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Stability in informal insurances: an approach by networks and overlapping coalitions

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Abstract

Based on empirical facts, we build a model of informal insurances where risk sharing groups are overlapping homogenous coalitions, originating from networks of historical trust relationships. We derive a general folk theorem under uncertainty and we identify the determinants of the stability of informal insurances. Our results are robust to social norms and provide theoretical explanations for empirical findings such as the puzzle that rich families in rural economies in developing countries consume less. Our model bridges the two traditional approaches of clubs and bilateral agreements.

Keywords : Informal insurance, stability, networks, overlapping coalitions
JEL classification : O17, Z13, D85, D71

1 Introduction

Most of the rural economies in developing countries including Sub-Saharan Africa are based on agriculture only. In such areas, individual incomes are so volatile due to negative shocks like poor weather, illness, crop diseases, wild animals' invasion, and crops damage by fire among others.[1] With such precarious incomes, the need for insurance on consumption is huge. Paradoxically, formal credit or insurance markets do no exist in such areas. Nevertheless, people share risk by mutually insuring each other through informal arrangements of reciprocal transfers known as *informal insurances*.[2] More than simple agreements, such arrangements are organized as institutions, governed by norms that determine the nature, the channels, and the amount of transfer to be made to anyone who suffers a negative idiosyncratic income shock. However, the big problem with such insurance is its informal nature. There is no signed paper, no formal collateral, no legal court to refer to in case of violation. The only way for these informal insurances to survive and perpetuate the mutual assistance is to be self enforced in the sense that the punishment any person suffers for violating the arrangement discourages him from doing so. In developing countries, policy makers are interested in the determinants of the stability of informal insurances because these are the central factors on which they can act to improve the conditions of life in rural economies. For example in a stable environment, helping some key people will be beneficial to the whole economy through ex-post transfers. But when stability fails, such help can be negative in the sense that it can destabilize the whole informal insurance institution. Stability deeply depends on the characteristics of the people, the norms that are in place and specifically on the structure of such institution. Consequently, a thorough study on the determinants of stability in informal insurance must properly identify the real characteristics of these institutions.

The aim of this paper is to analyze the stability of informal insurance institutions in rural economies (villages) in developing countries. Even though the topic has been widely visited in the literature, the particularity of our paper resides in the fact that our approach is more realistic. We base our model essentially on empirical findings and personal knowledge (because the author personally experienced informal insurance in his young age). In contrast, the bulk of the economic literature takes two extreme avenues to model the structure of such institutions in a village. The first one considers the whole village as one club with the same norm for everybody and where villagers make transfers to one another without any

1. See Postner (1980) for characteristics of agrarian economies.
2. For more information about informal insurance, see Rosenzweig (1989) and Udry (1990, 1994).

restriction. Empirical findings prove this structure wrong. Unlike the first one, the second approach considers informal insurance institutions as a collection of bilateral agreements and uses networks to model transfers. This approach also is not fully supported by empirical findings. Therefore, we question the accuracy of any policy which is based on unrealistic fundamentals. In this paper, we propose a new approach based on overlapping risk sharing groups which can only be obtained from a preexisting trust relationship network. This approach meets empirical facts and bridges the two extreme views of clubs and networks. In this realistic setting, we identify the characteristics of stable informal insurance institutions which are robust to social norms. Thus we contribute to the literature on self-enforcing insurance arrangements, pointed out since 1980 by Postner and established at first by Kimball (1988).

In this setting, we have three main results with our general model. First, we derive a general folk theorem under uncertainty. More precisely, we show that each informal insurance institution with enough patient individuals can be stable. The charm of this result resides in the fact that it is obtained in an environment of uncertainty and it is robust to the severity of the punishment. Second, we identify the complete graph as the network which is the most likely to be stable. Unlike previous attempts, we theoretically prove this wellknown empirical fact independently of the value of the discount factor and the severity of the punishment. We neither impose a specific punishment nor that the discount factor tends to one or zero as it is always done. The robustness of our result provides an explanation of the almost full risk risk pooling for consumption goods observed in Indian villages by Townsend (1994). Our third main result identifies external effects as determinants of stability. This result may help to explain the puzzling empirical conclusion of Ligon et al. (2002) that wealthier households tend to consume less. Finally, we apply our model to a specific economic environment to derive comparative statics for some exogenous parameters.

The remainder of the paper is organized as follows. In Section 2 we explore the backgrounds of informal insurance institutions. In Section 3, we set up the main features of the model and we present the results in Section 4. We conclude the paper in Section 5. Throughout the paper, we summarized the most important results in propositions and we relegated the proofs to the Appendix.

2 Background

2.1 Models of informal insurance

To the best of our knowledge, the economic literature follows two approaches to study informal insurances taking place in small villages in developing countries.

The first approach considers the whole village as a club where each villager is called to make transfers to the victims of idiosyncratic negative income shocks. Thus, the village is a single risk sharing group and any violator is excluded from this club. He must not expect any transfers from other villagers in the future. Proponents of this approach are Kimball (1988), Coate and Ravallion (1993), Ligon et al. (2002), Genicot and Ray (2003) to name but a few. But if this structure was real, and the villagers were rational, one should expect full risk sharing in the village. However, most of the empirical findings do not support this prediction. In fact, Deaton (1992) with data from Ghana and Thailand, Grimard (1997) with data from Côte d'Ivoire, and Fafchamps and Lund (2003) with data from Philippines, found that risk sharing groups are of smaller size than the whole village. Murgai et al. (2002) explain the limited size of the informal institutions by the high cost of links that convey transfers. But based on data from three Indian villages, the seminal paper by Townsend (1994) clearly states that risk sharing groups have limited size (although the evidence of almost full risk pooling for consumption goods) and suggests instead a structure of informal insurance institutions based on social networks.

Following the recommendations of Townsend (1994), the second approach models informal insurance institutions as a collection of bilateral agreements within villagers. Following some networks, some pairs of villagers set bilateral agreements to make reciprocal transfers to insure each other from negative idiosyncratic income shocks. For most of the models in this approach, any villager who violates the agreement will not receive future transfers from the victims of the violations. In contrary, Bloch et al. (2008) propose an alternative option where not only the victims but other villagers who are informed about the violation also sanction the violator. Proponents of this approach are De Weerdt and Dercon (2005), Fafchamps and Gubert (2007), Bramoullé and Kranton (2007), Bloch et al. (2008), Ambrus et al. (2010) to name but a few. Again, it is empirically documented that the size of a limited risk sharing group is generally greater than two.

Ultimately, neither the approach by clubs, nor the one by networks is fully supported by empirical facts. Therefore, we are concern that a policy which is based on the results of such models may be not fully effective.

2.2 Stylized facts

A foreign researcher who studies informal insurance may not be fully aware of some hidden realities, specific to rural communities in developing countries. Cultures and customs are different. But we know, coming from a small village in Western Africa where there is not a single formal bank even in 2014, that there are some common stylized facts surrounding informal insurances. Rural communities are made of families that belong to small homogenous groups like clans, castes or ethnic groups. For example, in the event of a funeral in group A, the social norms recommend that group A members help the bereaved family to alleviate expenses. This help mostly takes the form of direct monetary or nonmonetary transfers for the funerals which are very costly. There are also indirect transfers from other groups as well. If a relative of the bereaved family in group A is married to someone in a group B then some people from group B will send transfers through this relative. Thus, group B members do not necessarily make direct transfers to the bereaved family, but they channel them via their own members who belong also to group A and are linked to the bereaved family. One last important thing highly valued is the number or participants in the funeral, even if they do not contribute to its planning. Thus, even if a foreign researcher observes risk sharing in the whole village, he may not necessarily be aware of some specific codes. Therefore, based on personal anecdotes reinforced by empirical researches, we identify the following six stylized facts as mostly common to rural economies in developing countries.

