steady state, as shown by Turnovsky (1985) and Schmitt-Grohé and Uribe (2003).

[11] For the foreign economy, we do not detail relations but assume that similar definitions hold.

corresponding to the number of foreign firms:[12]

$$p_{c,t} = \left[(1-\alpha) \left(\int_{\omega \in \Omega_t} p_{d,t}(\omega)^{1-\theta} d\omega \right)^{\frac{1-\phi}{1-\theta}} + \alpha \left(\int_{\omega \in \Omega_t^*} p_{m,t}(\omega)^{1-\theta} d\omega \right)^{\frac{1-\phi}{1-\theta}} \right]^{\frac{1}{1-\phi}},$$

where $p_{d,t}(\omega)$ is the domestic price of good ω and $p_{m,t}(\omega)$ its import price. A simplifying assumption will be that the law of one price holds, so that:

$$p_{m,t}(\omega) = e_t p_{d,t}^*(\omega).$$

Optimal variety demands from domestic households are:

$$c_{d,t}(\omega) = (1-\alpha) \rho_t(\omega)^{-\theta} \left(\frac{p_{d,t}}{p_{c,t}} \right)^{-\phi} c_t \text{ and } c_{m,t}(\omega) = \alpha \rho_t^*(\omega)^{-\theta} \left(\frac{e_t p_{d,t}^*}{p_{c,t}} \right)^{-\phi} c_t,$$

where $\rho_t(\omega) = \frac{p_{d,t}(\omega)}{p_{d,t}}$, $\rho_t^*(\omega) = \frac{p_{d,t}^*(\omega)}{p_{d,t}^*}$, and,

$$p_{d,t} = \left(\int_{\omega \in \Omega_t} p_{d,t}(\omega)^{1-\theta} d\omega \right)^{\frac{1}{1-\theta}} \text{ and } p_{d,t}^* = \left(\int_{\omega \in \Omega_t^*} p_{d,t}^*(\omega)^{1-\theta} d\omega \right)^{\frac{1}{1-\theta}}.$$

To further simplify these demand functions, we define $s_t = \frac{e_t p_{d,t}^*}{p_{d,t}}$ as the terms of trade, and substitute to get:

$$c_{d,t}(\omega) = (1-\alpha) \rho_t(\omega)^{-\theta} \left((1-\alpha) + \alpha s_t^{1-\phi} \right)^{\frac{\phi}{1-\phi}} c_t \text{ and } c_{m,t}(\omega) = \alpha \rho_t^*(\omega)^{-\theta} \left((1-\alpha) s_t^{\phi-1} + \alpha \right)^{\frac{\phi}{1-\phi}} c_t.$$

We also introduce the real exchange rate $q_t = \frac{e_t p_{c,t}^*}{p_{c,t}}$, which relates to the terms of trade according to:

$$q_t = \frac{e_t p_{c,t}^*}{p_{c,t}} = \left(\frac{(1-\alpha) s_t^{1-\phi} + \alpha}{(1-\alpha) + \alpha s_t^{1-\phi}} \right)^{\frac{1}{1-\phi}}.$$

In spite of the assumption that the law of one price holds, purchasing power parity does not hold in equilibrium due to home bias in consumption – $q_t = 1$ (purchasing power parity holds) when $\alpha = \frac{1}{2}$, i.e. when there is no home bias in preferences –. An important implication is that, in general, the real exchange rate plays a non-trivial role in the external adjustment of the economy. Quadratic adjustment costs ac_t are paid in units of consumption goods and thus imply similar variety demands.

Finally, the budget constraint of the government is:

$$tax_t + \zeta_t w_t \ell_t = 0.$$

[12] Both Ω_t and Ω_t^* are subsets of Ω.

3.2 Firms

In each period t, there are two types of firms in the domestic economy: n_t firms that are already on the market at the beginning of the period and $n_{e,t}$ firms that are newly created during this period.[13] At the end of the period a fraction $\delta \in (0,1)$ of all existing firms is hit by an exit shock. We assume that entry is made one period ahead of production, so that:

$$n_t = (1-\delta)\left(n_{t-1} + n_{e,t-1}\right).$$

In period t, $n_{e,t}$ new firms enter the market. Each entrepreneur pays a sunk real and constant cost f_e paid in units of domestic goods to build the firm in period t and begins producing consumption goods in $t+1$. Entry in the market thus occurs until the expected present value of profits is equal to the cost of entry:[14]

$$v_t = f_e.$$

Each of the n_t firms is specialized in the production of a differentiated variety. In period t, the production function for the representative domestic firm specialized in variety ω is:

$$y_t(\omega) = z_t \ell_t^d(\omega).$$

The variable z_t denotes aggregate labor productivity, common to all domestic firms and subject to temporary shifts $\xi_{z,t}$ (see parametrization for more details) and $\ell_t^d(\omega)$ is the quantity of labor employed. We assume that prices are chosen subject to an adjustment cost, as in Rotemberg (1982). Denoting $p_{d,t}(\omega)$ as the price of good ω and $\rho_t(\omega) = \frac{p_{d,t}(\omega)}{p_{d,t}(\omega)}$ as its real price, the representative firm ω faces a quadratic cost:

$$\Gamma_t(\omega) = \frac{\kappa}{2}\left(\frac{p_{d,t}(\omega)}{p_{d,t-1}(\omega)} - 1\right)^2 \rho_t(\omega)\, y_t^d(\omega), \quad \kappa \geq 0.$$

The adjustment cost is paid in terms of domestic goods only. Consequently, the demand faced by the representative firm is:

$$y_t^d(\omega) = \rho_t(\omega)^{-\theta}\begin{pmatrix} (1-\alpha)\left((1-\alpha) + \alpha s_t^{1-\phi}\right)^{\frac{\phi}{1-\phi}}(c_t + ac_t) \\ +\alpha\left((1-\alpha)s_t^{1-\phi} + \alpha\right)^{\frac{\phi}{1-\phi}} c_t^* + f_e n_{e,t} + \Gamma_t \end{pmatrix}$$

[13] Similarly in the foreign economy.

[14] We do not consider entry shocks in this set-up. These shocks may be introduced by allowing the entry cost to vary over time. An extensive analysis of the effects of shocks on f_e are detailed in Bilbiie et al. (2007). Alternative specifications for the entry cost, such as expressing the entry cost in units of labor, may also be considered. However, this specification is known to result in a negative response of entries after monetary policy shocks, while the specification in terms of consumption goods delivers a positive response, in accordance with empirical evidence (see Lewis (2009)).

where $\Gamma_t = \int_{\omega \in \Omega_t} \Gamma_t(\omega)\, d\omega$. In period t, the representative firm ω chooses $p_{d,t}(\omega)$ to maximize $d_t(\omega) + v_t(\omega)$, where:

$$v_t(\omega) = E_t \left[\sum_{s=1}^{\infty} (\beta(1-\delta))^s \frac{u_{c,t+s}}{u_{c,t}} d_{t+s}(\omega) \right],$$

$$d_t(\omega) = \rho_t(\omega)\, y_t^d(\omega) - \frac{\kappa}{2} \left(\frac{p_t(\omega)}{p_{t-1}(\omega)} - 1 \right)^2 \rho_t(\omega)\, y_t^d(\omega) - \frac{\varpi_t}{z_t} y_t^d(\omega),$$

where $\varpi_t = \frac{w_t}{p_{d,t}}$ is the PPI-based real wage. The optimal pricing condition is:

$$\frac{p_{d,t}(\omega)}{p_{d,t}} = \rho_t(\omega) = \mu_t \frac{\varpi_t}{z_t} \tag{1}$$

where, after defining the average PPI inflation rate as $\widetilde{\pi}_t = \frac{p_{d,t}(\omega)}{p_{d,t-1}(\omega)} - 1$, the markup writes:

$$\mu_t = \frac{\theta}{(\theta - 1)\left(1 - \frac{\kappa}{2}\widetilde{\pi}_t^2\right) + \kappa\left(\widetilde{\pi}_t(1+\widetilde{\pi}_t) - \beta(1-\delta) E_t \left[\frac{\widetilde{\pi}_{t+1}(1+\widetilde{\pi}_{t+1})^2 y_{t+1} u_{c,t+1}}{(1+\pi_{t+1}) y_t u_{c,t}}\right]\right)}.$$

