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Cahier de Recherche / Working Paper
16-04

**Household Borrowing Constraints and
Residential Investment Dynamics**

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November 4, 2017

Abstract

It is well-known that residential investment leads output in the US economy. The main contribution of our paper is to highlight the role of household borrowing constraints in accounting for this fact. We study the role of home-equity loans used to boost consumption as a channel that affects residential investment. We consider a multi-agent model where some home-owning households face borrowing constraints that reflect home-equity loans or refinancing constraints. We show that the severity of the households' borrowing constraints in an economy can generate this stylized fact of US residential investment dynamics. Interestingly, the model correctly predicts coincident residential investment dynamics in countries with less severe borrowing constraints. This prediction is borne out when the model is calibrated to French data.

Key words: Home-Equity Loans, Borrowing Constraints, Residential Investment, Business Cycles
JEL classification: E22, E32, R21, R31

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We thank participants at the Rimini Conference in Economics and Finance 2016, and seminar participants at the Université de Sherbrooke.

1 Introduction

Residential investment in the housing sector has long been viewed as important for understanding fluctuations in US economic activity (Burns and Mitchell (1946)).¹ Recently, Leamer (2008, 2015) has reemphasized this view by noting that the decline of cumulative residential investment in recessions accounts for about half of the overall decline in US Gross Domestic Product (GDP). He writes ‘*Of the components of GDP, residential investment offers by far the best early warning sign of an oncoming recession*’ (Leamer (2008)). In this context, a well-known stylized fact is that US residential investment leads output over the business cycle.² The main contribution of this paper is to highlight the role of household borrowing constraints in accounting for this fact. In particular, we consider a model where home-equity loans are used to finance consumption and affect residential investment. In this environment, the severity of the borrowing constraint drives the lead of residential investment over output. Interestingly, the model correctly predicts coincident residential investment dynamics in countries with less severe borrowing constraints. This prediction is borne out when the model is calibrated to French data. Household borrowing constraints, therefore, provide an endogenous explanation for understanding residential investment dynamics.

Our focus on borrowing constraints as the underlying force for understanding residential investment dynamics builds on the recent literature that shows that such constraints have both relevant and important business cycle implications. Iacoviello (2005), for example, highlights the importance of collateral for households and firms to amplify the responses to housing price shocks. Iacoviello and Neri (2010) also find that fluctuations in house prices affect the borrowing capacity that can explain aggregate fluctuations. Liu, Wang and Zha (2013) study the importance of land in the firms’ collateral constraints and its business cycle effects. Justiniano, Primiceri and Tambalotti (2015) find that the boom in household debt that occur in 2000-2007 was transmitted through households collateral constraints directly from a change in house prices. Recently, using microdata,

¹Residential investment refers to investment in residential structures such as new houses, apartment buildings, and other dwellings

²This fact also applies to household investment which includes both residential investment and consumer durables. Fisher (2007) studies a separate stylized fact that household investment leads business investment over the business cycle in the US. In Fisher’s model, however, residential investment is coincident with output.

[Aladangady \(2017\)](#) finds that the borrowing constraint channel is key to understanding the reaction of consumption to changes in housing wealth.

We explicitly explore the role of home-equity loans to refinance consumption. We consider a multi-agent model with homeownership, where some home-owners face borrowing constraints that reflect home equity loans or refinancing constraints. To model this scenario we use the [Iacoviello \(2005\)](#) framework where homeowners facing borrowing constraints have a smaller discount factor relative to those who do not. In the baseline version of the model, business cycles are assumed to be driven by a standard unanticipated Total Factor Productivity (TFP) shock referred to as the technology shock.

We show that residential investment leads output by one quarter when the fraction of households who face borrowing constraints is matched to US data. The intuition for this result is as follows: there are two incentives that drive the housing investment decision. First is that housing services directly enter the households' utility function. Since the positive TFP shock induces a wealth effect, they decide to spend more on houses. However, this positive wealth effect alone is not sufficient to produce the leading pattern. Second is that a share of households are borrowing-constrained. The accumulation of the housing stock, therefore, allows them to borrow more contemporaneously and in future periods, so that they are also able to consume greater quantities of the non-durable good. The household borrowing constraint and its effect on residential investment are, therefore, important for understanding the lead of residential investment over output.

We find that the severity of the borrowing constraint measured as the share of households facing such a constraint determines whether or not residential investment leads output. In some eurozone countries, residential investment is coincident with output ([Kydland, Rupert and Sustek \(2016\)](#)). Our model is consistent with this fact. In eurozone countries, the share of households facing borrowing constraints is, on average, relatively smaller than the US, which according to the model implies that residential investment is coincident with the cycle. One piece of corroborative evidence at the aggregate level is the high household-debt-to-GDP ratio in the US relative to the eurozone. Data from the Bank for International Settlements show that US average household debt over the 1999 to 2015 time-period corresponds to 84.05 percent of annualized GDP, whereas for the

eurozone the same ratio is 56.35 percent.³ We calibrate the model to French data and show that the model generates coincident residential investment dynamics.

The rest of the paper is organized as follows. Section 2 presents the relation of our paper to the literature. Section 3 presents the model. Section 4 describes the calibration of parameters. Section 5 presents the quantitative results. Finally, section 6 concludes.

2 Relation to the literature

Previous research that examined the stylized fact mentioned above has considered other mechanisms than ours with limited success.⁴ [Kydland, Rupert and Sustek \(2016\)](#) present a model with long-term mortgage loans and multi-period time-to-build in residential construction to account for the lead of residential investment in the US versus coincident movement in eurozone countries. They model the loan structure as ‘first mortgage’ loans and abstract from the role of collateralized borrowing or refinancing loans against the home for consumption purposes. There are, however, two limitations that remain unaddressed. First is that since the 1980s US households increased their use of home equity loans and mortgage refinancing to cash out previously accumulated equity. As [Campbell and Hercowitz \(2005\)](#) note, the financial reforms of the early 1980s relaxed collateral constraints on household borrowing. Specifically, the federal government abandoned the New Deal regulatory system with the Monetary Control Act of 1980 and the Garn-St.Germain Act of 1982. These developments followed by sharp increases in debt-asset ratios reflecting the emergence of the subprime mortgage lending market, home-equity loans and mortgage refinancing activities. More recently, [Gorea and Midrigan \(2015\)](#) have pointed out that mortgage refinancing accounts for about one-third of the rise and fall in household spending during the 2001-2011 period in the US. Thus, home-equity loans and mortgage refinancing has been a salient feature of US housing market over the past three decades. Our model captures this in a parsimonious way.

Second is that, for the time period we study, relative to [Kydland, Rupert and Sustek \(2016\)](#), both residential investment and housing starts are in fact coincident in some European countries (for

³This difference might be due to cultural and institutional factors outside the scope of our model.