Fact 1 : Informal insurance is organized inside institutions which are governed by secular norms [Fafchamps and Lund (2003) with data from villages in Côte d'Ivoire]. Such norms determine not only the nature and the amount of transfers but also design the punishment that follows any violation.

Fact 2 : Informal insurance groups are not built strategically to share risk but are organized following a trust relationship [Fafchamps and Gubert, 2006, with data from villages in Philippines].

Fact 3 : Risk sharing groups are homogenous ethnic groups, families, clans, castes, and others [Grimard (1997) with data from Côte d'Ivoire].

Fact 4 : Risk sharing groups are not disjoint but mostly overlap [De Weerdt and Dercon (2005) with data from Tanzanian villages].

Fact 5 : There are spillover effect across risk sharing groups [Angelucci and De Giorgi (2009) with data from Mexican villages].

Fact 6 : Full insurance is achieved only within groups of reduced size but not necessarily for the whole village [Townsend (1994) and Ligon et al. (2002) with data from villages in southern India, Fafchamps and Lund (2003) with data from villages in Côte d'Ivoire].

In this paper, we model informal insurance based on those stylized facts. In each village there exists historical links between villagers. Such links reinforce trust between them and the collection of linked individuals in this network of trust relationships defines the population of an informal insurance institution. Directly linked individuals in the trust relationship network are considered homogenous, and form risk sharing groups. Such groups are not disjoint but they overlap. Thus, there are spillover effects across risk sharing groups since some individuals belong to several of them. The collection of such risk sharing groups constitutes an informal institution which is composed of less people than the whole village. Within such institution of limited size, villagers are fully insured against idiosyncratic negative income shocks.

2.3 Why networks are essential to form risk sharing groups

One of the fundamental ingredients of our model of informal insurances is the network of trust relationships. Such network exists for some traditional reasons that are beyond the scope of this paper and we take it as given. Each risk sharing group is derived from this network. In the event of a violation, the norm imposes the appropriate punishment. Whatever its severity, the punishment results in the severance of links and the resulting network is completely different from the original one. Consequently, the structure of the resulting risk sharing groups is completely modified. For these reasons, we must look more closely at the networks.

A network g is a list of unordered pairs of individuals $\{i, j\}$. In its graphical representation, the nodes are individuals and the edges represent the links between them. Two individuals are *directly linked* if their pair belong to the network. A *path* of length m between two individuals i and j in the network g, is a sequence $i_1, \ldots, i_{m+1}$ such that $\{i_l, i_{l+1}\} \in g$ for each $l \in \{1, \ldots, m\}$, with $i_1 = i$ and $i_{m+1} = j$. Two individuals i and j are *indirectly linked* in the network g if there exists a path of length $m > 1$ between them. The following Figure 1 shows some common networks of eight persons. [3]

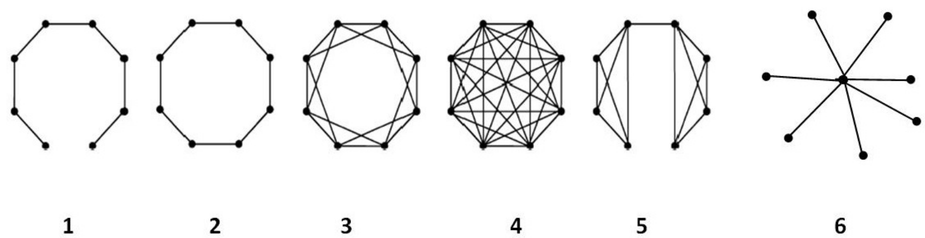

Figure 1 : 8-person networks

(1) Simple tree (2) Circle (3) Two-neighbors (4) Complete (5) Bridge (6) Star

If we take a snapshot of trust relationships in a village in a developing country, we observe several networks. Each of these networks constitute an informal insurance institution on its own and Figure 1 shows some different common structures for eight persons.

2.4 From networks to risk sharing groups

In network theory, a *clique* is a maximal set of directly linked players in a network g. Thus, each clique is a subset of the population, or simply a coalition. Obviously, such coalitions are overlapping. Agbaglah (2014) shows that each network is uniquely represented by the collection of all its cliques.

In our setting, we take the network of trust relationships as given, and we consider each clique as a risk sharing group. Such group is homogenous because all individuals in a clique

3. The first five networks are from Bloch et al. (2008).

are directly linked. Thus, our model respects the stylized Fact 3. Furthermore, our risk sharing groups are overlapping as required by stylized Fact 4. The collection of such overlapping risk sharing groups (coalitions for simplicity) defines uniquely the network g and represents the informal insurance institution I_g. In the following Example 1,[4] we construct the risk sharing groups and the corresponding informal insurance institutions for various networks.

Example 1 Let $N = \{x, y, z, a, b\}$ be the population of an informal insurance institution. For each network g in the Figure 2, we construct the corresponding risk sharing groups (cliques) and I_g. We use the following Table 1 for simplicity.

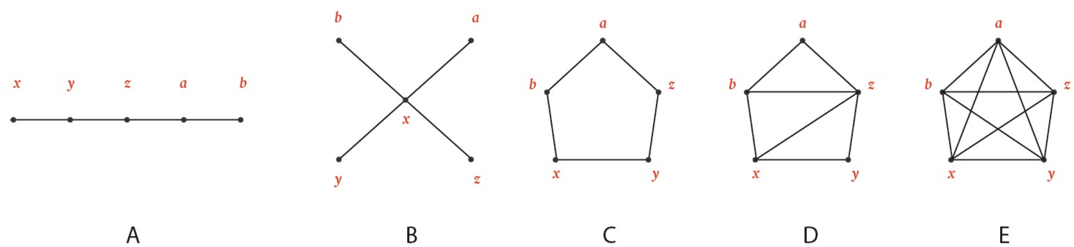

Figure 2 : examples of 5-person networks

Table 1 : Informal insurance institution, I_g, as a collection of risk sharing groups S_j.

Network g	Risk sharing groups S_j	I_g
A	$S_1 = xy; S_2 = yz; S_3 = za; S_4 = ab$	$I_A = \{xy, yz, za, ab\}$
B	$S_1 = xy; S_2 = xz; S_3 = xa; S_4 = xb$	$I_B = \{xy, xz, xa, xb\}$
C	$S_1 = xy; S_2 = yz; S_3 = za; S_4 = ab; S_5 = xb$	$I_C = \{xy, yz, za, ab, xb\}$
D	$S_1 = xyz; S_2 = xzb; S_3 = zab$	$I_D = \{xyz, xzb, zab\}$
E	$S = xyzab$	$I_E = \{xyzab\}$

Notice that if the trust relationship is the complete network (E), then we have a unique risk sharing group as in the approach by clubs. On the other hand, if we have a tree (A or B), then risk sharing groups are bilateral agreements. Far beyond these two extremes, we have intermediate structures as well. Thus our approach encompasses the traditional ones.

4. We borrow the Example 1 from Agbaglah (2014).

In the remainder of the paper, we use the term coalition at the place of risk sharing group for simplicity. We also we use the term *overlapping individual* to designate each person who belongs to more than one coalition in I_g. Thus, in the Example 1, y, z and a are overlapping individuals in I_A.

3 Main features

An informal insurance institution I_g is a collection of risk sharing groups in a population N of n individuals, linked through a network g of trust relationships in an agrarian village. At any date, a state of nature $\theta \in \Theta$ is realized with a probability $p(\theta)$. We assume that Θ is finite. In each state θ, an income distribution $y(\theta) = \left(y^i(\theta)\right)_{i \in N}$ is realized. Each such $y^i(\theta)$ belongs to a finite set Y of positive numbers. We assume that realized incomes are independent. [5]

In the remainder of the section, we describe the norms and the people, and we derive a definition of stability in our setting.