Importantly, firms entering the market price exactly like firms already on the market and behave as the (constant number of) price setters in Rotemberg (1982). Pricing conditions are the same for entrants as for firms operating on the market during period $t-1$. This is also consistent with the time-to-build structure of entries: new firms start producing after one period, have time to learn the pricing decisions made by 'old' firms in t and imitate them in period $t+1$.[15]

3.3 Aggregation and equilibrium

We solve the model by applying the symmetry across firms in each country:

$$\rho_t(\omega) = \rho_t,\ y_t^d(\omega) = y_t^d,\ d_t(\omega) = d_t,\ v_t(\omega) = v_t.$$

Aggregate real output is:

$$y_t = \int_{\omega \in \Omega_t} \rho_t(\omega)\, y_t(\omega)\, d\omega = n_t \rho_t y_t(\omega).$$

The structure of real price indices implies:

$$n_t \rho_t^{1-\theta} = 1.$$

Therefore the link between the individual (average) producer price index (PPI) inflation rate $\widetilde{\pi}_t$ and the aggregate PPI inflation rate π_t is:

$$\frac{1+\widetilde{\pi}_t}{1+\pi_t} = \frac{\rho_t}{\rho_{t-1}},$$

[15] See Bilbiie et al. (2007) for more discussion.

and the dynamics of the aggregate CPI is given by:

$$\pi_{c,t} = \pi_t \left(\frac{1-\alpha+\alpha s_t^{1-\phi}}{1-\alpha+\alpha s_{t-1}^{1-\phi}} \right)^{\frac{1}{1-\phi}}.$$

Finally, we introduce an average CPI that incorporates domestic and foreign average PPIs in the same proportions as the aggregate CPI. Its inflation rate is given by:

$$\widetilde{\pi}_{c,t} = \widetilde{\pi}_t \left(\frac{1-\alpha+\alpha \widetilde{s}_t^{1-\phi}}{1-\alpha+\alpha \widetilde{s}_{t-1}^{1-\phi}} \right)^{\frac{1}{1-\phi}},$$

where $\widetilde{s}_t = \frac{e_t p^*_{d,t}(\omega)}{p_{d,t}(\omega)}$.

In our benchmark scenario, following Chari et al. (2002), the model is closed by assuming that the central banks set nominal interest rates according to:

$$\begin{aligned} i_t &= \rho_i i_{t-1} + (1-\rho_i)\left(i + \phi_\pi \widetilde{\pi}_t + \phi_y \left(\tfrac{y_t^r}{y^r} - 1\right)\right) + \xi_{i,t}, \\ i_t^* &= \rho_i i_{t-1}^* + (1-\rho_i)\left(i + \phi_\pi^* \widetilde{\pi}_t^* + \phi_y \left(\tfrac{y_t^r}{y^r} - 1\right)\right) + \xi_{i^*,t}, \end{aligned}$$

where y_t^r is a measure of GDP that is consistent with the data, i.e. GDP is deflated by removing the variety effect, as explained in Bilbiie et al. (2007), and dropping the time index denotes steady state. Given that central banks do not have access to welfare-based measures of price indices, we consider that they stabilize average PPI inflation rates, the measures of price indices that are closer to the statistical measures they actually dispose of.

Imposing equity market clearing so that, $x_t = x_{t-1} = x_t^* = x_{t-1}^* = 1$, and assuming that domestic bonds are in zero net supply ($b_t = 0$), we define a competitive equilibrium as a sequence of quantities:

$$\{\mathcal{Q}_t\}_{t=0}^{\infty} = \{y_t, y_t^*, c_t, c_t^*, \ell_t, \ell_t^*, \ell_t^d, \ell_t^{d*}, n_t, n_t^*, n_{e,t}, n_{e,t}^*, ac_t, a_t, a_t^*\}_{t=0}^{\infty}$$

and a sequence of real prices and inflation rates:

$$\{\mathcal{P}_t\}_{t=0}^{\infty} = \{\rho_t, \rho_t^*, \varpi_t, \varpi_t^*, v_t, v_t^*, d_t, d_t^*, \pi_t, \pi_t^*, \pi_{c,t}, \pi_{c,t}^*, s_t, q_t\}_{t=0}^{\infty}$$

such that:

(*i*) For a given sequence of prices $\{\mathcal{P}_t\}_{t=0}^{\infty}$, a given realization of shocks $\{\mathcal{S}_t\}_{t=0}^{\infty} = \left\{\xi_{z,t}, \xi_{z^*,t}, \xi_{i,t}, \xi_{i^*,t}\right\}_{t=0}^{\infty}$, and a monetary policy $\{\mathcal{M}_t\}_{t=0}^{\infty} = \{i_t, i_t^*\}_{t=0}^{\infty}$, the sequence $\{\mathcal{Q}_t\}_{t=0}^{\infty}$ satisfies first-order conditions for domestic and foreign households and maximizes domestic and foreign firms' profits.

(*ii*) For a given sequence of quantities $\{\mathcal{Q}_t\}_{t=0}^{\infty}$, a given realization of shocks $\{\mathcal{S}_t\}_{t=0}^{\infty}$, and a monetary policy $\{\mathcal{M}_t\}_{t=0}^{\infty}$, the sequence $\{\mathcal{P}_t\}_{t=0}^{\infty}$ guarantees:

- labor markets equilibrium:

$$\ell_t = \int_0^1 \ell_t(j)\, dj = \int_0^{n_t} \ell_t^d(\omega)\, d\omega,$$
$$\ell_t^* = \int_0^1 \ell_t^*(j^*)\, dj^* = \int_0^{n_t^*} \ell_t^{*d}(\omega^*)\, d\omega^*,$$

- goods markets equilibrium:

$$y_t = (1-\alpha)\left((1-\alpha) + \alpha s_t^{1-\phi}\right)^{\frac{\phi}{1-\phi}} (c_t + ac_t) + \alpha\left((1-\alpha)\, s_t^{1-\phi} + \alpha\right)^{\frac{\phi}{1-\phi}} c_t^* + f_e n_{e,t} + \Gamma_t,$$

$$y_t^* = (1-\alpha)\left((1-\alpha) + \alpha s_t^{\phi-1}\right)^{\frac{\phi}{1-\phi}} c_t^* + \alpha\left((1-\alpha)\, s_t^{\phi-1} + \alpha\right)^{\frac{\phi}{1-\phi}} (c_t + ac_t) + f_e^* n_{e,t}^* + \Gamma_t^*,$$

- foreign bonds market equilibrium:

$$e_t a_t + a_t^* = 0.$$

Finally, we use labor markets equilibrium and aggregate production functions, as well as other structural relations to simplify aggregate profits:

$$d_t = \frac{y_t}{n_t}\left(1 - \frac{\kappa}{2}\widetilde{\pi}_t^2 - \frac{1}{\mu_t}\right) \quad \text{and} \quad d_t^* = \frac{y_t^*}{n_t^*}\left(1 - \frac{\kappa}{2}\widetilde{\pi}_t^{*2} - \frac{1}{\mu_t^*}\right),$$

and to simplify the labor market equilibrium:

$$z_t \ell_t \rho_t = y_t \quad \text{and} \quad z_t^* \ell_t^* \rho_t^* = y_t^*. \tag{2}$$

In a closed economy version of the model, the aggregation of budget constraints yields the aggregate accounting relation, determining the dynamics of aggregate labor. Equations (2) would thus be simply redundant. In an open economy, however, the aggregation of budget constraints yields the dynamics of net foreign assets, and Equations (2) determine the dynamics of aggregate hours.