⁴The early literature on home production features studies that do not reproduce the lead in residential investment over the business cycle (see, for example, [Benhabib et al. \(1991\)](#); [Gomme et al. \(2001\)](#); [Greenwood and Hercowitz \(1991\)](#); [McGrattan et al. \(1997\)](#)).

example, France and Belgium). This fact makes it difficult to justify the differences in time-to-build between US and eurozone countries.⁵

Our paper is also related to [Ren and Yuan \(2014\)](#) which attempts to explain why residential investment leads the cycle in the US, using a partial equilibrium model with collateral constraints, agent heterogeneity, and TFP news shocks. Their model, however, has the odd implication that agents prefer to purchase houses instead of consumption goods after a positive endowment shock. This is necessarily the case in their model because the mortgage interest rate is lower than the credit card rate, both of which are calibrated parameters. In actual economies, even though the mortgage rate is lower than the credit card rate, the use of credit card for consumer goods purchases is common place. Another aspect that is ignored in the partial equilibrium setting of [Ren and Yuan \(2014\)](#) is that they do not have the labour supply decision in the model. This assumption is not innocuous because TFP news shocks, which are essential to their explanation, can produce wealth effects on labour supply that mitigate business cycle comovements ([Jaimovich and Rebelo \(2009\)](#)). Unlike the model in [Ren and Yuan \(2014\)](#), we consider a general equilibrium framework.

3 The model

The model economy is populated by three categories of agents: impatient and patient households, and entrepreneurs. These agents are denoted by I , P , and E , respectively. The shares of these agents in the economy differ and are denoted as ω_I , ω_P , and ω_E , respectively. Agents are infinitely-lived and maximize a discounted sum of time-separable utilities. We assume that impatient households and entrepreneurs borrow from patient households (the lenders), therefore the discount factors of the two borrowing agents, θ_I and θ_E , are lower than the lenders' discount factor, β . The impatient households' loans are collateralized by the expected value of their stock of housing. Both types of households are employed by firms that produce non-durable goods and houses—a durable good. They derive utility from the consumption of non-durable goods and housing services. We assume that house developers operate in a perfectly competitive environment, and that entrepreneurs undertake the production of non-durable goods. They also make non-residential

⁵For the period 1974Q1-2006Q4 studied in [Kydland, Rupert and Sustek \(2016\)](#) housing starts leads the cycle in France. However, for the 1983Q1-2016Q4 period that we study, it is coincident.

and residential investments and face a collateral constraint. Specifically, their inter-period debt and working capital loans to cover the wage bill cannot exceed a fraction of their capital stock. In the appendix, we present a fully detailed description of the model's stationary version.

3.1 Households

All impatient households face the same maximization problem which is the following:

$$\max E_0 \sum_{t=0}^{\infty} \theta_I^t \left(\ln(c_{It} - \zeta c_{It-1}) + \psi \ln h_{It} + \eta_I \ln \left(1 - (n_{Ict}^{1+\epsilon} + n_{Iht}^{1+\epsilon})^{1/(1+\epsilon)} \right) \right) \quad (1)$$

subject to

$$c_{It} + (1 + \tau_h)q_{ht}h_{It} - q_{ht}(1 - \delta_h)h_{It-1} = b_{Ht} - R_{t-1}b_{Ht-1} + p_{lt}l_{It} + \sum_{i=c,h} w_{it}n_{Iit} - \tau_n \left(\sum_{i=c,h} w_{it}n_{Iit} + \tau_m r_{t-1}b_{Ht-1} \right) + \xi_{It}, \quad (2)$$

$$b_{Ht} \leq m_H E_t q_{ht+1} h_{It}. \quad (3)$$

First, note that variables c_{It} , h_{It} , n_{Ict} , n_{Iht} , b_{Ht} , l_{It} , and ξ_{It} are variables that are expressed per impatient household. Their period utility function is in logarithms and weighs their consumption of the non-durable good, c_{It} , their housing stock, h_{It} , and their hours worked in the production of non-durable goods, n_{Ict} , and houses, n_{Iht} . They have habits in consumption of non-durable goods measured by parameter ζ . We follow [Horvath \(2000\)](#) and [Iacoviello and Neri \(2010\)](#) and allow for an imperfect substitution of labor between sectors that is governed by parameter ϵ . A higher value of ϵ implies a decreased reallocation of hours worked by impatient households from one sector to another in response to wage differentials.

On the right-hand-side of their budget constraint, equation (2), impatient households have different sources of revenues: (i) they earn labor income that is taxed at rate τ_n , so that their total after-tax labor income is $(1 - \tau_n) \sum_{i=c,h} w_{it}n_{Iit}$ where wages w_{it} are taken as given, (ii) they borrow b_{Ht} at the interest rate $R_t = 1 + r_t$, so that net borrowing is $b_{Ht} - R_{t-1}b_{Ht-1}$, (iii) they are endowed with one unit of land l_{It} that they sell to house developers at price p_{lt} , and (iii) they receive transfers from the government: ξ_{It} . We also assume that mortgage debt carries a tax advantage such that a fraction τ_m of interest payments $r_{t-1}b_{Ht-1}$ can be deducted from taxable income. We consider

adjustable rate mortgages, as they are renegotiated every period. The left-hand-side consists of expenditures that these households have, (i) for the consumption of non-durable goods, and (ii) the accumulation of housing stock from which a fraction τ_h is taxed. The relative price of housing is q_{ht} , and housing depreciates at rate δ_h .

The second constraint, equation (3), is a collateral constraint. Specifically, the level of debt contracted every period cannot exceed a fraction m_H of the expected value of the households' stock of housing, *i.e.* $E_t q_{ht+1} h_{It}$. Since impatient households have a lower discount factor, this constraint is binding in the steady state. We consider small shocks and a sufficiently low discount factor so that it is also always binding in the neighborhood of the steady state.

The representative patient household's problem departs from the impatient households' problem as their higher discount factor makes it suboptimal for them to carry debt ($\beta > \theta_I$). However, the utility function of patient households takes the same form as is shown by the following equations:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left(\ln(c_{Pt} - \zeta c_{Pt-1}) + \psi \ln h_{Pt} + \eta_P \ln \left(1 - (n_{Pct}^{1+\epsilon} + n_{Pht}^{1+\epsilon})^{1/(1+\epsilon)} \right) \right) \quad (4)$$

subject to

$$c_{Pt} + (1 + \tau_h) q_{ht} h_{Pt} - q_{ht} (1 - \delta_h) h_{Pt-1} = \frac{1}{\omega_P} (\omega_I (R_{t-1} b_{Ht-1} - b_{Ht}) + \omega_E (R_{t-1} b_{Kt-1} - b_{Kt})) + (1 - \tau_n) \sum_{i=c,h} w_{it} n_{Pit} + p_{lt} l_{Pt} + \xi_{Pt}. \quad (5)$$

Note that the variables with subscript P are the counterparts for patient households of variables described below for the impatient households. All variables are expressed per patient household. This category of agents are the savers in the economy as they lend to the impatient households, $\omega_I/\omega_P b_{Ht}$, and entrepreneurs, $\omega_E/\omega_P b_{Kt}$.