3.1 Norms

Traditional norms play a crucial role in informal insurance institutions. Should an individual face an idiosyncratic negative income shock, such norms impose not only the scheme of the transfers he receives, but also the nature and the severity of the punishment that any violator will suffer. Such severity results in the number of links that will be severed with the deviator. The punishment can be as harsh as to completely isolate the violator from the whole community, or as weak as to prevent the victim alone from future interactions with the violator. Bloch et al. (2008) define the former punishment as the *strong punishment* the later one as the *weak punishment*. Although we discuss such extreme cases in this paper as benchmarks, we do not impose any specific punishment because norms are different from one community to another. As a result, almost all our results are robust to the severity of the punishment.

According to stylized Fact 6, full insurance is achieved within groups of reduce size which

5. All our results hold under the weaker condition that realized incomes are not perfectly correlated.

are represented by informal insurance institutions in our setting. In an utilitarian point of view, full insurance implies equal sharing. This is, each individual in the informal insurance institution must end up with the same amount of consumption good. This amount is the average of the realized income. Notice that each individual can only make transfers to, or receive transfers from someone in the same risk sharing group. The only possibility to make or receive transfers across coalitions is via overlapping individuals.

Formally, we represent received transfers by positive numbers and sent transfers by negative ones. For each coalition $S \in I_g$ and state $\theta \in \Theta$, we denote by $z^i_S(\theta)$ the net transfer that individual $i \in S$ makes within S. The overall net transfer for i in the informal insurance institution is $z^i(\theta) = \sum_{i \in S,\ S \in I_g} z^i_S(\theta)$ and he consumes $c^i(\theta) = y^i(\theta) + z^i(\theta)$.

Notice that if i is not an overlapping individual, then he belongs to only one coalition S and $z^i(\theta) = z^i_S(\theta)$. For simplicity in the remainder of the paper, we write variables without their argument θ.

As overlapping individuals make or receive transfers across coalitions, the norm needs to be *consistent* in the distribution of individual consumptions within coalitions. This is, if i belongs to $S_1 \cup S_2$, his consumption as a member of S_1 must be the same as if he is considered instead as a member of S_2. The following proposition determines the consumption within each coalition that is consistent with the equal sharing in I_g.

Proposition 1 *The equal sharing is consistent if and only if each individual in a risk sharing group S consumes a fixed amount $\bar{c}_S$ of consumption good.*

Notice that in the Proposition 1, $\bar{c}_S$ is not necessarily the average income within S because of transfers outside of S.

We prove the proposition by firstly showing its evidence for coalitions without overlapping individuals, and after we extend the proof to more than one coalitions.

3.2 People

Villagers in agrarian economies attach a special value to respect, visits, helps in one's farm, high influence in decision committees, high audience in village assembly, high honor during ceremonies and other social privileges. These privileges increase with the acquaintanceship. Such intangible goods are not consumable products and we do not find any route how they can be converted into consumption. According to the stylized Fact 6, there are spillover effects across risk sharing groups. One way to include such externalities in the model is to covert them into consumption as in Ambrus et al. (2010). In this paper instead, we avoid this shortcut by using the number of direct links to represent the spillover effects. The idea behind is that, within the same risk sharing group all individuals have the same number of direct links. But an overlapping individual has additional direct links from other risk sharing groups he belongs to and consequently he has additional social privileges. Since insurance is about consumption, we view such additional social privileges as spillover effects. Therefore, we endow each individual with a utility function that depends not only on the consumption good, but also on the architecture of the underlying network of trust relationships.

Formally, for each individual i, the utility function is

$$U_i(c, g) = u(c) + f(d_i(g))$$

where u is a smooth function, strictly concave and increasing, f is a smooth and increasing function, and $d_i(g)$ is the total number number of i's direct links in the network g. We make the additional assumption that $f(0) = 0$. This is, one who lives in autarky does not enjoy any spillover effects.

Finally, we assume that each individual discounts the future at the same rate $\delta \in [0, 1]$.

3.3 Stability

For two individuals i and j in the same coalition, we say that i *deviates* from j, if i reneges on the transfer to j. We say i deviates from a set $D \subseteq N$ if i deviates from all $j \in D$. Such sets D are included in $\mathcal{S}^i$, the union of all the coalitions i belongs to.

If i deviates from D, he suffers a punishment (imposed by the norm) which results in severance of links in g and a residual network $g_R \subsetneq g$ is obtained. Notice that the number of

i's links to be severed from g to obtain g_R depends on the severity of the punishment in I_g. For the weak punishment, each individual in D severs his link with i, whereas for the strong punishment, all the links with i within the coalitions containing D members are severed.
We assume that each risk sharing group, attempts to implement some symmetric and stationary risk-sharing arrangement throughout the time. Genicot and Ray (2003) discuss the extension to non-symmetric, non-stationary, and history dependent arrangements.
Let c^i be i's consumption at each date if he abides by the norm. If i deviates from D, let c^i_D be his current consumption and c^i_R his future consumption after being punished and let g_R be the residual network. Notice that in a stationary setting, c^i_R and g_R are constant in time.

First, if i and all others abide by the norm, his lifetime utility is

$$U_i\big(c^i, g\big) + \frac{\delta}{1-\delta}\sum_{\theta} p(\theta)U_i\big(c^i(\theta), g\big)$$

Second, if i deviates from a set D, his lifetime utility is

$$U_i\big(c^i_D, g\big) + \frac{\delta}{1-\delta}\sum_{\theta} p(\theta)U_i\big(c^i_R(\theta), g_R\big)$$

We say I_g is *stable* if it is *immune to any individual deviation*. This, is

$$U_i\big(c^i, g\big) + \frac{\delta}{1-\delta}\sum_{\theta} p(\theta)U_i\big(c^i(\theta), g\big) > U_i\big(c^i_D, g\big) + \frac{\delta}{1-\delta}\sum_{\theta} p(\theta)U_i\big(c^i_R(\theta), g_R\big)$$

for all $i \in N$, $D \subseteq \mathcal{S}^i$, and $g_R \subset g$. If we replace U_i by its value and rearrange slightly, we obtain the following formal definition of stability.

Definition 1 *Let $\theta_0 \in \Theta$ be the current state of the nature and let $\beta \equiv (\theta_0, i, D, g_R)$, and $\mathcal{B}$ the collection of all such tuples.*
I_g is stable if and only if, for all $\beta \in \mathcal{B}$,

$$u\big(c^i_D(\theta_0)\big) - u\big(c^i(\theta_0)\big) < \frac{\delta}{1-\delta}\Big\{\sum_{\theta} p(\theta)\big[u\big(c^i(\theta)\big) - u\big(c^i_R(\theta)\big)\big]\Big\} + \frac{\delta}{1-\delta}\big\{f\big(d_i(g)\big) - f\big(d_i(g_R)\big)\big\} \tag{1}$$

Notice that we do not impose any specific punishment in the definition of stability. For the benchmark punishments, we say that I_g is *weak-stable* (respectively *strong-stable*) if it is stable under the strong (respectively weak) punishment. Similar definitions of stability are obtained in different contexts by Coate and Ravallion (1993), and also by Bloch et al. (2008). The Definition 1 shows the existence of two consequences following deviation.

The first *direct* consequence,

$$\frac{\delta}{1-\delta}\Big\{\sum_{\theta} p(\theta)\big[u\big(c^i(\theta)\big) - u\big(c^i_R(\theta)\big)\big]\Big\} ,$$

is the loss of consumption smoothing coverage.