4 Dynamics

In this section, we assign numerical values to the parameters of the model and analyze the dynamics of our international economy after productivity and monetary policy shocks.

4.1 Parameterization

First, we assume the following utility function:

$$u(c_t, \ell_t) = \frac{\left(c_t^{\eta}(1-\ell_t)^{1-\eta}\right)^{1-\gamma}}{1-\gamma}.$$

Second, we assign numerical values to the parameters of the economy. The frequency of the model is quarterly. The discount factor is set to $\beta = 0.988$, implying a steady-state annual real interest rate of 5%. The proportion of firms affected by the exit shock each period in the economy is $\delta = 0.025$ (see Bergin and Corsetti (2008)). The price stickiness parameter, which affects the size and persistence of inflation, is set to $\kappa = 77$, as suggested by Ireland (2001). As in Bilbiie et al. (2007), the elasticity of substitution between varieties is set to $\theta = 3.8$. The value of the elasticity of substitution between domestic and foreign goods is probably the most critical parameter. The international real business cycles literature traditionally follows Backus et al. (1993) and sets the elasticity of substitution to 1.5. However, Heathcote and Perri (2002) calibrate this elasticity to values as low as 0.8 and suggest that lower values, such as 0.5, may actually improve the ability of macroeconomic models to match empirical evidence, which is confirmed by Thoenissen (2011). We choose to set $\phi = 1.05$ because counter-cyclicality of net exports requires low values of this elasticity of substitution. We adjust the level of trade openness α to match the average level of trade openness in OECD countries over the whole period, i.e. 52%. As α is the steady-state share of imports in GDP, we set $\alpha = 0.25$. In terms of preferences, we set $\eta = 0.34$, and $\gamma = 2$ as in Backus et al. (1993). The premium paid by domestic households to access international financial markets is set to $\Phi = 0.001$, as in Schmitt-Grohé and Uribe (2003), which represents an average 0.36% annual premium. Concerning exogenous shocks, without loss of generality, we normalize the steady-state value of shocks to $z = f_e = 1$. In the benchmark case, parameters governing the dynamics of productivity shocks are taken from Chari et al. (2002) with persistence $\rho_z = 0.95$, standard deviation of innovations $\sigma(\xi_z) = \sigma(\xi_{z^*}) = 0.7\%$ and $corr(\xi_z, \xi_{z^*}) = 0.25$.[16] Finally, parameters of the monetary policy rules are $\rho_r = 0.5$, $\phi_\pi = 1.5$, and $\phi_y = 0.125$. The

[16] The processes of productivity shocks are thus $\log z_t = \rho_z \log z_{t-1} + \xi_{z,t}$ and $\log z_t^* = \rho_z \log z_{t-1}^* + \xi_{z^*,t}$.

volatility of monetary innovations is set to match the absolute volatility of GDP in the baseline calibration, and we assume that $corr\left(\xi_i, \xi_{i^*}\right) = 0.5$. The implied volatility of monetary innovations is $\sigma\left(\xi_i\right) = \sigma\left(\xi_{i^*}\right) = 0.25\%$, in line with most empirical estimates.

4.2 Linearization and data-consistency

The model is linearized around the steady state described in Appendix A. Equilibrium relations are summarized in Appendix B. However, the resulting aggregate dynamics must be modified to be consistent with the data presented in Section 2, or when calculating second-order moments, as below. More precisely, as shown in Bilbiie et al. (2007), the effect of endogenous varieties must be removed from quantities, since this effect is not accounted for in the data. We thus define,

$$u_t^r = \frac{u_t}{\rho_t}, \text{ or equivalently, } \widehat{u}_t^r = \widehat{u}_t - \widehat{\rho}_t$$

as the dynamics of an aggregate variable u_t that is consistent with the data. A hat on variables denotes the logdeviation of this variable from the steady state. For what concerns price indices, we argue that average PPIs and CPIs inflation rates ($\widetilde{\pi}_t$ and $\widetilde{\pi}_{c,t}$ respectively) even though there are not welfare-based, are closer to actual statistic measures of PPIs and CPIs, and therefore are more plausible policy targets. Similarly, real exchange rate, we compare the dynamic properties of $\widetilde{q}_t$ to the empirical properties of the real exchange rate.

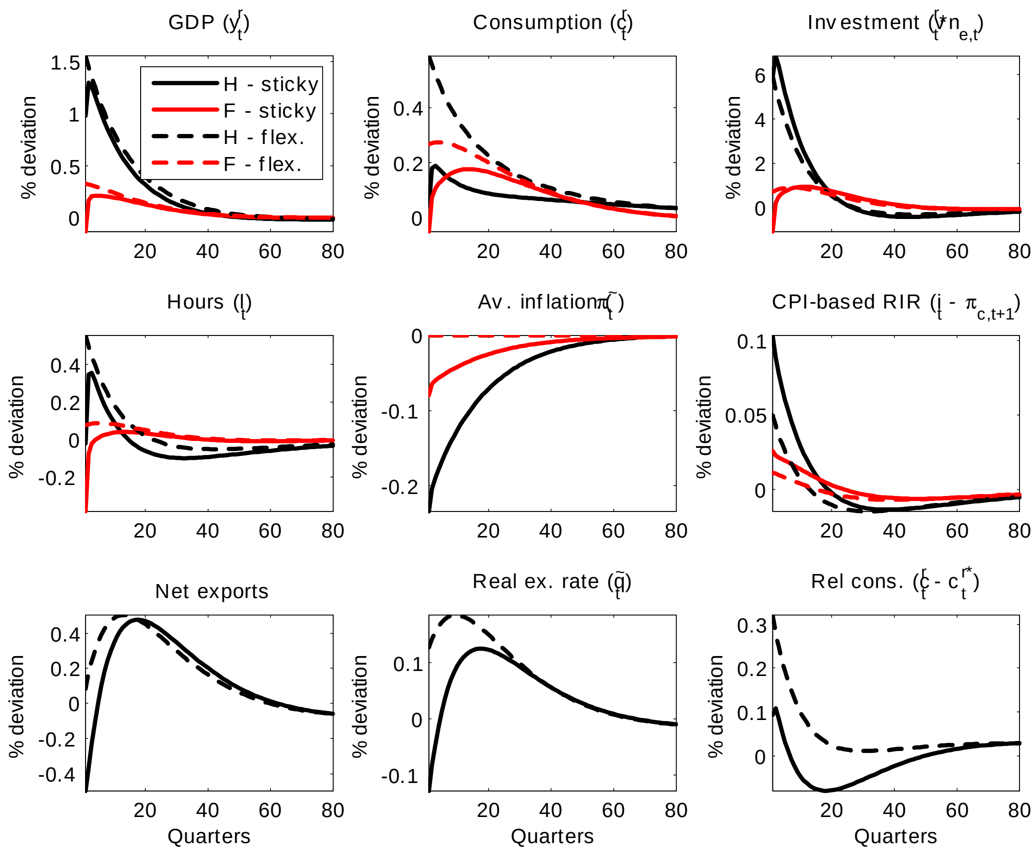
4.3 Impulse response functions

In this section, we discuss the qualitative properties of our baseline model after productivity and monetary policy shocks.