3.2 Entrepreneurs

The entrepreneurs maximize the following problem:

$$\max E_0 \sum_{t=0}^{\infty} \theta_E^t \ln(c_{Et} - \zeta c_{Et-1}) \quad (6)$$

subject to

$$c_{Et} + x_{ct} + k_{bt} + \frac{w_{ct}}{\omega_E} \sum_{j=I,P} \omega_j n_{jct} + \tau_k r_{ct} k_{ct-1} = \frac{y_t}{\omega_E} + b_{Kt} - R_{t-1} b_{Kt-1} + (1 - \tau_k) p_{bt} k_{bt} + p_{lt} l_{Et} + \xi_{Et}, \quad (7)$$

$$y_t = z_t (\omega_E k_{ct-1})^{\alpha_C} \left(\sum_{j=I,P} \omega_j n_{jct} \right)^{1-\alpha_C}, \quad (8)$$

$$x_{ct} = k_{ct} - (1 - \delta_c) k_{ct-1}, \quad (9)$$

$$b_{Kt} + \frac{w_{ct}}{\omega_E} \sum_{j=I,P} \omega_j n_{jct} \leq m_K k_{ct}. \quad (10)$$

Note that variables c_{Et} , x_{ct} , b_{Kt} , k_{ct} , l_{Et} , k_{bt} , and ξ_{Et} are variables that are expressed per entrepreneur. Entrepreneurs are borrowing-constrained agents that invest and produce the non-durable goods. Equation (7) shows the production of these goods involves labor provided by impatient and patient households, that are paid at wage w_{ct} , and capital k_{ct} . In addition to the Cobb-Douglas production function, y_t , the exogenous source of fluctuations is ensured by a neutral technology shock z_t . This shock follows an AR(1) process:

$$\ln z_t = \rho_z \ln z_{t-1} + \epsilon_{zt}, \quad \epsilon_{zt} \sim N(0, \sigma_z^2) \quad (11)$$

where ρ_z corresponds to the persistence parameter and σ_z^2 to the variance of the innovation. Entrepreneurs accumulate capital according to the law of motion that is given by equation (9) where x_{ct} corresponds to the investment in non-durable goods. They also allocate resources for residential investment k_{bt} , which is effectively an investment in the materials that are used in the construction of houses. In contrast to capital used in the production of non-durable goods, there is no accumulation of materials. We assume that their imputed rental income of capital $r_{ct} k_{ct}$ is taxed at rate τ_k , where the rental r_{ct} corresponds to the marginal product of capital. In similar fashion to the households, they have habits in consumption of non-durable goods c_{Et} . However, for simplicity, we assume that they do not work nor do they own houses.

The revenues of the entrepreneurs are shown in the right-hand-side of equation (7). First, a part of the final goods are sold for the consumption of impatient and patient households, and the remaining part is allocated to their own consumption and investments. Second, entrepreneurs sell

k_{bt} , to the firms at price p_{bt} . Similar to the imputed rental income of capital, the sale of materials is taxed at rate τ_k . Third, entrepreneurs are endowed with l_t acreage in land every period that they sell to firms at price p_{lt} . Fourth, since they have a lower discount factor, they borrow b_{Kt} at the interest rate R_t , so that their net borrowing is $b_{Kt} - R_{t-1}b_{Kt-1}$. Fifth, they receive governmental transfers ξ_{Et} .

Entrepreneurs have two types of loans: inter-period debt b_{Kt} and working capital loans that are contracted to cover their wage bill. These loans have an intra-period maturity, as they are reimbursed at the end of the period once non-durable goods are sold.⁶ As seen in equation (10), the sum of the inter-period debt and working capital loans cannot exceed a fixed fraction m_K of the capital stock.

3.3 House developers

House developers operate in a perfectly competitive market to produce new houses, ih_t , from a Cobb-Douglas production that aggregates materials, land, and labor. Their problem is static and is as follows:

$$\max q_{ht}ih_t - w_{ht} \sum_{j=I,P} \omega_j n_{jht} - p_{lt}l_t - \omega_E p_{bt}k_{bt} \quad (12)$$

subject to

$$ih_t = z_t (\omega_E k_{bt})^{\alpha_B} l_t^{\alpha_L} \left(\sum_{j=I,P} \omega_j n_{jht} \right)^{1-\alpha_B-\alpha_L} \quad (13)$$

$$l_t = \sum_{j=I,P,E} \omega_j l_{jt}. \quad (14)$$

Every period, the firms pay factors of production from the sale of houses. The labor inputs and materials have to be adjusted to account for the difference in population shares of the three categories of agents. The assumption of competitive markets ensures that profit is null in equilibrium. The exogenous source of fluctuations in the production of houses, z_t , is perfectly correlated with the one in the production of non-durable goods.

⁶For simplicity, we assume that working capital loans do not incur any interest rates.

3.4 Government

In this economy, the government does not play any productive role. It levies distortionary taxes on households, and entrepreneurs, and makes lump-sum transfers, so that the total value of transfers corresponds exactly to the amount that these two categories of agents pay in taxes:

$$\xi_{It} = \omega_I \left(\tau_n \sum_{i=c,h} w_{it} n_{Iit} + \tau_h q_{ht} h_{It} - \tau_m \tau_n r_{t-1} b_{Ht-1} \right), \quad (15)$$

$$\xi_{Pt} = \omega_P \left(\tau_n \sum_{i=c,h} w_{it} n_{Pit} + \tau_h q_{ht} h_{Pt} \right), \quad (16)$$

$$\xi_{Et} = \omega_E \tau_k [r_{ct} k_{ct-1} + p_{bt} k_{bt}]. \quad (17)$$

3.5 Market clearing conditions

There is a resource constraint in the economy, so that output corresponds to the sum of consumption and investment:

$$y_t = \sum_{i=I,P,E} \omega_i c_{it} + \omega_E (x_{ct} + k_{bt}). \quad (18)$$

In accordance with the definition of GDP provided by the NIPA, we measure GDP generated by the model from a value-added approach, so that:

$$GDP_t = y_t + q_{ht} i h_t - \omega_E k_{bt}. \quad (19)$$

Additionally, the law of motion for houses is as follows:

$$i h_t = h_t - (1 - \delta_H) h_{t-1} \quad (20)$$

where $h_t = \omega_I h_{It} + \omega_P h_{Pt}$. We assume that land supply is equal to one for all periods.⁷

4 Calibration

Table 1 shows the calibration of the baseline model. Based on data from the Survey of Consumer Finances (SCF), [Justiniano, Primiceri and Tambalotti \(2015\)](#) find that 61% of U.S. households are

⁷In similar fashion to [Davis and Heathcote \(2005\)](#), we do not model land dynamics over the business cycle.