The second *indirect* consequence,

$$\frac{\delta}{1-\delta}\big\{f\big(d_i(g)\big) - f\big(d_i(g_R)\big)\big\} ,$$

is due to the spillover effect which is a loss of social privileges following the severance of links.

4 Results

4.1 General results

An informal insurance institution I_g is stable if none of its risk sharing group is broken. This is, a stable I_g is immune to any individual deviation. In this section we investigate the determinants of the stability of I_g.

We first compare the stability of I_g under the benchmark punishments. Let $\mathcal{WS}$, $\mathcal{SS}$, and $\mathcal{PS}$ be the set of all networks g such that I_g is respectively weak-stable, strong-stable, and stable under an intermediate punishment $\mathcal{P}$. The following propositions compares the sets $\mathcal{WS}$, $\mathcal{SS}$, and $\mathcal{PS}$.

Proposition 2 $\mathcal{SS} \subseteq \mathcal{PS} \subseteq \mathcal{WS}$

We prove the Proposition 2 by contraposition in the Appendix. It states that there exists a set interval of underlying networks in stable informal insurance institutions. The lower bound is $\mathcal{SS}$ and the upper, $\mathcal{WS}$. For any other punishment scheme, the set of networks g such that I_g is stable, lies between these bounds.

Notice that if g is a tree, then strong and weak punishments result in the same residual network g_R and therefore all kinds of stability are equivalent. But generally, the strong-stability implies the weak-stability but the converse is not necessarily true as we show it in the counterexample below.

Counterexample

The counterexample is obtained when g is the complete 3-person network, with $f(x) = x$, and two possible income realizations 0 and 1 with equal probability, $u(c) = c - \lambda c^2$, $\lambda = 0.4$, $\delta = 0.2$. $\square$

Our first series of result concerns discounting. Since the decision to deviate or not is an intertemporal arbitrage, the discount factor is fundamental for stability. As most of the papers in this strand of the economic literature, we first characterize the stability assuming that δ is close to unity. Furthermore, we go beyond this limitation and provide a more general result.

Proposition 3 *Everything else equal, if δ tends to unity, then I_g is stable for all network g.*

The Proposition 3 states that stability is guaranteed for each informal insurance institution where people are patient enough. Kimball (1988), Ligon et al., 2002, Bloch et al. (2008) obtain similar results.

Our first main result, the following Proposition 4, provides a characterisation of the stability that is robust to the value of the discount factor.

Proposition 4 *For all network g, there exists $\delta_0 \in (0,1)$ such that for all $\delta \in (\delta_0, 1)$, I_g is stable.*

The Proposition 4, that we prove in the Appendix, is a folk theorem operating in an environment of uncertainty since future incomes are not known. Besides, it is robust to the severity of the punishment. Under the strong punishment and certainty, we obtain the popular folk theorem.

Notice that the threshold δ_0 depends on the punishment and the characteristics of the underlying network g. However, as we do not impose any parametrization for the basics of the model, it is not obvious how this dependence functions. In the sequel, we simulate the stability of the six distinct 8-person networks in Figure 1 and we summarize the results in Table 2 and Table 3. Our goal is to identify the characteristics of networks that affect δ_0. [6] For this purpose, we vary δ_0 in the range of 0.18 to 0.62 and we compute the stability result for each network, keeping everything else constant. We use three common characterizations of networks, the *sparseness*, the *clustering* and the *density*. The sparseness coefficient of a network quantifies how sparse it is. The density coefficient keeps track of the relative fraction of links that are present in the network. The clustering coefficient is a measure of cliquishness, that accounts for the number and size of cliques of a network. [7] We remind the reader that each clique is a risk sharing group in our setting.

As we can see from Tables 2 and 3, sparseness seems to be irrelevant for stability.
For weak stability, apart from the circle, the number of links and density tend to be negatively correlated to δ_0. The combined effect of these characteristics places the circle (8 overlapping individuals and 8 links) below the star (1 overlapping individual and 7 links) and the simple tree (7 overlapping individual and 7 links).
For strong stability, apart from the bridge (2 overlapping individuals and 13 links) and the circle, the number of links, density, and clustering tend to be negatively correlated to δ_0.

To conclude, the simulations confirm that there is no obvious relation between the characteristics of networks and the threshold δ_0. However, the complete network seems to match the lowest δ_0, for both strong and weak stability. By Proposition 2, this matching may work for all possible punishments. Our second main result, the following Corollary 1, confirms this observation formally.

6. Details of the simulations are available upon request.

7. See Bloch et al. (2008) for details on sparseness and for more details on the characteristics of networks, see Jackson (2008).

Table 2 : Weak stability

Network	**Characteristics**				δ_0
	Sparseness	**Links**	**Clustering**	**Density**	
Complete	1	28	1	1.00	0.18
Two-neighbors	2	16	0.5	0.57	0.28
Bridge	1	13	0.8	0.46	0.53
Star	0	7	0	0.25	0.56
Simple tree	0	7	0	0.25	0.58
Circle	6	8	0	0.29	0.62

Table 3 : Strong stability

Network	**Characteristics**				δ_0
	Sparseness	**Links**	**Clustering**	**Density**	
Complete	1	28	1	1.00	0.26
Two-neighbors	2	16	0.5	0.57	0.54
Star	0	7	0	0.25	0.56
Simple tree	0	7	0	0.25	0.58
Bridge	1	13	0.8	0.46	0.58
Circle	6	8	0	0.29	0.62

Corollary 1 *I_g is the most likely to be stable if and only if g is the complete network.*

Empirically, it is a common knowledge in this strand of the economic literature that the complete network is more likely to induce full risk sharing. Theoretically, Bloch et al. (2008) found a similar result in the bilateral setting for low values of δ. Our result in the Proposition 4 is robust to both the values of the discount factor and the severity of the punishment. It is therefore a more general result. Furthermore, the complete network in our setting corresponds to the situation where villagers are so homogenous and so trusting that they form together a single risk sharing group. Thus, the result is very intuitive although the setting is very general. Ultimately, the robustness of this result may help explain the almost full risk pooling for consumption goods that Townsend (1994) found for Indian villages.

Our third main result, the Proposition 5, relates to the influence of the spillover effects on stability. Individual decisions to deviate or not also affect other's wellbeing through spillover effects. The best way to understand the importance of such effects, is to compare our setting with the situation without externalities.

Suppose that we do not account for spillover effects. This is, $f \equiv 0$ in the utility function. Thus, I_g is stable if and only if for all $\theta_0 \in \Theta$, $i \in N$, and $D \subseteq \mathcal{S}^i$

$$u\big(c^i_D(\theta_0)\big) - u\big(c^i(\theta_0)\big) < \frac{\delta}{1-\delta}\Big\{\sum_{\theta} p(\theta)\big[u\big(c^i(\theta)\big) - u\big(c^i_R(\theta)\big)\big]\Big\}, \tag{2}$$

Thus, under the weak punishment, the incentive to deviate is very high because it is sufficient to be minimally linked in the network g to be fully insured. Therefore, the only possible strong-stable informal insurance institution is I_g such that g is a tree. This result is also obtained by Bloch et al. (2008) and Bramoullé and Kranton (2007). On the other hand, under the strong punishment, each violator loses all informal insurance coverage even if he deviates from only one person. Thus, only overlapping individuals may deviate since they can deviate in one risk sharing group and still be insured in others. But transfers across coalitions will be jeopardized if overlapping individuals deviate and this is so bad for insurance.