Figure 1 reports the impulse response functions (IRFs, hereafter) of data-consistent GDP, consumption, investment, hours worked, average PPI inflation, net exports, real exchange rate, and relative consumptions after a domestic productivity shock. We also report the IRFs under price stability policies ($\widehat{\widetilde{\pi}}_t = \widehat{\widetilde{\pi}}_t^* = 0$), that mimic the flexible prices equilibrium, to evaluate the role of price stickiness in the dynamics of the model and provide intuition on the key mechanism at work.

After a positive productivity shock, we observe a rise in GDP, consumption, hours and investment in the domestic economy. The responses of these three variables are persistent, and consumption exhibits a hump-shape when prices are sticky. Higher productivity in the

Figure 1: IRFs to a one percent domestic productivity shock – sticky vs. flexible prices.



domestic economy causes a drop in the production cost, an increase in expected dividends and induces households to invest in the creation of new varieties. The shock is transmitted to the foreign economy through trade flows, and depends on the price setting assumption.

Under flexible prices, the drop in the relative price of domestic goods increases their international competitiveness, and induces an expenditure switching towards them in both economies, which results in positive net exports.[17] Indeed, the real depreciation more than compensates the increase in imports driven by the rise in domestic consumption relative to foreign consumption. Since both relative consumptions and the (data-consistent) real exchange rate increase after a domestic productivity shock, their correlation with output is

[17] We find that a domestic productivity shock depreciates the data-consistent real exchange rate under flexible prices, while Ghironi and Melitz (2005), in a related framework, document an appreciation. In Ghironi and Melitz (2005)'s model, labor supply is inelastic, which induces terms of labor to appreciate after a productivity shock, leading the real exchange rate to appreciate. Labor supply is endogenous in our model, which allows terms of labor to depreciate, leading the real exchange rate to depreciate as well.

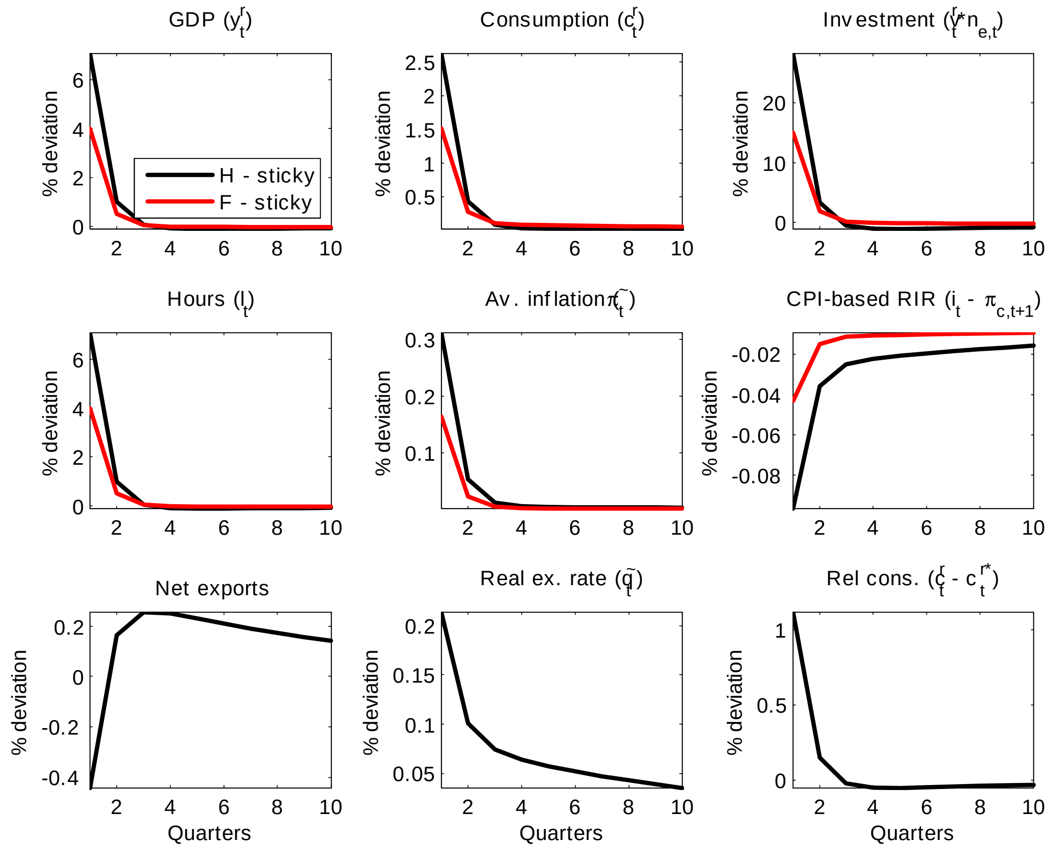
positive in the case of productivity shocks under flexible prices. This adjustment scheme, while in accordance with other studies (see Chari et al. (2002) for instance), is contradicted by the pattern of the data. Similarly, productivity shocks generate pro-cyclical net exports under flexible prices, in contrast with empirical evidence.

Under sticky prices, on the other hand, productivity shocks generate counter-cyclical net exports, an appreciated real exchange rate and a negative correlation between relative consumptions and the real exchange rate, at least for a few periods. Sticky prices, because they affect the expected returns of investing in the creation of new ventures, are the key to these changes. Indeed, after a domestic productivity shock, firms temporarily increase their markups, inducing a disinflation. The latter increases the CPI-based real interest rate and dividends (since the value of firms remains constant through the free-entry condition) through the arbitrage relation between bonds and shares. Figure 1 shows very clearly the overshooting of investment, and the increased consumption smoothing induced by sticky prices. These higher returns to investment in new firms in the domestic economy push domestic households to reduce their net foreign assets. But as the domestic nominal interest rate is cut by the central bank to fight the disinflation, equilibrium in the market of foreign bonds requires expectations of nominal appreciation of the domestic currency. This nominal appreciation changes the response of the real exchange rate – although for a few periods only – and the latter appreciates on impact. Because relative consumptions still react positively after the shocks – although more smoothly – this real appreciation leads to negative net exports, again for a few periods only.

Thus, under sticky prices, while the responses of output, consumption, investment and hours worked are qualitatively preserved, the dynamics of international adjustment is modified, on impact and for a few periods: the real exchange rate appreciates, leading to negative net exports, and relative consumptions are still positive on impact. Therefore, driven by productivity shocks and under sticky prices, the model produces a negative correlation between relative consumptions and the real exchange rate and leads to counter-cyclical net exports. These dynamic features are highly encouraging in the perspective of reproducing important features of the data such as the Backus-Smith puzzle.

Figure 2 reports the IRFs of the same variables after an expansionary monetary policy shock in the domestic economy. Notice that under flexible prices, monetary policy shocks do no affect the economy.

Figure 2: IRFs to a one percent monetary policy shock – sticky prices.



When the economy experiences a monetary policy shock in the domestic economy, GDP, consumption, hours worked and investment increase due to the increase in aggregate demand for consumption goods. To accommodate the increase in aggregate consumption, firms increase real wages to allow for more production, which induces a rise in the average inflation of production prices. Transmission to the foreign economy occurs through increasing domestic imports due to higher consumption. The nominal exchange rate depreciates alongside with the drop in the nominal interest rate, leading the real exchange rate to depreciate. However, while this affects positively net exports, the increase in domestic consumption is so high on impact that net exports remain negative for two quarters. After that, the long lasting real exchange rate depreciation generates an increase in net exports. Thus, monetary policy shocks deliver slightly counter-cyclical or a-cyclical net exports, and a positive comovement between relative consumptions and the real exchange rate.

