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On the Role of Product Quality in Product Reallocation and Macroeconomic Dynamics

Ako Viou BAHUN-WILSON

UNIVERSITÉ DE SHERBROOKE

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Abstract

Recent empirical investigations by Argente et al. (2018) reveal that product reallocation, i.e. creation and destruction of products, happens through two leading margins: entry and exit of production modules within firms, the so-called "extensions", and changes in the characteristics of products within incumbent production modules, the so-called "improvements". This paper develops a DSGE model in which product reallocation involves these two margins and examines the impact on macroeconomic dynamics. I show that relative to the standard model that only accounts for extensions, the model augmented with improvements does a better job at explaining the dynamics of products and the firm-level TFP. A recession facilitates the production of low-quality/low-cost products, which allows the survival of low-productivity modules that would not survive in a fixed-quality environment. Thus, the firm-level TFP decreases. As the recessionary shock dissipates, the share of production modules that use costly technology and manufacture high-quality goods increases, thereby also increasing the firm-level TFP. My results illustrate the importance of recognizing the dynamics of product characteristics within firms' production lines in addition to the dynamics of production lines per se to understand business cycles.

Keywords : product creation and destruction, multi-line firm, quality switching, business cycles.

JEL Codes : D24, E23, E32, L15, L60.

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†Ph.D. student, department of economics, Université de Sherbrooke. E-mail address: ako.viou.bahun-wilson@usherbrooke.ca.

1 Introduction

Several studies have identified the reallocation of products, i.e. the creation and destruction of products, as an important source of economic growth and macroeconomic fluctuations.[1] Regarding the origins of product reallocation, empirical investigations show that most product creation and destruction happen within the boundaries of the firm and that firm flows only explain a negligible fraction of aggregate product turnover (Argente et al., 2018; Bernard et al., 2010; Bernard and Okubo, 2016; Broda and Weinstein, 2010).[2] Recently, Argente et al. (2018), using detailed product and firm-level data, provided further information on how product reallocation happens within firms. They show that it hinges on two leading margins: the creation and destruction of production modules, the so-called "extensions", and changes in the quality of products within the modules that firms already have in their portfolio, the so-called "improvements".[3]

While the macroeconomic impact of product reallocation has received valuable attention in the literature, business cycle models embedding multi-line firms assume that the characteristics of products manufactured within the firm's production lines (or modules) are fixed over time. As such, the dynamics of products are caused by the dynamics of production lines.[4] In this paper, and consistent with new evidence by Argente et al. (2018), I relax the standard assumption of fixed-quality products. Instead, I propose a dynamic and stochastic general equilibrium (DSGE) model that accounts for both the entry and exit of production modules within a large firm and endogenous changes in the quality of products within each production module over the business cycle. The proposed model helps rationalize important business cycle features, espe-

[1]For instance, Argente et al. (2018) show that the decrease in the rate of product reallocation can explain around 20% to 25% of the total decline in TFP observed during the Great Recession. There is also a consensus in the economic literature that product reallocation is one of the key mechanisms through which innovation translates into economic growth, as new and better products replace obsolete ones (e.g., Acemoglu et al., 2018; Aghion et al., 2014; Aghion and Howitt, 1992; Grossman and Helpman, 1991; Schumpeter et al., 1939, among others).

[2]For instance, using micro-level data collected at the household level over the years 194 and 1999 to 2003, Broda and Weinstein (2010) find that 92 percent of product creation and 97 percent of product destruction happen within existing manufacturers.

[3]Argente et al. (2018) observe around 1.64 million different products collected from stores over the period 2007Q1 to 2013Q4. Each product is identified by a unique Universal Product Code (UPC). The UPC is a 12-digit number that is the finest disaggregation level at the product level. The authors have obtained UPC data from the Nielsen Retail Measurement Services (RMS) scanner data set. Then, they match product information with firm information obtained from GS1 US. Notably, their data offers the advantage of observing a large set of products rather than the products consumed by a sample of households as in Broda and Weinstein (2010). They found that extensions account for around 12 percent of total product reallocation, and improvements account for more than 80 percent of total product reallocation.

[4]See, for instance, Hamano and Zanetti (2017).

cially the procyclicality of the firm-level TFP, which are challenging to replicate using standard assumptions. Thus, recognizing the dynamics of product quality simultaneously with the dynamics of production lines within firms is essential to understanding business cycles.

The paper builds on the DSGE framework in Hamano and Zanetti (2017), henceforth HZ17, in which product reallocation is based on entry and exit of production lines within a representative firm. As in HZ17, my model embeds a large firm with multiple production modules, each identified by its idiosyncratic productivity level and producing a single variety of a specific good. Product creation and destruction are drained by entries and exits of production modules within the firm (i.e. entry and exit extensions). Entry extension is endogenous since each new module must pay sunk entry costs, limiting the number of newly created products. Also, exit extension is endogenous since modules that are not productive enough to afford fixed operational costs must discontinue production until a favourable shock make them profitable again. To the HZ17 original framework, I add two ingredients to include the creation and destruction of products due to changes in the quality of products within existing production modules over the business cycle.

First, I assume that households value product quality so that a variety receives a favourable demand when produced with high-quality, *ceteris paribus*. Second, I allow for the possibility that upon entry and once entry costs are paid, each module can choose from a menu of two available technologies, as in Gervais (2015): a cheap technology to produce a low-quality variety or an expensive technology to produce a high-quality variety. Despite the attractiveness of the high-quality market, only the most productive modules can operate there due to the presence of relatively high fixed and marginal production costs. Thus, each module endogenously chooses its product quality over the business cycle, guided by its idiosyncratic productivity level (fixed over time) and the productivity cutoff required to enter each quality market (which fluctuates with aggregate shocks). In a specific period, modules that are unable to afford at least the fixed operating costs associated with a low-quality production shut down and stay idle until a new technological improvement makes them profitable again. However, high-quality modules that become unprofitable in a specific period can switch to a low-quality variety to survive the destruction. The quality switching over the business cycles (or entry and exit improvements) adds a new margin for products to adjust to shocks.