TABLE 1: Calibration of the baseline model

POPULATION		
ω_I	0.6	fraction of impatient households
ω_P	0.31	fraction of patient households
ω_E	0.09	fraction of entrepreneurs
DISCOUNT FACTORS, PREFERENCES AND LOAN-TO-VALUES		
β	0.9925	discount factor (savers and patient households)
θ_W	0.973	discount factor (impatient households)
θ_E	0.973	discount factor (entrepreneurs)
ζ	0.7	habits in consumption
ψ	0.122	weight on housing
η_I	1.93	weight on leisure (impatient households)
η_P	1.66	weight on leisure (patient households)
ϵ	0.28	sectoral labor mobility
m_H	0.8	imp. households' loan-to-value
m_K	0.5	entrepreneurs' loan-to-value
TAXATION		
τ_n	0.22	labor income tax
τ_k	0.29	capital tax
τ_h	0.00625	property tax
τ_m	1	fraction of deductible mortgage interest payments
DEPRECIATION AND TECHNOLOGY		
δ_m	0.025	depreciation of capital
δ_h	0.004	depreciation of housing
α_C	0.28	elasticity with respect to capital (non-durable goods)
α_B	0.53	elasticity with respect to material inputs (housing)
α_L	0.1	elasticity with respect to land (housing)
ρ_z	0.974	persistence of the technology shock

borrowing-constrained. Consistent with this evidence, we calibrate $\omega_I = 0.6$. Also from the Survey of Consumer Finances (SCF), [Cagetti and De Nardi \(2006\)](#) report that the percentage of business owners in the population is 9.1 percent, therefore we set $\omega_E = 0.09$. The share of patient households corresponds to the remaining share, *i.e.* $\omega_P = 0.31$. The value of the patient households and savers' quarterly discount factor is set so that the real interest rate is 3 percent in the steady state. Since impatient households and entrepreneurs are borrowing-constrained, they face a shadow cost of borrowing in addition to interest payments. This cost can be considered as an interest premium. Similar to [Bernanke, Gertler and Gilchrist \(1999\)](#), we choose the impatient households' discount factor to match a two percent interest premium, *i.e.* $\theta_W = \theta_E = 0.973$. For a reasonable size of the TFP shock, this value of the discount factor ensures that the borrowing constraint is always binding. As for the parameterization of habits in consumption, [Havranek, Rusnak and Sokolova \(2015\)](#) conducted a meta-analysis of the estimated values in the literature. Based on 251 studies on the US economy, they report a median value of 0.72 for macro estimates. Therefore, we set ζ to 0.7.

The weight on housing in the households' utility function is governed by ψ and is set to match the average household market value of real-estate-to-GDP-ratio in the US from 1983 to 2016 which is 4.68. This value is based on the Federal Reserve Board data (Table B.101, line 3). The weights assigned to leisure, η_I and η_P , are picked so that households work 30 percent of their allocated time in the steady state. The parameter that governs the degree of sectoral labor mobility, ϵ , is the only one that is determined outside the steady state. It is set to match the relative volatility of residential investment to GDP. As for the households' loan-to-value parameter, we follow [Justiniano, Primiceri and Tambalotti \(2015\)](#) calibrated value based on the average loan-to-value for purchases and refinancing based on the SCF data of 1992, 1995, and 1998, *i.e.* $m_H = 0.8$. For the entrepreneurs' loan-to-value m_K , we follow [Bernanke, Gertler and Gilchrist \(1999\)](#) and set it to 0.5.

Taxation distorts the decisions of households and entrepreneurs. For labor and capital taxes, τ_n and τ_k , we take the values estimated by [Gomme and Rupert \(2007\)](#) who use [Mendoza, Razin and Tesar \(1994\)](#) methodology to calculate effective average tax rates. We set the property tax rate, $\tau_h = 0.00625$, that is the quarterly value reported for the US in [Girouard, Kennedy, Van den](#)

Noord and André (2006) based on their survey of housing taxation across OECD countries. Since mortgage interest payments are fully deducted from taxable income in the US, we set $\tau_m = 1$.

The depreciation rate of capital is standard, *i.e.* $\delta_m = 0.025$. We follow Gomme and Rupert (2007) in computing the housing depreciation rate from BEA data and set $\delta_h = 0.004$. We set the capital share in the production of houses, $\alpha_C = 0.28$ in accordance with the estimations of the same study. We follow Davis and Heathcote (2005) and Iacoviello and Neri (2010) and set the share of land in the production of new houses to $\alpha_L = 0.1$. In the production function of houses, we do not distinguish between capital in structures and intermediate goods. Yet material costs compose a large share of total house production costs. Specifically, in the US input-output tables from 1997 to 2014, this share is 15% greater than workers' compensation. We set α_H to match this ratio to which we add a 10% share for structures in similar fashion to the two aforementioned studies. Finally, we choose the persistence parameter of the technology shock, $\rho = 0.974$. It corresponds to the average between estimates of the persistence of technology shock that affects the housing production and the non-durable goods production as reported by Iacoviello and Neri (2010).

5 Quantitative results

In this section, we first present the main quantitative results for the baseline calibration of the model that matches moments of the US data (values presented in Table 2). The key finding is that the presence of borrowing constraints is critical to replicate the lead in residential investment and many co-movements and relative volatilities for the US that a standard real business cycle model fails to deliver.

5.1 Results

In Table 2 we report the relative volatilities and co-movements of consumption, residential investment, non-residential investment, housing starts, and housing prices. We have set the degree of sectoral labor mobility measured by ϵ to match the relative standard deviation of residential investment, so we have imposed the model to match this moment. There are, however, no restrictions on the values of all other moments. The model reproduces quite well the relative volatility of non-residential investment. Specifically, the volatility of investment is significantly higher in the

TABLE 2: **Baseline Model Properties**

CO-MOVEMENTS			RELATIVE VOLATILITIES		
STATISTIC	DATA	MODEL	STATISTIC	DATA	MODEL
$\rho(c_t, GDP_t)$	0.87	0.99	σ_c/σ_{GDP}	0.83	0.6
$\rho(k_{bt}, GDP_t)$	0.67	0.85	σ_{kb}/σ_{GDP}	6.55	6.55
$\rho(x_{ct}, GDP_t)$	0.86	0.98	σ_{xc}/σ_{GDP}	4.22	3.42
$\rho(x_{ct}, k_{bt})$	0.39	0.83	σ_{kb}/σ_{xc}	1.55	1.92
$\rho(ih_t, GDP_t)$	0.58	0.86	σ_{ih}/σ_{GDP}	10.56	6.11
$\rho(q_{ht}, GDP_t)$	0.51	0.73	σ_{qh}/σ_{GDP}	2.94	0.45

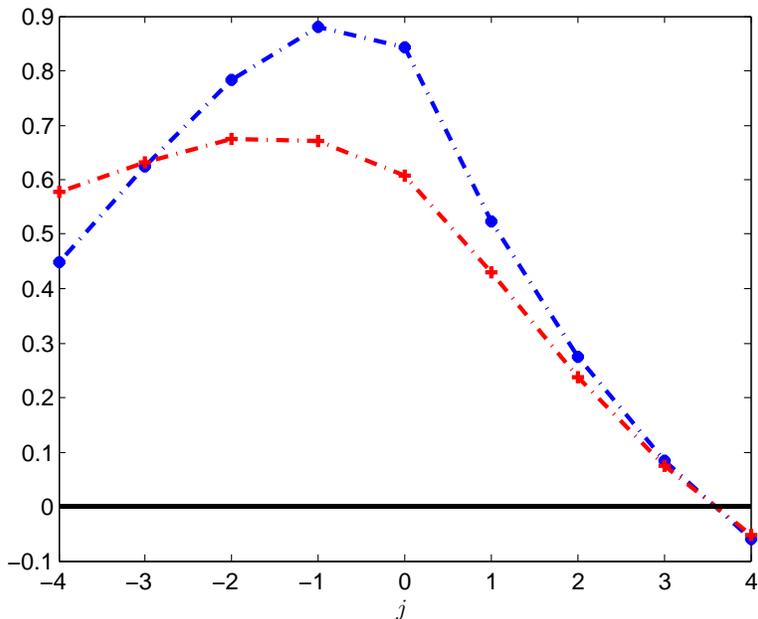
NOTE—The U.S. data spans from 1983Q1 to 2017Q2. All series are logged and detrended with the HP-filter ($\lambda=1600$). Data sources are in the appendix. The variable c_t corresponds to total consumption consistent with NIPA, *i.e.* $c_t = \sum_{i=I,P} \omega_i (c_{it} + r_{hit}h_{it}) + \omega_{ECEt}$ where r_{hit} correspond to the imputed rental housing prices ($r_{hit} = u'_{hit}/u'_{cit}$). The variables k_{bt} , x_{ct} , ih_t , and q_{ht} are residential investment, non-residential investment, new houses, and housing prices, respectively.