A combination of productivity and monetary policy shocks could thus help to solve the Backus-Smith puzzle since both shocks have opposite consequences on the correlation between relative consumptions and the (data-consistent) real exchange rate under sticky prices. Importantly, this aspect of the data can be reproduced consistently with counter-cyclical net exports.

5 Business cycle moments

In this section, we evaluate the ability of our model to reproduce business cycle moments. Simulated series are HP-filtered and transformed appropriately to be consistent with the data. The top part of Table 3 reports the standard deviations of key variables relative to the standard deviation of GDP, as well as autocorrelations. The bottom part of Table 3 reports the correlations of key variables with GDP, the cross-country correlations and international business cycle moments. We present the results for three different situations and compare them to the data over the sample 1975Q1–2007Q4 (first panel). First, panel (b) presents the results when prices are flexible, so as to check the exact role of prices stickiness in our economy. The situation of flexible prices is obtained by substituting the following condition $\widehat{\widetilde{\pi}}_t = \widehat{\widetilde{\pi}}_t^* = 0$, to the monetary policy rules. As a consequence, markups are constant and real prices move only if productivity changes. Panel (b) contrasts the implications of considering an equilibrium with sticky prices and productivity shocks only. Finally, panel (c) presents business cycle moments with productivity and monetary policy shocks.

We start by contrasting the business cycle properties of the flexible prices model, displayed in panel (a). The model performs quite well in terms of volatility of investment. Private consumption, hours, the real exchange rate, as well as trade flows and net exports exhibit too little volatility, however. In addition, the persistence displayed in the data is well reproduced, except for net exports, that are too persistent. Consumption, hours worked and investment are too much correlated with GDP and, more importantly, net exports are pro-cyclical, as shown in the analysis of IRFs. This feature arises also because the pattern of cyclicality of trade flows is reversed compared to the data: exports are more strongly positively correlated with GDP than imports. Cross-country correlations are poorly matched in this first experiment, except for private consumptions. Concerning the real exchange rate and relative consumptions, the correlation is 0.77, which is far from the data but outperforms the correlation found in most standard real business cycle models, where the correlation is

Table 3: National and international business cycle moments

	Data	**(a)**	**(b)**	**(c)**
Standard deviations*				
Consumption ($\widehat{c}_t^r$)	1.01	0.39	0.16	0.31
Investment ($\widehat{v}_t^r \widehat{n}_{e,t}$)	3.10	3.87	5.59	4.62
Hours ($\widehat{\ell}_t$)	0.88	0.36	0.37	0.84
Exports	2.38	0.39	0.40	0.37
Imports	2.96	0.39	0.40	0.37
Net exports	0.61	0.34	0.76	0.47
Real Exch. Rate ($\widehat{q}_t$)	2.79	0.17	0.19	0.12
Autocorrelations				
Consumption ($\widehat{c}_t^r$)	0.79	0.71	0.80	0.14
Investment ($\widehat{v}_t^r \widehat{n}_{e,t}$)	0.78	0.70	0.77	0.42
GDP ($\widehat{y}_t^r$)	0.78	0.71	0.80	0.31
Hours ($\widehat{\ell}_t$)	0.87	0.70	0.60	0.10
Exports	0.63	0.76	0.77	0.36
Imports	0.74	0.76	0.77	0.36
Net exports	0.67	0.96	0.76	0.71
Real Exch. Rate ($\widehat{q}_t$)	0.81	0.82	0.74	0.67
Corr. with GDP				
Consumption ($\widehat{c}_t^r$)	0.68	0.96	0.82	0.92
Investment ($\widehat{v}_t^r \widehat{n}_{e,t}$)	0.72	0.99	0.99	0.98
Hours ($\widehat{\ell}_t$)	0.62	0.99	0.79	0.90
Exports	0.40	0.78	-0.33	0.35
Imports	0.61	0.60	0.73	0.87
Net exports	-0.26	0.21	-0.56	-0.40
Real Exch. Rate ($\widehat{q}_t$)	0.08	0.54	-0.53	-0.16
Cross-country corr.				
Consumption ($\widehat{c}_t^r$)	0.46	0.63	0.44	0.61
Investment ($\widehat{v}_t^r \widehat{n}_{e,t}$)	0.51	-0.01	-0.28	0.14
GDP ($\widehat{y}_t^r$)	0.55	0.18	-0.05	0.39
Hours ($\widehat{\ell}_t$)	0.50	0.02	-0.14	0.57
Corr. with RER				
Exports	0.34	0.34	0.94	0.51
Imports	-0.03	-0.34	-0.94	-0.51
Net exports	0.30	0.78	1.00	0.80
Relative cons.	-0.09	0.77	-0.94	0.02

$*$: Relative to standard deviation of GDP
(a): Flexible prices with TFP shocks
(b): Sticky prices with TFP shocks
(c): Sticky prices with TFP and monetary shocks

close to unity even when financial markets are incomplete (see e.g. Chari et al. (2002)). These first findings are of importance because they prove that it is worth considering models where the number of varieties is endogenous when addressing this feature of the data. However, as suggested by Chari et al. (2002), introducing price stickiness might be relevant when trying to match important aspects of the data, such as the counter-cyclicality of net exports or the Backus-Smith puzzle.

As a matter of fact, panel (b) shows that price stickiness improves the model's performance along many dimensions. First, the model does as well as the flexible prices version in terms of persistence and does better in terms of imports, exports and net exports cyclicality. The volatility of net exports is closer to its empirical counterpart than in the flexible-price model. However, price stickiness tends to lower dramatically the volatility of consumption and to increase that of investment, consistently with the IRFs reported in the previous section. Notice that the persistence of the real exchange rate is almost perfectly matched while its volatility is poorly reproduced. As depicted in the IRFs, the situation reported in panel (b) delivers together counter-cyclical net exports and a negative correlation between the real exchange rate and relative consumptions. Unfortunately, the correlation is too negative compared to the data. Therefore, we analyze the business cycle properties of our economy when driven by both productivity and monetary policy shocks.

Panel (c) indeed appears as the best model to explain the data. In this case, the cyclicality of trade flows and net exports is almost perfectly matched. Identically, the combination of productivity and monetary shocks yields a correlation between the real exchange rate and relative consumptions that is almost zero, very close to the data. Volatilities are acceptably reproduced, except the volatility of the real exchange rate, and cross-country correlations are correctly matched, except the cross-country correlation of investment. However, the overall good performance of this specification in explaining the data is reached at the cost of a significant drop in persistence, due to the low persistence implied by monetary shocks (see IRFs).

Summarizing the main findings of Table 3, the assumption of flexible prices is relevant to match the persistence observed in the data but is not sufficient to reproduce the cyclicality of exports, imports and net exports and fails in solving the Backus-Smith puzzle. Adding sticky prices is important to explain the counter-cyclicality of net exports, preserves the ability of the model to deliver the same persistence as in the data, and crucially affects the correlation of the real exchange rate with relative consumptions. In addition, monetary policy shocks are essential to reproduce almost exactly the cyclicality of trade flows and net exports, and to balance the excessively negative correlation between the real exchange rate and relative consumptions implied by productivity shocks. On balance, a model combining endogenous entry, incomplete financial markets, sticky prices and two sources of disturbances is a good compromise to reproduce most features of the data.