Back to our setting, we notice that under the weak punishment it is no more sufficient to be minimally linked because social privileges are also valued. Thus, the incentive to deviate is lowered. This is also true for overlapping individuals under the strong punishment. Ultimately, the existence of the spillover effects compensates overlapping individuals for spreading insurance across risk sharing groups. Thus it is rational for them in our setting to collect transfers from one coalition and deliver it in another without the incentive to deviate. This may explain why it is empirically observed that risk sharing groups overlap.

Formally, consider the difference in the utility of an individual who loses one link while his consumption is unchanged. For an individual i, such utility cost is $U_i(c, g) - U_i(c, g_R)$

where g_R is a subnetwork of g such that $d_i(g) - 1 = d_i(g_R)$. We have

$$U_i(c, g) - U_i(c, g_R) = u(c) + f(d_i(g)) - (u(c) + f(d_i(g) - 1) = f(d_i(g)) - f(d_i(g) - 1.$$

Obviously, this utility cost depends on $d_i(g)$. Define the *external utility cost*, $\mathcal{C}$, as the maximal value of such utility cost. Formally, $\mathcal{C} \equiv Max_{x \in \{1,2,\ldots,n-1\}}\{f(x) - f(x-1)\}$

Proposition 5 *If the external utility cost is high enough, then I_g is stable for all network g.*

We prove the Proposition 5 in the Appendix. This proposition helps to explain the puzzling situation pointed out by Ligon et al. (2002) that wealthier households tend to consume less. In fact, we view wealthier households as the most likely to be overlapping individuals. Thus, if we consider consumption alone, it is puzzling why they consume like every one else in I_g. But if they value social privileges having high external utility costs, then consuming like every one else in I_g becomes rational. Thus, this seemingly puzzling behavior in the setting of consumption alone, finds a rational explanation in our setting.

4.2 Application

In this section we use a specific functional form to study how exogenous parameters like the level of income, the income dispersion, the probability of income realizations, and the risk aversion affect the stability of an informal insurance institution.

In this application, similarly to De Weerdt and Dercon (2005), the network g of trust relationships is composed of two complete subnetworks connected by one overlapping individual. This specification of g is simple but sufficient to identify the role of distinct risk sharing groups and overlapping individuals. The corresponding informal insurance institution is $I_g = \{S_1, S_2\}$ with $S_1 \cap S_2 = \{o\}$. Individual o is the only overlapping individual in this specification. There are two possible incomes, $y + d$ and y, such that $y > 0$, $d > 0$, and $y + d$ is realized with probability p. The parameter d measures the dispersion in income and y the level of income. The utility over consumption is the same as in Bramoullé and Kranton (2007), $u(c) = c - \lambda c^2$. Notice that there is a positive relation between λ and the coefficient of risk aversion. For u to be strictly increasing and concave, we need $0 < \lambda < \frac{1}{2y_h}$. Finally, $f(x) = \alpha x$, with $\alpha > 0$.

In the following, we derive comparative statics for the parameters y, p and λ. For each $x \in \{y, p, \lambda\}$, we use the notation (I_g, x_k) to designate the informal insurance institution where, everything equal, the parameter x takes the value x_k.

Proposition 6 ***Comparative statics***

1. *For two values of realized income y_1 and y_2 such that $y_2 > y_1$, if (I_g, y_1) is stable, then (I_g, y_2) is also stable.*
2. *For two values of risk aversion, λ_1 and λ_2, such that $\lambda_2 > \lambda_1$, if (I_g, λ_1) is stable, then (I_g, λ_2) is also stable.*
3. *For two probabilities of the high income realization, p_1 and p_2, such that $p_2 > p_1$,*
   - *For $0.5 > p_2 > p_1$, if (I_g, p_1) is stable, then (I_g, p_2) is also stable.*
   - *For $p_2 > p_1 > 0.5$, if (I_g, p_2) is stable, then (I_g, p_1) is also stable.*

We prove Proposition 6 in the Appendix. The proposition states that the stability of the informal insurance institution is reinforced if the population becomes wealthier, as long as the dispersion in income remains the same. Furthermore, an increase in the aversion over risk preserves stability. This is very intuitive because, as individuals become more averse to risk, they value more insurance and therefore avoid to deviate. Finally, for low probabilities, an increase in the probability of the realization of the highest income preserves stability. On the other hand, if this probability is high enough, then a slight decrease does not affect the stability.

In the remainder of the section, we simulate several values of the exogenous parameters. For each simulation, we vary δ on a grid and we compute the proportion of stable informal insurance institution corresponding to the parameter of interest. We compute our results graphically in Figure 3 and Figure 4 respectively for weak-stability and strong-stability.

In Figure 3, we isolate the values of the discount factor corresponding to a non-ambiguous ranking of the exogenous parameters. We obtain the following ranking from the most to the least pertinent parameter for weak-stability. We have in this order, the level of income, the risk aversion, the dispersion in income, and the probability of the realization of the highest income. Notice that the rankings of risk aversion and dispersion in income are reversed for $\delta > 0.55$. The result in Figure 4 is obtained for strong-stability. As we can see from the graph, the ranking is reversed and the probability of high income realization becomes the most pertinent parameter for stability. It is followed in this order by the dispersion in income, the level of income , and finally risk aversion. This reversion is very intuitive since under weak punishment, the incentive to deviate is higher. Deviation is avoided only if the dispersion in income is high for wealthier individuals and the high income is more probable to be realized.