housing construction sector both in the data and generated by the model.⁸ This success is shared with previous work, *e.g.* [Davis and Heathcote \(2005\)](#), [Kydland, Rupert and Sustek \(2016\)](#), and [Fisher \(2007\)](#), however, a standard multi-sector real business model as in [Greenwood and Hercowitz \(1991\)](#) fails at replicating this relative volatility. The effects of borrowing constraints are not at work here. There are two reasons. First, a lower depreciation rate of houses than capital used in the production of non-durable goods is key to explain the volatility. In fact, the effects of investing in housing production last longer with a lower depreciation rate. Therefore, the response of investment to a TFP shock is greater. Second, contrary to non-residential investment, residential investment is affected twice by the TFP shock. Its supply increases because the TFP shock affect the production of non-durable goods that is used for residential investment, and its demand increases from a greater marginal product of residential investment in the production of new houses. In similar fashion, the house production is very volatile, because the technology shock affects, simultaneously and equally, both sectors.

⁸As stated by equation (19), GDP’s definition also includes the value-added of the housing construction sector. However, since that sector accounts for only 1.2% of GDP, relative volatilities and co-movements are essentially the same if we remove that sector from GDP’s definition.

The model is able to replicate the positive co-movements in all aggregate quantity variables as well as in housing prices. Accounting for the high volatility of housing prices in the data, however, remains a challenge for the model. In the literature, it appears that technology shocks are not sufficient to explain the radical shifts in housing prices.

Figure 1: **Cross-correlation function between GDP and residential investment conditional on the TFP shock.**

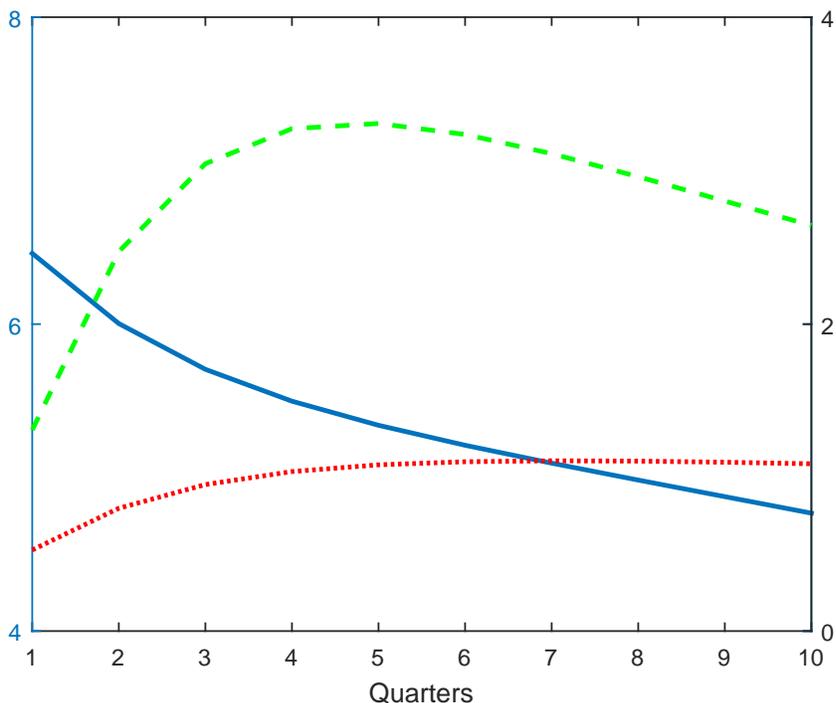


NOTE— Specifically, this function corresponds to the correlation between residential investment at time $t + j$ and output at time t . The blue line corresponds to the correlations generated by the benchmark model and the red line to the correlations estimated from US data between 1983Q1 and 2017Q2 based on calibration in Table 1. All series are logged and detrended with the HP-filter ($\lambda=1600$).

In Figure 1, we present the cross-correlation functions between output and residential investment computed from the actual data and those generated by the benchmark model. Residential investment leads output by one quarter. The one period lead generated by the model overshoots its data counterpart, but the predictions for the remaining leads and lags are consistent with the data.

The impulse responses displayed in Figure 2 gives evidence of the borrowing constraint mech-

Figure 2: **Impulse responses to a one percent technology shock.**



Notes: The solid blue line corresponds to the response of residential investment (left axis), the green dashed line to non-residential investment (right axis), and the dotted red line to GDP (right axis).

anism at play. There are two incentives for purchasing larger quantity of houses. First, housing services directly enter the households' utility function. Since the positive technology shock induces a wealth effect, they decide to spend more on houses. However, this positive wealth effect alone is not sufficient to produce the leading pattern. The key characteristic is that a fraction of these households are borrowing-constrained. Therefore, the accumulation of the housing stock allows them to borrow more contemporaneously and in future periods, so that they are also able to consume greater quantities of the non-durable good.

Habit formation in consumption plays a role in the delayed and hump-shaped response of GDP to technology shocks.⁹ Specifically, preferences that feature habits give additional incentives for

⁹This result is shared with [Pintus and Wen \(2013\)](#) who show that habits in consumption coupled with borrowing constraints can create large boom-bust cycles.

TABLE 3: **Housing starts and output**

CORRELATIONS BETWEEN HOUSING STARTS AT TIME $t + j$ AND OUTPUT AT TIME t									
	$j = -4$	-3	-2	-1	0	1	2	3	4
US DATA	0.46	0.56	0.62	0.64	0.58	0.4	0.22	0.08	-0.05
BASELINE CALIB.	0.44	0.62	0.78	0.88	0.86	0.54	0.29	0.1	-0.05
FRENCH DATA	0.36	0.44	0.44	0.44	0.45	0.31	0.16	0.02	-0.15
FRENCH CALIB.	0.49	0.67	0.8	0.84	0.87	0.59	0.34	0.14	-0.02