6 Monetary policy and welfare analysis

Finally, we proceed to a welfare evaluation of various monetary policies in the model. The welfare is computed using a second-order approximation of the sequence of utility streams. We then calculate the permanent equivalent consumption loss Λ such that[18]

$$\Lambda = 1 - \left(1 + \Upsilon_c\left(\frac{var(\widehat{c}_t)+var(\widehat{c}_t^*)}{2}\right) + \Upsilon_\ell\left(\frac{var\left(\widehat{\ell}_t\right)+var\left(\widehat{\ell}_t^*\right)}{2}\right)\right)^{\frac{1}{\eta(1-\gamma)}}$$

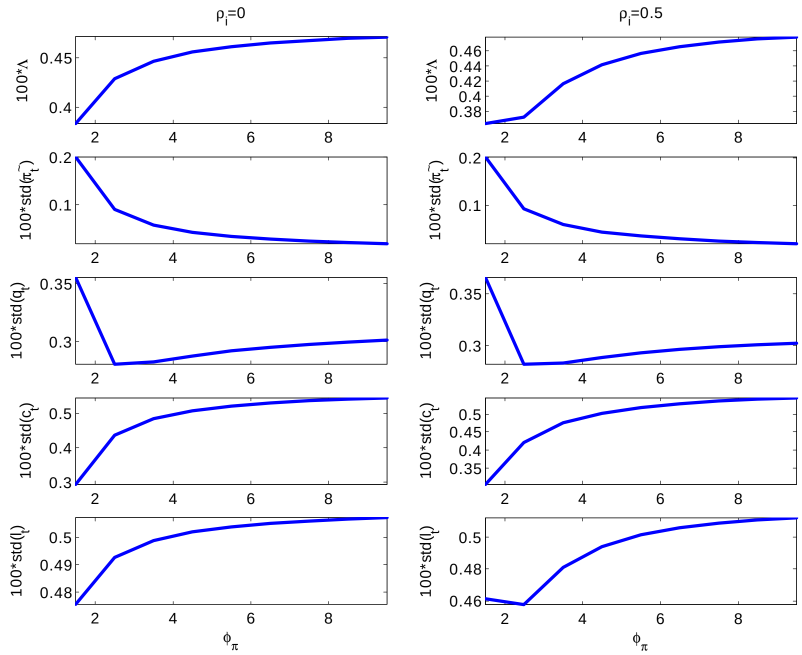
where $\Upsilon_c = \frac{1}{2}\eta^2\left(1-\gamma\right)^2$ and $\Upsilon_\ell = \frac{1}{2}\left(1-\eta\right)^2\left(1-\gamma\right)^2\left(\frac{\ell}{1-\ell}\right)^2$. Λ measures the amount of steady state consumption that agents would be willing to give up to live in a world without macroeconomic fluctuations, i.e. where $var\left(\widehat{c}_t\right) = var\left(\widehat{\ell}_t\right) = 0$. Therefore, higher Λ means higher welfare losses from fluctuations and lower Λ means lower losses (or higher welfare). Based on this welfare measure, we analyze the desirability of price stability in an open economy with endogenous entry and sticky prices.

First, we let ϕ_π vary significantly in two cases: when monetary policy responds to the output and when it does not. Results are reported in Figures 3 and 4. These charts display the evolution of the welfare measure, and the volatilities of average PPI inflation, welfare-based real exchange rate, welfare-based consumptions and hours worked. Simulations are performed under interest rate smoothing (right panel) and without smoothing (left panel), and abstract from exogenous monetary perturbations, i.e. $\sigma\left(\xi_i\right) = \sigma\left(\xi_{i^*}\right) = 0$.

When monetary policy responds to the output, fighting too hard against average inflation damages the welfare (first row of Figure 3). The reason for this comes from the investment overshooting documented in the analysis of IRFs. Indeed, when responding strongly to average inflation, the central bank, say in the case of a domestic productivity shock, lowers the

[18] See Appendix C for the derivation.

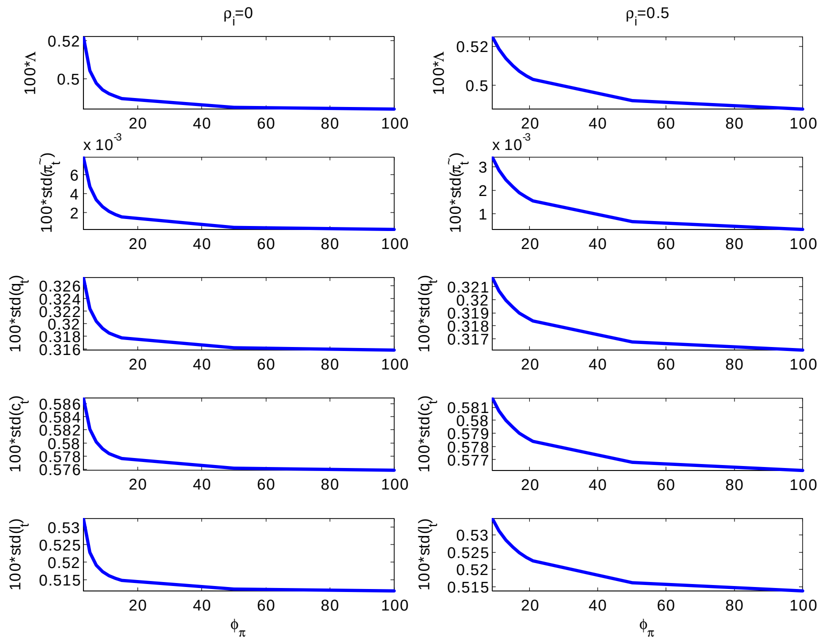
Figure 3: Welfare and volatilities for varying ϕ_π with $\phi_y = 0.125$.



nominal interest rate more and reduces the disinflation, thereby dampening the magnitude of the increase in the real interest rate. Investment booms less, which has two important consequences. First, households smooth consumption less (eventhough the extensive margin of consumption increases less), and second, the real exchange rate appreciates less. In the case of a domestic productivity shock, the real appreciation is welfare enhancing in both countries: (i) it allows domestic households to work less – through the so-called terms of trade spillover – and (ii) it depreciates the foreign currency in real terms, inducing foreign households to reduce their labor supply less. The volatility of hours thus drops in both countries. Therefore, reacting too much to inflation lowers the volatility of investment, increases the volatility of consumption through the consumption-saving trade-off, and renders hours worked more volatile through a less volatile (and counter-cyclical) real exchange rate.

The importance of responding to the output is confirmed by Figure 4. When central banks do not respond to output, the level of welfare losses is systematically above the level obtained when responding to output. Welfare losses range from 0.004% to 0.0045% when central banks respond to the output, while it ranges from 0.0048% to 0.0052% when they do not. Figure 4 also shows that, when not responding to output fluctuations, monetary policy should be aimed at reducing average inflation. When monetary policy responds to inflation only – as

Figure 4: Welfare and volatilities for varying ϕ_π when $\phi_y = 0$.

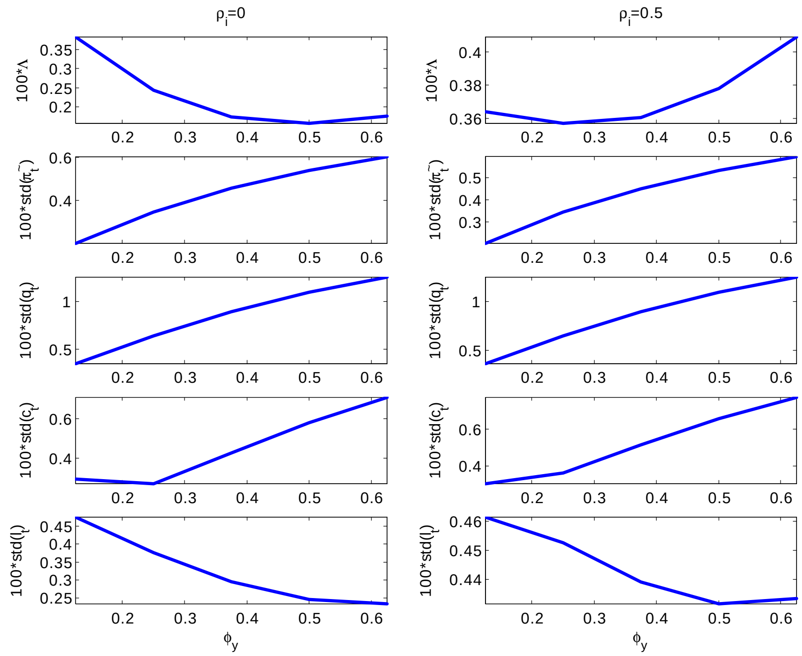


in the limiting case where prices are flexible – the model does not lead to a counter-cyclical real exchange rate. For example, after a domestic productivity shock, the real exchange rate depreciates, with negative consequences on hours worked in both countries and on domestic consumption. In this case, the best policy is to fight against inflation to lower the magnitude of the depreciation through lower disinflation.