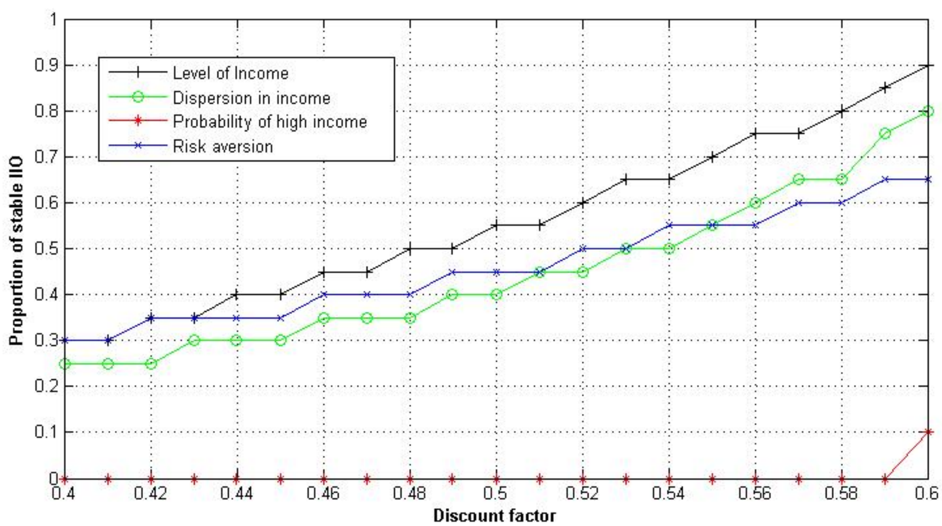

Figure 3 : weak-stability

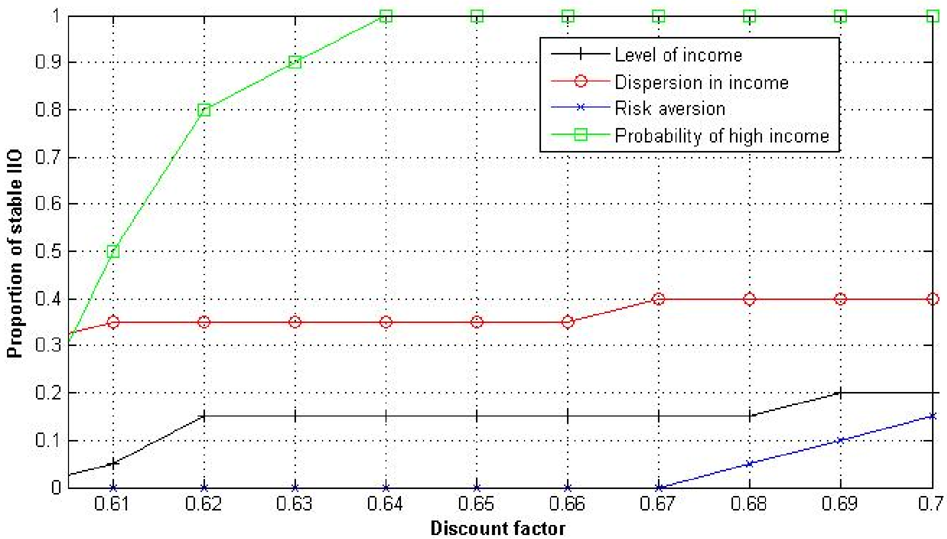

Figure 4 : strong-stability

5 Conclusion

This paper develops the first model of informal insurance which bridges the traditional approaches by clubs and bilateral agreements. Based on stylized facts observed in villages in developing countries, we model informal insurance institutions as risk sharing groups which result from historical trust relationships between villagers. The social norms impose not only the transfers but also the punishment that any violator suffers. In this setting, we investigate the stability in informal insurances and we derive several results which are robust to norms and also to some other individual specifications. Such norms are rigid and the policy maker will hardly change them. Therefore, our robust results can be used to formulate policy recommendations independently of the rigidity of social norms.

First, we show that an informal insurance institution can be stable whatever the underlying network, the severity of the punishment, and the discount rate. However, the policy maker needs to act such as to lower a specific threshold value (the δ_0 in our paper). Any policy that can create partnership between distinct risk sharing groups will lower that threshold. For example, the funding of cultural and sportive events between risk sharing groups in villages may increase the creation of additional links. Thus, if the underlying trust relationships tend to the complete network, full insurance will be achieved in short term.

Second, since overlapping individuals play a crucial role in the transfers across coalitions, a policy maker may identify such key individuals and honor them by distinctions for example. Such social privileges for overlapping individuals will reinforce the stability. Furthermore, the best way to put money in informal insurance institutions and keep stability, is to finance the activities of overlapping individuals. Through the transfers, this policy will increase the level of consumption in the whole village in a stable environment.

Third, from our simulations, the best policy that guarantees the stability of informal insurance institutions is the creation of an environment where incomes are high and dispersions are low. We think of subsidies for fertilizers, modernization of agriculture, and technical assistance for the conservation of crops. The idea is that, if conditions for stability are established, the norms will help mitigate risks, in the absence of formal insurance and credit markets.

Thus, we hope that policy makers can design effective policies to reduce the volatility in consumption based on our results and the specificities of informal insurance institutions. The next step for us is to test the robustness of our results with data.

Appendix

Proof of Proposition 1.

The if part of the proposition is obvious because by equal sharing, each individual consumes the average realized income which is $\overline{c} = \frac{1}{n}\sum_{i \in N} y^i$. Therefore, consumption is identical in each coalition.

In the following we show the only if part.

- If $I_g = \{N\}$

  Suppose each individual in N consumes the same amount of consumption good, $\overline{c}_N$. There is only one coalition and no overlapping individual. Thus, the net transfer within N is null. As a result, $\overline{c}_N = \overline{c}$ and equal sharing obtains. There is no consistency problem here because there is only one coalition.

- If $I_g = \{S, S'\}$, then by construction, there exists at least one overlapping individual

$i \in S \cap S'$.

Suppose each individual in S consumes $\bar{c}_S$ and each one in S' consumes $\bar{c}_{S'}$. Since $i \in S \cap S'$, we have $\bar{c}_S = \bar{c}_{S'}$. Since $S \cup S' = N$, we conclude that $\bar{c}_S = \bar{c}_{S'} = \bar{c}$. This is, equal sharing is consistent.

– This reasoning easily extends to more than two coalitions because each coalition contains at least one overlapping individual. Thus, coalition by coalition, the consumption is the same and since there is no transfer outside of N, the equal sharing obtains for I_g.

$\square$

Proof of Proposition 2.

The proof is by contraposition. Suppose that I_g is not weak-stable. There exists $\beta = (\theta_0, i, D, g_R) \in \mathcal{B}$, such that

$$u\big(c^i_D(\theta_0)\big) - u\big(c^i(\theta_0)\big) \geq \frac{\delta}{1-\delta}\Big\{\sum_\theta p(\theta)\big[u\big(c^i(\theta)\big) - u\big(c^i_R(\theta)\big)\big]\Big\} + \frac{\delta}{1-\delta}\{f_i(g) - f_i(g_R)\},$$

where g_R is the network obtained after the strong punishment following i's deviation to D. Let $g_{R'}$ denote the network that obtains in the same context with the weak punishment. Obviously, $g_R \subseteq g_{R'}$, and the expected residual consumption for i following the weak punishment, $\sum_\theta p(\theta)u\big(c^i_{R'}(\theta)\big)$, is obviously not less than what obtains in the strong punishment case. Therefore, $f\big(d_i(g_R)\big) \leq f\big(d_i(g_{R'})\big)$, and $\sum_\theta p(\theta)u\big(c^i_R(\theta)\big) \leq \sum_\theta p(\theta)u\big(c^i_{R'}(\theta)\big)$ since incomes realizations are independent. Thus,

$$u\big(c^i_D(\theta_0)\big) - u\big(c^i(\theta_0)\big) \geq \frac{\delta}{1-\delta}\Big\{\sum_\theta p(\theta)\big[u\big(c^i(\theta)\big) - u\big(c^i_{R'}(\theta)\big)\big]\Big\} + \frac{\delta}{1-\delta}\{f(d_i(g)) - f(d_i(g_{R'}))\}.$$

Hence, I_g is not strong-stable.

Furthermore, for any punishment scheme $\mathcal{P}$ and the residual network $g_{R''}$, we have $g_R \subseteq g_{R''} \subseteq g_{R'}$. $\square$

Proof of Proposition 3 and Proposition 4.

Fix $\beta \in \mathcal{B}$, and let

$F_\beta(\delta) \equiv (1-\delta)\big[u\big(c^i_D(\theta_0)\big) - u\big(c^i(\theta_0)\big)\big] - \delta\Big\{\sum_\theta p(\theta)\big[u\big(c^i(\theta)\big) - u\big(c^i_R(\theta)\big)\big] + f\big(d_i(g)\big) -$

$f\big(d_i(g_R)\big)\Big\}$.

Notice that $u\big(c^i_D(\theta_0)\big) - u\big(c^i(\theta_0)\big) > 0$ because if i deviates, he consumes more than the prescribed amount of consumption. Also, $\sum_\theta p(\theta)\big[u\big(c^i(\theta)\big) - u\big(c^i_R(\theta)\big)\big] > 0$ because the residual expected utility over consumption is less than the expected utility corresponding to the whole set N. Finally, $f\big(d_i(g)\big) - f\big(d_i(g_R)\big) > 0$ because f is decreasing. By definition, I_g is stable iff for all $\beta \in \mathcal{B}$, $F_\beta(\delta) < 0$.

Proposition 3 :

For a fixed β, F_β is differentiable and decreasing. Furthermore, by the continuity of F_β in the interval $[0,1]$ and the fact that and $F_\beta(1) < 0$, there exists $\delta_\beta < 1$ such that $F_\beta(\delta_\beta) < 0$. As $\mathcal{B}$ is finite, let δ_1 be the maximal value of such δ_β. For each $\delta \geq \delta_1$, I_g is stable.