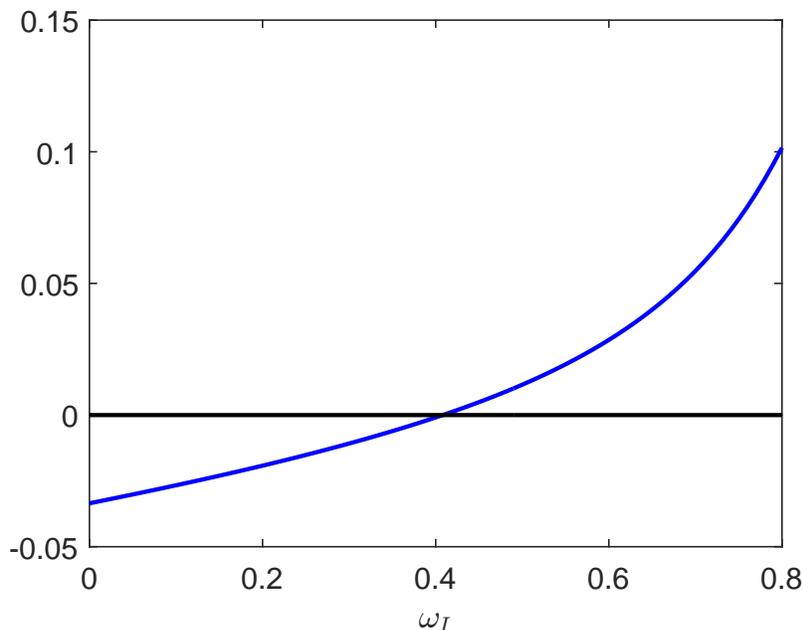
NOTE—The U.S. data spans from 1983Q1 to 2017Q2 and the French data from 1983Q1 to 2016Q3. All series are logged and detrended with the HP-filter ($\lambda=1600$). Data sources are in the appendix. The highest correlations of each row are marked in bold.

households to demand houses relative to non-durable goods when the shock impacts the economy. However, as will be seen in the next section, for a lower share of impatient households in the economy, habit formation is not sufficient to ensure the lead of residential investment.

Table 3 reports the crosscorrelations between housing starts (new houses) and output. Even though the correlations generated by the baseline calibration overshoot their empirical counterparts in the first two rows, housing starts lead output in the US data and for the baseline calibration. This success of our model is related to the same household borrowing constraint channel described in the previous paragraphs for the lead of residential investment over output.

We now demonstrate a key property of the model. The share of impatient households in the economy affects the lead of residential investment over output. Figure 3 presents the difference between the correlation of residential investment at time $t - 1$ and output at time t and the correlation of these two variables at time t for a wide range of the shares of impatient households (ω_I) in the economy. Specifically, when the share of impatient households is increased, the share of patient households is decreased, and vice versa. All the other parameters remain the same as reported in Table 1. A positive gap indicates that the absolute magnitude of the leading correlation is greater than contemporaneous correlation. The gap between the leading and contemporaneous correlations is unambiguously a positive function of the fraction of borrowing-constrained households.

Figure 3: **GDP and residential investment (net) crosscorrelations conditional on TFP shocks**



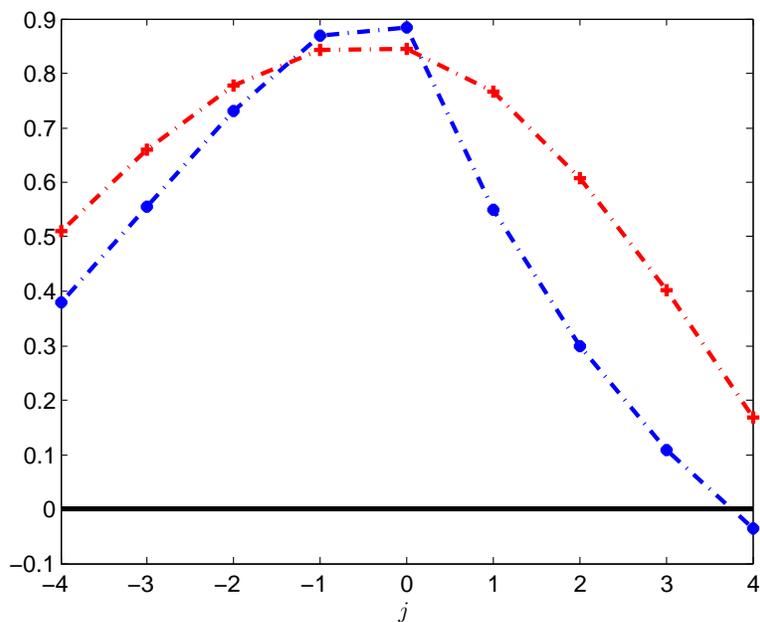
NOTE—The figure shows the difference between the cross-correlation in output at t and residential investment at time $t - 1$ and t , respectively, for different shares of impatient households (ω_I). All series are logged and detrended with the HP-filter ($\lambda=1600$).

5.2 Alternative calibration for household borrowing

After having stressed the role played by borrowing constraints for the lead of residential investment for the US, we calibrate the financial frictions parameters to match the data of eurozone countries. According to a survey conducted by the [ECB \(2016\)](#) in 2014, the share of households that had a mortgage was 20 percent in the eurozone. This smaller share is the only parameter that we re-calibrate relative to the benchmark calibration, *i.e.* $\omega_I = 0.2$.

Since the share of households facing the borrowing constraint is relatively small the additional incentive to acquire housing is significantly reduced. This increases the correlation between aggregate quantity variables. Consequently, the lead in residential investment vanishes as can be seen in [Figure 4](#). This result is consistent with the cross-correlation pattern for France, indicating a very

Figure 4: Cross-correlation between residential investment and GDP conditional on TFP shocks: French calibration



NOTES—Specifically, this function corresponds to the correlation between residential investment at time $t + j$ and output at time t . The blue line corresponds to the correlations generated by the benchmark model and the red line to the correlations estimated from French data between 1983Q1 and 2016Q4. All series are logged and detrended with the HP-filter ($\lambda=1600$).

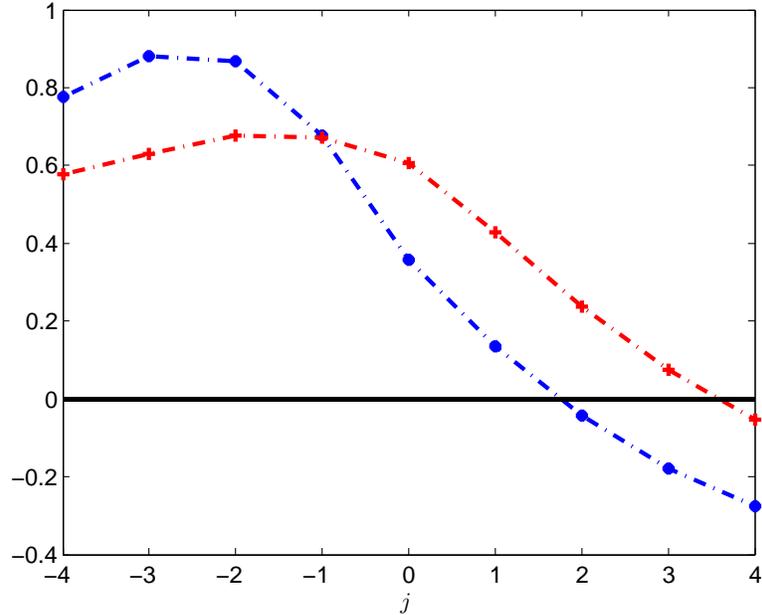
good fit of the model.¹⁰ Moreover, a lower share of impatient households in the economy make housing starts (new houses) coincident with GDP. This result is consistent with French data as presented by the last two rows of Table 3.