Responding to the output thus changes the dynamics of external adjustment by allowing real interest rates to increase more, and creates a positive externality in the economy by generating a counter-cyclical real exchange rate. However, responding too much to the output might also be damaging. It can indeed result in too volatile investment in the creation of new ventures, which can increase the volatility of varieties, and therefore the volatility of the extensive margin of private consumption. To a lesser extent and for less plausible values of the response to the output, varying the real interest rate too much can result in reversing the sign of the response of consumption after productivity shocks. Thus, by how much should central banks react to the output? Figure 5 presents the evolution of the welfare measure and the volatility of key variables when varying the reaction of central banks to the output.

Figure 5 shows that central banks should react moderately to output (roughly 0.5 when $\rho_i = 0$ and 0.25 when $\rho_i = 0.5$) to maximize the intensity of the real exchange rate externality

Figure 5: Welfare and volatilities for varying ϕ_y when $\phi_\pi = 1.5$.



without increasing the volatility of the extensive margin of consumption too much. Indeed, the volatility of hours is a decreasing function of ϕ_y because of the positive effect of a counter-cyclical real exchange rate on hours, but the volatility of welfare-based consumption is an increasing function of ϕ_y because more volatile interest rates imply more volatile investment, which may increase the volatility of the extensive margin of private consumption too much.

According to our simulations, the best monetary policy in terms of welfare is thus to react moderately to the output without smoothing the interest rate. The corresponding welfare losses from business cycles are less 0.002 % of permanent consumption.

7 Conclusion

This paper introduces changes in the number of varieties and endogenous entry in an international business cycle model with sticky prices and incomplete financial markets. This framework is able to reproduce several features of the data that are challenging for standard models. In particular, the model is successful at delivering a negative correlation of net exports with GDP, persistent dynamics of international variables, such as the real exchange rate, and a negative correlation of relative consumption with the real exchange rate. Including monetary policy shocks improves the fit of the model along some dimensions. Importantly, these results are obtained together with satisfactory statistical predictions for domestic variables. Endogenous entry and price stickiness are the key ingredients that allow for the joint replication of the business cycles statistics. The key mechanism at work is the overshooting of investment in the creation of new ventures, that changes the response of net exports and exchange rates together, through reversed patterns of international capital flows. In terms of monetary policy implications, we highlight the importance of responding to output fluctuations to generate a counter-cyclical real exchange rate, with positive spillovers on hours worked, and thus on welfare. However, the magnitude of the response to output fluctuations should not be too large, between 0.25 and 0.5. Too much response to output fluctuations would indeed result in too volatile investment, magnifying the volatility of the extensive margin of private consumption, with negative welfare implications.

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Appendix

A Steady State

We solve the model in logdeviation from the Pareto-optimal symmetric steady state equilibrium without inflation where, $c = c^*$, $y = y^*$, $\ell = \ell^*$, $n = n^*$, $n_e = n_e^*$, $s = q = e = 1$ and $\widetilde{\pi} = \widetilde{\pi}^* = \widetilde{\pi}_c = \widetilde{\pi}_c^* = 0$. Pareto-optimality requires that $\zeta = (1-\theta)^{-1}$. Assuming $z = f_e = 1$, the steady state of the economy is such that:

$$\ell = \frac{\eta\theta\left(1-(1-\delta)\beta\right)}{\theta\left(1-(1-\delta)\beta\right)-(1-\eta)\beta\delta}.$$

Other variables are given by:

$$i = \frac{1}{\beta}-1, \quad v = 1, \quad d = \frac{\left(1-(1-\delta)\beta\right)}{(1-\delta)\beta}, \quad \mu = \frac{\theta}{\theta-1},$$
$$n_e = \frac{\delta}{1-\delta}\varphi^{\frac{\theta-1}{2-\theta}}, \quad \rho = \varphi^{\frac{1}{2-\theta}}, \quad n = \varphi^{\frac{\theta-1}{2-\theta}}, \quad y = \theta d\varphi^{\frac{\theta-1}{2-\theta}},$$
$$c = \left(\theta d - \frac{\delta}{1-\delta}\right)\varphi^{\frac{\theta-1}{2-\theta}},$$

where $\varphi = \frac{\theta(1-(1-\delta)\beta)-(1-\eta)\beta\delta}{\eta(1-\delta)\beta}$.

B Loglinearization

Loglinearized conditions for households are:

$$\widehat{u}_{c,t} - E_t\left\{\widehat{u}_{c,t+1}\right\} - E_t\left\{\widehat{i}_t - \widehat{\pi}_{c,t+1}\right\} = 0,$$
$$\widehat{u}_{c^*,t} - E_t\left\{\widehat{u}_{c^*,t+1}\right\} - E_t\left\{\widehat{i}_t^* - \widehat{\pi}_{c,t+1}^*\right\} = 0,$$
$$\widehat{u}_{c,t} - E_t\left\{\widehat{u}_{c,t+1}\right\} - E_t\left\{\frac{r+\delta}{1+r}\widehat{d}_{t+1}\right\} = 0,$$
$$\widehat{u}_{c^*,t} - E_t\left\{\widehat{u}_{c^*,t+1}\right\} + \widehat{v}_t^* - E_t\left\{\frac{r+\delta}{1+r}\widehat{d}_{t+1}^*\right\} = 0,$$
$$\frac{\ell}{1-\ell}\widehat{\ell}_t + \widehat{c}_t - \widehat{\varpi}_t + \alpha\widehat{s}_t = 0,$$
$$\frac{\ell}{1-\ell}\widehat{\ell}_t^* + \widehat{c}_t^* - \widehat{\varpi}_t^* - \alpha\widehat{s}_t = 0,$$

where

$$\widehat{u}_{c,t} = \left(\eta(1-\gamma)-1\right)\widehat{c}_t - \frac{\ell}{1-\ell}(1-\eta)(1-\gamma)\widehat{\ell}_t,$$
$$\widehat{u}_{c^*,t} = \left(\eta(1-\gamma)-1\right)\widehat{c}_t^* - \frac{\ell}{1-\ell}(1-\eta)(1-\gamma)\widehat{\ell}_t^*.$$

Loglinear conditions for firms are:

$$\begin{aligned}
\widehat{n}_t - (1-\delta)\,\widehat{n}_{t-1} - \delta\widehat{n}_{e,t-1} &= 0, \\
\widehat{n}_t^* - (1-\delta)\,\widehat{n}_{t-1}^* - \delta\widehat{n}_{e,t-1}^* &= 0, \\
\widehat{\rho}_t - \widehat{\mu}_t - (\widehat{\varpi}_t - \widehat{z}_t) &= 0, \\
\widehat{\rho}_t^* - \widehat{\mu}_t^* - (\widehat{\varpi}_t^* - \widehat{z}_t^*) &= 0, \\
\widehat{\widehat{\pi}}_t - \beta(1-\delta)\,E_t\left\{\widehat{\widehat{\pi}}_{t+1}\right\} + \frac{\theta-1}{\kappa}\widehat{\mu}_t &= 0, \\
\widehat{\widehat{\pi}}_t^* - \beta(1-\delta)\,E_t\left\{\widehat{\widehat{\pi}}_{t+1}^*\right\} + \frac{\theta-1}{\kappa}\widehat{\mu}_t^* &= 0.
\end{aligned}$$