Proposition 4 :

For a fixed β, F_β is continuous and decreasing on $[0,1]$, and $F_\beta(0) > 0$, and $F_\beta(1) < 0$. By the intermediate value theorem, there exists $\delta_\beta \in [0,1]$ such that $F_\beta(\delta_\beta) = 0$. As $\mathcal{B}$ is finite, let δ_0 be the maximum value of such δ_β. F_β being decreasing, then for all $\delta > \delta_0$, $F_\beta(\delta) < 0$ and this holds for all β. Therefore, I_g is stable for all $g \in G$.

□

Proof of Corollary 1.

We show in the proof of the Proposition 4 that, given a network g, $\delta_0 = Max_\beta\{\delta_\beta\}$.

First, let g be the complete network and $g' \neq g$. For all $i \in N$, $d_i(g) \geq d_i(g')$. As f is decreasing, for all $i \in N$, $f(d_i(g)) \geq f(d_i(g'))$. Since D defines c^i_R, and $d_i(g_R)$, everything equal, $\frac{u\left(c^i_D(\theta_0)\right)-u\left(c^i(\theta_0)\right)}{u\left(c^i_D(\theta_0)\right)-u\left(c^i(\theta_0)\right)+\sum_\theta p(\theta)\left[u\left(c^i(\theta)\right)-u\left(c^i_R(\theta)\right)\right]+f\left(d_i(g)\right)-f\left(d_i(g_R)\right)} \leq \frac{u\left(c^i_D(\theta_0)\right)-u\left(c^i(\theta_0)\right)}{u\left(c^i_D(\theta_0)\right)-u\left(c^i(\theta_0)\right)+\sum_\theta p(\theta)\left[u\left(c^i(\theta)\right)-u\left(c^i_R(\theta)\right)\right]+f\left(d_i(g')\right)-f\left(d_i(g_R)\right)}$. Therefore by maximizing over all $\beta \in \mathcal{B}$, we obtain a lower δ_0 with g. As g' is arbitrary, then δ_0 is the lowest possible.

Second, $\delta_\beta = \frac{u\left(c^i_D(\theta_0)\right)-u\left(c^i(\theta_0)\right)}{u\left(c^i_D(\theta_0)\right)-u\left(c^i(\theta_0)\right)+\sum_\theta p(\theta)\left[u\left(c^i(\theta)\right)-u\left(c^i_R(\theta)\right)\right]+f\left(d_i(g)\right)-f\left(d_i(g_R)\right)}$.

Let $A \equiv u\big(c^i_D(\theta_0)\big) - u\big(c^i(\theta_0)\big)$, $B \equiv \sum_\theta p(\theta)\big[u\big(c^i(\theta)\big) - u\big(c^i_R(\theta)\big)\big]$, and $C \equiv f\big(d_i(g)\big) - f\big(d_i(g')\big)$. Thus, I_g is stable if the network g is such that t A is minimal and $B+C$ is maximal because δ_β is increasing in A and decreasing in $B+C$ and A, B, and C are independent. As D defines c^i_R, and $d_i(g_R)$, if we maximize over θ_0 and D, we obtain $c^i_D = y_h$, the highest income in Y, for θ_0 such that only i gets y_h and all others get y_l, the lowest income in Y. Thus $d_i(g_R) = 0$ and the residual set is the singleton set. Therefore, the lowest value of δ_0 is obtained for the network g such that for all i, $d_i(g)$ is maximal. This is $d_i(g) = n-1$ and this is nothing but the complete network where each individual has exactly $n-1$ direct links. $\square$

Proof of Proposition 5.

Let $\mathcal{C} \equiv Max_{x\in\{1,2,\ldots,n-1\}}\big\{f(x) - f(x-1)\big\}$ be the maximal utility cost of losing one link for a fixed amount of consumption.

The stability condition

$u\big(c_D^i(\theta_0)\big) - u\big(c^i(\theta_0)\big) < \frac{\delta}{1-\delta}\Big\{\sum_\theta p(\theta)\big[u\big(c^i(\theta)\big) - u\big(c_R^i(\theta)\big)\big] + f\big(d_i(g)\big) - f\big(d_i(g_R)\big)\Big\}$

implies that

$u\big(c_D^i(\theta_0)\big) - u\big(c^i(\theta_0)\big) + \frac{\delta}{1-\delta}\sum_\theta p(\theta)u\big(c_R^i(\theta)\big) < \frac{\delta}{1-\delta}\big\{\sum_\theta p(\theta)u\big(c^i(\theta)\big) + f\big(d_i(g)\big) - f\big(d_i(g_R)\big)\big\}$.

The LHS is a functional of the realized state of the nature. Since the number of states is finite, let $M \equiv Max_{\theta_0}\Big\{u\big(c_D^i(\theta_0)\big) - u\big(c^i(\theta_0)\big) + \frac{\delta}{1-\delta}\sum_\theta p(\theta)u\big(c_R^i(\theta)\big)\Big\}$. Also notice that $L = \sum_\theta p(\theta)u\big(c^i(\theta)\big)$ is constant. Thus the stability condition becomes $f\big(d_i(g)\big) - f\big(d_i(g_R)\big) > K$ where $K = \frac{\delta}{1-\delta}(M-L)$. If there is a deviation by i, $d_i(g_R) \leq d_i(g) - 1$. Thus, $f\big(d_i(g)\big) - f\big(d_i(g_R)\big) \geq f\big(d_i(g)\big) - f\big(d_i(g) - 1\big)$. The RHS of this inequality is constant if f is a linear functional. Otherwise, let the external effect be determined by the maximum value of the RHS computed over $1, \ldots, n$. This is $\mathcal{C}$. It is sufficient to have $\mathcal{C} > K$ to obtain stability. $\square$

Proof of Proposition 6

Let n_1 and n_2 be the respective sizes of the coalitions S_1 and S_2, with $n \equiv n_1 + n_2$ the size of the population.

The following lemma establishes the stability condition for the application.

Lemma 1

Given $g \in G$, I_g is stable if and only if for all $k \in \{n, n_1, n_2\}$:

For weak-stability,

$$\textbf{(i)} \quad d(\tfrac{k}{n} - \tfrac{1}{n})\Big[1 - \lambda\big(2y + d(\tfrac{k}{n} + \tfrac{1}{n})\big)\Big] < \tfrac{\delta}{1-\delta}\Big\{\lambda(\tfrac{1}{n-k+1} - \tfrac{1}{n})d^2p(1-p) + \alpha(n_j - 1)\Big\}.$$

For strong-stability, (i) and the additional condition,

(ii) $\quad d(\frac{1}{n})\big[1-\lambda\big(2y+d(\frac{3}{n})\big)\big] < \frac{\delta}{1-\delta}\alpha$

Proof of Lemma 1.

The consumption dictated by the equal consumption rule is $c^i(\theta_0) = \frac{1}{n}\sum y^i(\theta_0)$.

• Weak stability

Weak-stability corresponds to strong punishment.

If $i \in S_j$ and $i \neq o$:

$d_i(g) = n_j - 1$ and $d_i(g_R) = 0$

R is autarky. Thus, $c^i_R(\theta) = \begin{cases} y+d & \text{with probability } p \\ y & \text{with probability } 1-p \end{cases}$

In case of deviation, the best strategy is to deviate from all in order to keep y^i.

Thus, $c^i_D(\theta_0) = y^i(\theta_0)$.

Notice that i deviates if and only if her income realization is $y+d$. And if $0 < m < n$ individuals obtain $y+d$, $F(m) \equiv u(y+d) - u\big(\frac{m(y+d)+(n-m)y}{n}\big) = d(1-\frac{m}{n})\big[1-\lambda\big(2y+d(1+\frac{m}{n})\big)\big]$.

F is decreasing in m and therefore is maximized for $m = 1$. Thus, i has high income realization, and all the other individuals have low income realization.