5.3 Cross-correlations conditional on preference shocks

In this section we examine the effects of a shock to the households and entrepreneurs' intertemporal preferences. Specifically, their discount factors are all affected identically, *i. e.* $\theta_I A_t$ for impatient

¹⁰We choose to report the cross-correlation pattern for France for two reasons. First, it is the second-largest economy in the eurozone. Second, its statistical agency reports a long time span of residential investment data. Moreover, from 1983 to 2016, its household-debt-to-GDP ratio is approximately half of the US's (38.8 percent to be precise).

Figure 5: **Cross-correlation between residential investment and GDP conditional on preference shocks: Baseline calibration**



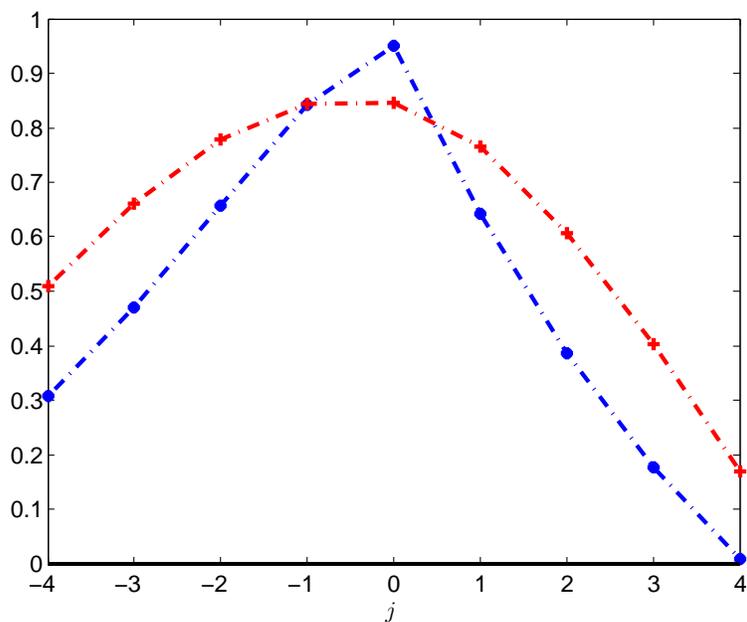
households, βA_t for patient households, and $\theta_E A_t$ for entrepreneurs, where

$$\ln A_t = \rho_A \ln A_{t-1} + \epsilon_{At} \quad \epsilon_{At} \sim N(0, \sigma_A^2).$$

Following [Iacoviello and Neri \(2010\)](#), we set $\rho_A = 0.97$. In Figures 5 and 6, we present the cross-correlation function between residential investment and GDP for the baseline and French calibrations, where preference shocks are the only source of exogenous disturbances. Similarly to TFP shocks, lowering the share of impatient households in the economy makes a substantial impact on the cross-correlation function. When 60% of households are impatient, residential investment leads GDP, while the two aggregate variables coincide for 20% share of impatient households.

We consider negative preference shocks, because it is easier to compare their effects to the effects of positive technology shocks. Following negative shocks, all agents become relatively more impatient, and therefore have more pressure to consume housing services and non-durable goods as early as possible. For impatient households, the incentives to accumulate housing stock are even greater because houses are collateral assets that allow them to borrow more. Therefore, a greater

Figure 6: Cross-correlation between residential investment and GDP conditional on preference shocks: French calibration

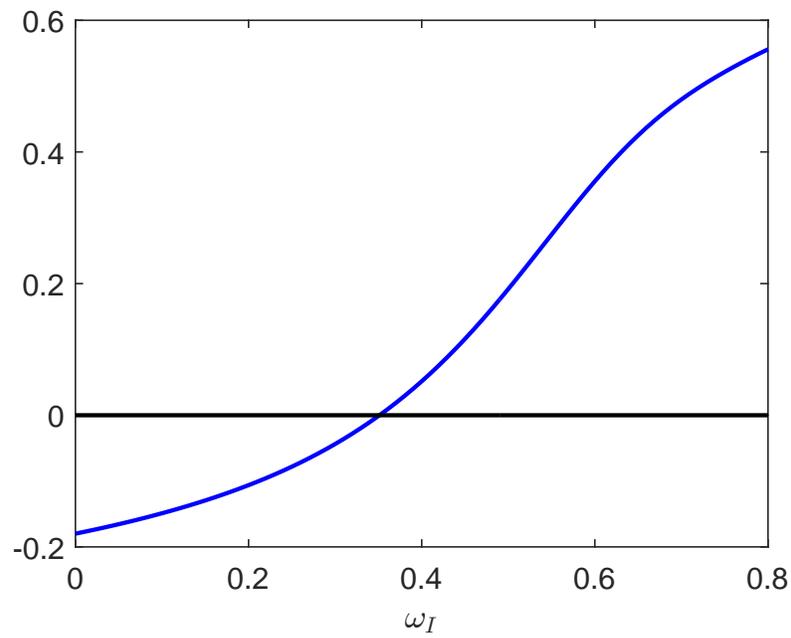


share of borrowing-constrained households in the economy leads to larger positive shifts in the demand for houses relative to non-durable goods for the period when the shock occurs. The larger the share of impatient households the more the increase in the production of non-durable goods is delayed and that contributes to residential investment leading the cycle.

Similar to Figure 3, Figure 7 presents the size of the lead of residential investment conditional on preference shocks for a wide range of the share of impatient households ω_I . For lower values of ω_I , the effects of borrowing constraints are diluted, and residential investment does not lead output.

Figure 7: **GDP and residential investment (net) crosscorrelations conditional on preference shocks:**

$$Corr(y_t, k_{bt-1}) - Corr(y_t, k_{bt})$$



NOTE—The figure shows the difference between the cross-correlation in output at t and residential investment at time $t - 1$ and t , respectively, for different shares of impatient households (ω_I) conditional on preference shocks. All series are logged and detrended with the HP-filter ($\lambda=1600$).

6 Conclusion

Residential investment leads the business cycle in the US. The main contribution of this paper is to demonstrate that home-equity loans used for consumption can influence residential investment dynamics in ways that can account for this stylized fact. When the share of constrained borrowers is relatively large, residential investment leads output. When this proportion is relatively small, as in eurozone countries, residential investment becomes coincident with the output and no longer serves as a leading indicator in the economy. Household borrowing constraints, therefore, provide an endogenous explanation to help account for a key feature of residential investment dynamics.

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A Data

A.1 US

The series for output, consumption, residential investment, and non-residential investment are from NIPA, Table 1.1.3. New houses are from the US Bureau of Census, specifically this series “Housing Starts: Total: New Privately Owned Housing Units Started”. The housing price index is the S&P/Case-Shiller U.S. National Home Price Index which is divided by the CPI index from the BLS.

A.2 France

The series for output and residential investment are from the Quarterly National Accounts published by the INSEE. Housing starts are from the Main Economic Indicators of the OECD.