Other loglinear equilibrium conditions are:

$$\begin{aligned}
\widehat{n}_t - (\theta-1)\,\widehat{\rho}_t &= 0, \\
\widehat{n}_t^* - (\theta-1)\,\widehat{\rho}_t^* &= 0, \\
\widehat{y}_t - (1-\tau)\left((1-\alpha)\,\widehat{c}_t - \alpha\widehat{c}_t^* - 2\phi\alpha(1-\alpha)\,\widehat{s}_t\right) + \tau\widehat{n}_{e,t} &= 0, \\
\widehat{y}_t^* - (1-\tau)\left((1-\alpha)\,\widehat{c}_t^* - \alpha\widehat{c}_t + 2\phi\alpha(1-\alpha)\,\widehat{s}_t\right) + \tau\widehat{n}_{e,t}^* &= 0, \\
\widehat{z}_t + \widehat{\ell}_t - (\widehat{y}_t - \widehat{\rho}_t) &= 0, \\
\widehat{z}_t^* + \widehat{\ell}_t^* - (\widehat{y}_t^* - \widehat{\rho}_t^*) &= 0, \\
\widehat{n}_t + \widehat{d}_t - \widehat{y}_t - (\theta-1)\,\widehat{\mu}_t &= 0, \\
\widehat{n}_t^* + \widehat{d}_t^* - \widehat{y}_t^* - (\theta-1)\,\widehat{\mu}_t^* &= 0, \\
\widehat{\widehat{\pi}}_t - \widehat{\pi}_t - \widehat{\rho}_t + \widehat{\rho}_{t-1} &= 0, \\
\widehat{\widehat{\pi}}_t^* - \widehat{\pi}_t^* - \widehat{\rho}_t^* + \widehat{\rho}_{t-1}^* &= 0, \\
\widehat{\pi}_{c,t} - \widehat{\pi}_t - \alpha(\widehat{s}_t - \widehat{s}_{t-1}) &= 0, \\
\widehat{\pi}_{c,t}^* - \widehat{\pi}_t^* + \alpha(\widehat{s}_t - \widehat{s}_{t-1}) &= 0, \\
\widehat{\widehat{\pi}}_{c,t} - \widehat{\widehat{\pi}}_t - \alpha\left(\widehat{\widehat{s}}_t - \widehat{\widehat{s}}_{t-1}\right) &= 0, \\
\widehat{\widehat{\pi}}_{c,t}^* - \widehat{\widehat{\pi}}_t^* + \alpha\left(\widehat{\widehat{s}}_t - \widehat{\widehat{s}}_{t-1}\right) &= 0, \\
\widehat{s}_t - \widehat{s}_{t-1} - (\widehat{e}_t - \widehat{e}_{t-1}) - \widehat{\pi}_t^* + \widehat{\pi}_t &= 0, \\
\widehat{\widehat{s}}_t - \widehat{\widehat{s}}_{t-1} - (\widehat{e}_t - \widehat{e}_{t-1}) - \widehat{\widehat{\pi}}_t^* + \widehat{\widehat{\pi}}_t &= 0, \\
\widehat{q}_t - (1-2\alpha)\,\widehat{s}_t &= 0, \\
\widehat{\widehat{q}}_t - (1-2\alpha)\,\widehat{\widehat{s}}_t &= 0,
\end{aligned}$$

where $\tau = \frac{f_e n_e}{y} = \frac{\delta\beta}{\theta(1-(1-\delta)\beta)}$. The loglinearized dynamics of (home) net foreign assets is given by:

$$\widehat{nfa}_t - \beta^{-1}\widehat{nfa}_{t-1} = \widehat{y}_t - (1-\tau)\,\widehat{c}_t - \tau\widehat{n}_{e,t} - \frac{\alpha(\theta-1)}{\theta}\widehat{s}_t,$$

where $\widehat{nfa}_t = \frac{nfa_t}{y}$ and $nfa_t = \frac{e_t a_t}{p_{c,t}}$. Finally, the loglinearized interest rate parity writes:

$$\widehat{i}_t = \widehat{i}_t^* + E_t\left\{\widehat{e}_{t+1}\right\} - \widehat{e}_t - y\Phi\widehat{nfa}_t.$$

C Welfare criterion

Let Λ denote the percentage of permanent steady state consumption that agents would be willing to give up to live in a world free of fluctuations, i.e.

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(u\left(c_t, \ell_t\right) + u\left(c_t^*, \ell_t^*\right) \right) = 2E_0 \sum_{t=0}^{\infty} \beta^t u\left(\left(1-\Lambda\right) c, \ell\right). \tag{3}$$

We use a second-order approximation of the utility function after posing $h_t = 1 - \ell_t$ and neglecting cross-products:

$$u\left(c_t, \ell_t\right) \simeq u + (1-\gamma)\, u \left(\eta \widehat{c}_t + (1-\eta)\, \widehat{h}_t + (1-\gamma) \left(\frac{\eta^2}{2} \widehat{c}_t^2 + \frac{(1-\eta)^2}{2} \widehat{h}_t^2 \right) \right).$$

Plugging into (3) and recalling $E\left(\widehat{c}_t\right) = E\left(\widehat{h}_t\right) = 0$, we get:

$$E_0 \sum_{t=0}^{\infty} \beta^t (1-\gamma)^2 \left(\frac{\eta^2}{2} \left(\widehat{c}_t^2 + \widehat{c}_t^{*2} \right) + \frac{(1-\eta)^2}{2} \left(\widehat{h}_t^2 + \widehat{h}_t^{*2} \right) \right) = \frac{2\left((1-\Lambda)^{\eta(1-\gamma)} - 1 \right)}{1-\beta}.$$

We finish by using the fact that $\sum_{t=0}^{\infty} \beta^t E\left(\widehat{x}_t^2\right) \simeq \frac{var(\widehat{x}_t)}{1-\beta}$, $\forall \widehat{x}_t$, which implies:

$$\Lambda = 1 - \left(1 + \frac{1}{2} \eta^2 (1-\gamma)^2 \left(\frac{var(\widehat{c}_t) + var(\widehat{c}_t^*)}{2} \right) + \frac{1}{2} (1-\eta)^2 (1-\gamma)^2 \left(\frac{var\left(\widehat{h}_t\right) + var\left(\widehat{h}_t^*\right)}{2} \right) \right)^{\frac{1}{\eta(1-\gamma)}}.$$

Finally, given that $h_t = 1 - \ell_t$:

$$var\left(\widehat{h}_t\right) = \left(\frac{\ell}{1-\ell} \right)^2 var\left(\widehat{\ell}_t\right),$$

and

$$\Lambda = 1 - \left(1 + \Upsilon_c \left(\frac{var(\widehat{c}_t) + var(\widehat{c}_t^*)}{2} \right) + \Upsilon_\ell \left(\frac{var\left(\widehat{\ell}_t\right) + var\left(\widehat{\ell}_t^*\right)}{2} \right) \right)^{\frac{1}{\eta(1-\gamma)}},$$

where $\Upsilon_c = \frac{1}{2} \eta^2 (1-\gamma)^2$ and $\Upsilon_\ell = \frac{1}{2} (1-\eta)^2 (1-\gamma)^2 \left(\frac{\ell}{1-\ell} \right)^2$.