Furthermore,

$\sum_\theta p(\theta)u\big(c^i_R(\theta)\big) = pu(y+d) + (1-p)u(y) = pd + y - \lambda(y^2 + 2pdy + pd^2)$, and

$\sum_\theta p(\theta)u\big(c^i(\theta)\big) = y + dp - \lambda\big(y^2 + 2dpy + \frac{d^2p}{n}(1+(n-1)p)\big)$.

Thus, the stability condition for i is

$d(1-\frac{m}{n})\big[1-\lambda\big(2y+d(1+\frac{m}{n})\big)\big] < \frac{\delta}{1-\delta}\big\{\lambda\frac{n-1}{n}pd^2(1-p) + \alpha(n_j-1)\big\}$ for all m such that $0 < m < n$.

Hence for $m = 1$, $d(1-\frac{1}{n})\big[1-\lambda\big(2y+d(1+\frac{1}{n})\big)\big] < \frac{\delta}{1-\delta}\big\{\lambda\frac{n-1}{n}pd^2(1-p) + \alpha(n_j-1)\big\}$.

For o, the overlapping individual, $d_o(g) = n-1$. Either $D \subset S_j$ and $d_o(g_R) = n - n_j$, or $D \subset S_1 \cup S_2$ and $d_o(g_R) = 0$.

If $D \subset S_j$, by deviating only to D, o still be with $S_{j'}$. The stability condition is $d(1-\frac{m}{n})\big[1-\lambda\big(2y+d(1+\frac{m}{n})\big)\big] - \epsilon_{j'} - \lambda\big(-2\epsilon_{j'}(y+d) + \epsilon^2_{j'}\big) < \frac{\delta}{1-\delta}\big\{\lambda(\frac{1}{n-n_j+1)} - \frac{1}{n)})pd^2(1-p) + \alpha(n_j-1)\big\}$ for all m such that $0 < m < n$. Where $\epsilon_{j'}$ is the transfer that o makes in the other coalition.

Thus for $m=1$, $\epsilon_{j'}=(n_{j'}-1)\frac{d}{n}=(1-\frac{n_j}{n})d$, and the stability condition for o is
$d(1-\frac{1}{n})\big[1-\lambda\big(2y+d(1+\frac{1}{n})\big)\big]-d(1-\frac{n_j}{n})\big[1-\lambda\big(2y+d(1+\frac{n_j}{n})\big)\big] < \frac{\delta}{1-\delta}\big\{\lambda(\frac{1}{n-n_j+1)}-\frac{1}{n)})pd^2(1-p)+\alpha(n_j-1)\big\}$, what is equivalent to
$d(\frac{n_j}{n}-\frac{1}{n})\big[1-\lambda\big(2y+d(\frac{n_j}{n}+\frac{1}{n})\big)\big] < \frac{\delta}{1-\delta}\big\{\lambda(\frac{1}{n-n_j+1}-\frac{1}{n})pd^2(1-p)+\alpha(n_j-1)\big\}$.

If $D\subset S_1\cup S_2$, the stability condition is
$d(1-\frac{m}{n})\big[1-\lambda\big(2y+d(1+\frac{m}{n})\big)\big] < \frac{\delta}{1-\delta}\big\{\lambda\frac{n-1}{n}pd^2(1-p)+\alpha(n-1)\big\}$ for all m such that $0<m<n$.

Thus, for $m=1$, $d(1-\frac{1}{n})\big[1-\lambda\big(2y+d(1+\frac{1}{n})\big)\big] < \frac{\delta}{1-\delta}\big\{\lambda\frac{n-1}{n}pd^2(1-p)+\alpha(n-1)\big\}$.

To summarize, weak-stability is equivalent to

(i) $d(1-\frac{1}{n})\Big[1-\lambda\big(2y+d(1+\frac{1}{n})\big)\Big] < \frac{\delta}{1-\delta}\Big\{\lambda\frac{n-1}{n}d^2p(1-p)+\alpha(n_j-1)\Big\}$

(ii) $d(\frac{n_j}{n}-\frac{1}{n})\Big[1-\lambda\big(2y+d(\frac{n_j}{n}+\frac{1}{n})\big)\Big] < \frac{\delta}{1-\delta}\Big\{\lambda(\frac{1}{n-n_j+1}-\frac{1}{n})d^2p(1-p)+\alpha(n_j-1)\Big\}$

(iii) $d(1-\frac{1}{n})\Big[1-\lambda\big(2y+d(1+\frac{1}{n})\big)\Big] < \frac{\delta}{1-\delta}\Big\{\lambda\frac{n-1}{n}d^2p(1-p)+\alpha(n-1)\Big\}$

Notice that $n>n_j$. Therefore, (i) implies (iii).

• Strong stability

For strong-stability, either $i\neq o$ deviate from n_j-1 others and stays alone (corresponding to strong punishment) or deviates from $0<l<n_j-1$ others except o and still be in connection the whole community (so i will not face the direct effect of punishment). As for o, either he deviates from $n-1$ or to n_j-1(corresponding to strong punishment), or he deviates from $0<l<n-1$ (with $D\subset S_1\cup S_2$) and he is still linked with the whole community.

As we pointed out earlier, the state where the deviation gain is maximal corresponds to $m=1$. Thus, strong-stability condition is
$d(1-\frac{1}{n})\big[1-\lambda\big(2y+d(1+\frac{1}{n})\big)\big]-\epsilon_l-\lambda\big(-2\epsilon_l(y+d)+\epsilon_l^2\big) < \frac{\delta}{1-\delta}\alpha l$.

Where ϵ_l is the transfer that the deviator fails to make to l individuals. In the state where $m=1$, $\epsilon_l=\frac{n-1-l}{n}d$ (because if the transfer to o is not made, $i\neq o$ will not benefit from the consumption smoothing) for $i\neq o$, and $\epsilon_l=\frac{n-1-l}{n}d$ for o.

Thus strong-stability is equivalent to the weak-stability conditions plus
$d(\frac{l}{n})\big[1-\lambda\big(2y+d(\frac{l+2}{n})\big)\big] < \frac{\delta}{1-\delta}\alpha l$. But $l>0$, so $d(\frac{1}{n})\big[1-\lambda\big(2y+d(\frac{l+2}{n})\big)\big] < \frac{\delta}{1-\delta}\alpha$.

The LHS is a decreasing function of l. For all l the condition must be verified, then it must be the case for the maximum value of LHS. This value is obtained with the minimum value of l which is $l = 1$. Thus, the stability condition is $d(\frac{1}{n})\big[1 - \lambda\big(2y + d(\frac{3}{n})\big)\big] < \frac{\delta}{1-\delta}\alpha$. $\square$

Remaining proof of the Proposition 6.

1) All the stability conditions can be written as $A[1 - \lambda(2y + B)] < C$, where $A > 0$, and $\lambda > 0$. Therefore, $y_2 > y_1 \Rightarrow A[1 - \lambda(2y_2 + B)] < A[1 - \lambda(2y_1 + B)]$

2) All the stability conditions can be written as $A - \lambda B < \lambda C + D$, where $B > 0$, and $C \geq 0$. We can rewrite the stability conditions as $-\lambda(B + C) < D - A$, where $B + C > 0$. Therefore, $\lambda_2 > \lambda_1 \Rightarrow -\lambda_2(B + C) < -\lambda_1(B + C)$.

3) All the stability conditions (i) can be written as $A < Bp(1 - p) + C$, where $B > 0$. But the functional $F(p) = p(1-p)$ defined on $(0, 1)$, increases in $(0, 0.5)$ and decreases in $(0.5, 1)$. The probability p is irrelevant for condition (ii). $\square$

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