B The model equations

B.1 Impatient households

$$\max E_0 \sum_{t=0}^{\infty} \theta_I^t \left(\ln(c_{It} - \zeta c_{It-1}) + \psi \ln h_{It} + \eta_I \ln \left(1 - (n_{Ict}^{1+\epsilon} + n_{Iht}^{1+\epsilon})^{1/(1+\epsilon)} \right) \right) \quad (21)$$

subject to

$$c_{It} + (1 + \tau_h) q_{ht} h_{It} - q_{ht} (1 - \delta_h) h_{It-1} = b_{Ht} - R_{t-1} b_{Ht-1} + p_{lt} l_{It} + \sum_{i=c,h} w_{it} n_{Iit} - \tau_n \left(\sum_{i=c,h} w_{it} n_{Iit} - \tau_m r_{t-1} b_{Ht-1} \right) + \xi_{It}, \quad (22)$$

$$b_{Ht} \leq m_H E_t q_{ht+1} h_{It}, \quad (23)$$

$$n_t = (n_{Ict}^{1+\epsilon} + n_{Iht}^{1+\epsilon})^{1/(1+\epsilon)}. \quad (24)$$

First-order conditions

c_{It}:

$$\frac{1}{c_{It} - \zeta c_{It-1}} - E_t \frac{\zeta}{c_{It+1} - \zeta c_{It}} = \lambda_{1It} \quad (25)$$

b_{Ht}:

$$\lambda_{1It} - \lambda_{2It} = \theta_I (1 + r_t (1 - \tau_n \tau_m)) E_t \lambda_{1It+1} \quad (26)$$

h_{It}:

$$q_{ht}\lambda_{1It}(1 + \tau_h) - m^H \lambda_{2It} E_t q_{ht+1} = \theta_I (1 - \delta_h) E_t \lambda_{1It+1} q_{ht+1} + \frac{\Psi}{h_{It}} \quad (27)$$

n_{Ijt} j = c, h:

$$\lambda_{1It}(1 - \tau_n) w_{jt} = \frac{\eta_I}{1 - n_{It}} \left(\frac{n_{Ijt}}{n_{It}} \right)^\epsilon \quad (28)$$

B.2 Patient households

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left(\ln(c_{Pt} - \zeta c_{Pt-1}) + \psi \ln h_{Pt} + \eta_P \ln \left(1 - (n_{Pct}^{1+\epsilon} + n_{Ph_t}^{1+\epsilon})^{1/(1+\epsilon)} \right) \right) \quad (29)$$

subject to

$$c_{Pt} + (1 + \tau_h) q_{ht} h_{Pt} - = \frac{1}{\omega_P} (\omega_I (R_{t-1} b_{Ht-1} - b_{Ht}) + \omega_E (R_{t-1} b_{Kt-1} - b_{Kt})) + q_{ht}(1 - \delta_h) h_{Pt-1} \quad (30)$$

$$n_t = (n_{Pct}^{1+\epsilon} + n_{Ph_t}^{1+\epsilon})^{1/(1+\epsilon)}. \quad (31)$$

First-order conditions

c_{Pt}:

$$\frac{1}{c_{Pt} - \zeta c_{Pt-1}} - E_t \frac{\zeta}{c_{Pt+1} - \zeta c_{Pt}} = \lambda_{1Pt} \quad (32)$$

h_{Pt}:

$$q_{ht}\lambda_{1Pt}(1 + \tau_h) = \beta(1 - \delta_h) E_t \lambda_{1Pt+1} q_{ht+1} + \frac{\Psi}{h_{Pt}} \quad (33)$$

n_{Pjt} j=c,h:

$$\lambda_{1Pt}(1 - \tau_n) w_{jt} = \frac{\eta_P}{1 - n_{Pt}} \left(\frac{n_{Pjt}}{n_{Pt}} \right)^\epsilon \quad (34)$$

b_{Ht} and b_{Kt}:

$$\lambda_{1Pt} = \beta R_t E_t \lambda_{1Pt+1} \quad (35)$$

B.3 Entrepreneurs

$$\max E_0 \sum_{t=0}^{\infty} \theta_E^t \ln(c_{Et} - \zeta c_{Et-1}) \quad (36)$$

subject to

$$c_{Et} + x_{ct} + k_{bt} + \frac{w_{ct}}{\omega_E} \sum_{j=I,P} \omega_j n_{jct} + \tau_k r_{ct} k_{ct-1} = \frac{y_t}{\omega_E} + b_{Kt} - R_{t-1} b_{Kt-1} + (1 - \tau_k) p_{bt} k_{bt} + p_{lt} l_{Et} + \xi_{Et}, \quad (37)$$

$$y_t = z_t (\omega_E k_{ct-1})^{\alpha_C} \left(\sum_{j=I,P} \omega_j n_{jct} \right)^{1-\alpha_C}, \quad (38)$$

$$x_{ct} = k_{ct} - (1 - \delta_c) k_{ct-1}, \quad (39)$$

$$b_{Kt} + \frac{w_{ct}}{\omega_E} \sum_{j=I,P} \omega_j n_{jct} \leq m_K k_{ct}. \quad (40)$$

First-order conditions

c_{Et}:

$$\frac{1}{c_{Et} - \zeta c_{Et-1}} - E_t \frac{\zeta}{c_{Et+1} - \zeta c_{Et}} = \mu_{1Et} \quad (41)$$

b_{Kt}:

$$\mu_{1t} - \mu_{2t} = \theta_E R_t E_t \mu_{1t+1} \quad (42)$$

k_{ct}:

$$\mu_{1t} - m_K \mu_{2t} = \theta_E E_t (\mu_{1t+1} (1 - \delta_c + (1 - \tau_k) r_{ct+1})) \quad (43)$$

$$\text{where } r_{ct} = \frac{\alpha_C y_t}{k_{ct-1}} \quad (44)$$

k_{bt}:

$$\mu_{1t} = \mu_{1t} (1 - \tau_k) p_{bt} \quad (45)$$

n_{jct}:

$$\frac{\mu_{1t} (1 - \alpha_C) y_t}{\sum_{j=I,P} \omega_j n_{jct}} = (\mu_{1t} + \mu_{2t}) w_{ct} \quad (46)$$

B.4 House developers

$$\max q_{ht}ih_t - w_{ht} \sum_{j=I,P} \omega_j n_{jit} - p_{lt}l_t \quad (47)$$

subject to

$$ih_t = z_t (\omega_E k_{bt})^{\alpha_B} l_t^{\alpha_L} \left(\sum_{j=I,P} \omega_j n_{jht} \right)^{1-\alpha_B-\alpha_L}. \quad (48)$$

Land supply:

$$l_t = \sum_{j=I,P,E} \omega_j l_{jt} = 1 \quad (49)$$

Law of motion of houses:

$$ih_t = h_t - (1 - \delta_h)h_{t-1} \quad (50)$$

$$h_t = \omega_I h_{It} + \omega_P h_{Pt}. \quad (51)$$

First-order conditions

\mathbf{n}_{jht} :

$$w_{ht} = (1 - \alpha_B - \alpha_L) \frac{q_t ih_t}{\sum_{j=I,P} \omega_j n_{jht}} \quad (52)$$

\mathbf{k}_{bt} :

$$\omega_E p_{bt} = \alpha_B \frac{q_t ih_t}{k_{bt}} \quad (53)$$

\mathbf{l}_t :

$$p_{lt} = \alpha_L \frac{q_t ih_t}{l_t} \quad (54)$$