The model is calibrated using statistics on product reallocation in Argente et al. (2018) and establishes three main results. First, similar to HZ17, when product reallocation hinges on entry and exit of production modules, product destruction as well as the firm-level TFP are counter-cyclical, which is at odds with Argente et al. (2018)s' empirical findings.[5] This happens because a recessionary technology shock raises the productivity threshold required to maintain the profitability of modules. Thus, only the most productive modules remain in activity within the firms, which increases the firm-level TFP.

Second, in the augmented HZ17 model, a recessionary technology shock specific to the high-quality market raises production costs for high-quality modules, eliminating the less efficient ones. To survive destruction, these modules switch to low-quality production. At the same time, the shock increases entry costs for modules, limiting the creation of new modules and decreasing labor demand and wages. The fall in wages ultimately favours the production of low-quality products so that even less-productive modules are retained in the firm, reducing exit extensions and decreasing the firm-level TFP, as documented by Argente et al. (2018) for the period of the Great Recession. As the recessionary shock dissipates, the destruction of low-quality/low-productivity modules increases moderately, providing a new explanation to the slow recovery of firm-level TFP, as observed in the aftermath of the Great Recession.

Third, I show that subsidizing the fixed costs of high-quality technologies to attract high-productivity modules and stimulate the economy will not necessarily increase firms' TFP. Indeed, the subsidy lowers the productivity threshold required to produce high-quality products, but has a minimal effect on the productivity threshold required to produce low- quality products. Thus, the endogenous destruction of the lowest productivity modules operating on the low-quality market remains practically unchanged, as does the average productivity of the firm. However, due to the subsidy, the highest-productivity modules producing low-quality products can now maximize their profits by producing high-quality products, taking advantage of lower production costs. Thus, the proportion of high-quality products increases.

The paper is related to several strands of the literature. First, it is related to business cycle models that investigate the implications of product reallocation for macroeco-

[5]Note, however, that Hamano and Zanetti (2017) replicate a procyclical pattern of product destruction for a relatively low degree of product heterogeneity and persistence in the aggregate productivity shock.

nomic dynamics. Besides HZ17, Minniti and Turino (2013) propose a DSGE model in which the dynamics of product creation is driven not only by the traditional mechanism of firms' entry but also by the process of product adding within firms, which generates a strong procyclicality of product creation. In a recent study, Hamano and Oikawa (2022) develop a general equilibrium model in which product reallocation involves firm entry and exit, and also an endogenous product mix within firms over the business cycle given different income elasticities across products in consumer preferences. Although I abstract from the dynamics of firms, I account for both variety and quality differentiation of products.

Second, the paper is related to business cycles models that study the role of product quality and variety in macroeconomic volatilities. For instance, Hamano and Zanetti (2018) develop a model to disentangle the contribution of changes in product quality and variety in the volatility of aggregate prices. In their model, product quality is linked to the firm's productivity in a fixed relationship. In my model, I allow products of different qualities to be produced with the same productivity level over the business cycle.

Third, my analysis relates to Schumpeterian growth models that explain economic growth through a process of creative destruction of products (e.g., Acemoglu et al., 2018; Aghion et al., 2014; Aghion and Howitt, 1992; Grossman and Helpman, 1991; Schumpeter et al., 1939, among others). Unlike the growth literature, I focus on the cyclical properties of product creation and destruction.

Fourth, my work borrows from international trade studies that use product quality to explain the positive correlation between productivity and price. A non-exhaustive list of work in that field includes those of Antoniades (2015), Baldwin and Harrigan (2011), Crozet et al. (2012), Dinopoulos and Unel (2013), Hallak and Sivadasan (2013), Manova and Zhang (2012), Kugler and Verhoogen (2011), Verhoogen (2008), etc. However, unlike the trade literature, I am interested in the short-term dynamics of the economy and I circumscribe my analysis in a closed economy space.

Finally, the paper is related to the literature that studies the impacts of industrial policy of R&D subsidy on firms incitative to innovate (Acemoglu et al., 2018; Goolsbee, 1998; Romer, 2000; Wilson, 2009). Unlike these studies, in my paper, production lines select their production technology endogenously.

The remainder of the paper is organized as follows: section 2 presents the model, section 3 describes the calibration strategy, section 4 presents the results and section 5 concludes.

2 The model

The benchmark economy consists of one unit mass of atomistic, identical and infinitely lived households and a large firm composed of a multitude of production modules in monopolistic competition. Every period, the representative household supplies a quantity of labor to the firm to purchase a basket of available goods; each production module chooses its quality of production to maximize profits, given its idiosyncratic productivity level, consumer preferences, and the aggregate state of the economy. The economic environment of households and production modules is stochastic, and aggregate labor productivity shocks drive the short-run dynamics of the economy.

2.1 Households

At time t, the representative household chooses the aggregate consumption level, C_t, and total labor supply, L_t, to maximize the expected intertemporal utility

$$\mathbb{E}_t \sum_{i=t}^{\infty} \beta^{i-t} \left(lnC_t - \chi \frac{L_t^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}} \right) \tag{1}$$

where $\beta \in (0,1)$ is the discount factor, $\chi > 0$ is the degree of disutility in supplying labor, and φ is the Frisch elasticity of labor supply. C_t is defined over a continuum of goods, Ω, and during each period, only a subset of goods, $\Omega_t \subset \Omega$, is available. Each good in the economy has a unique variety indexed by ω. The consumption aggregator is given by the standard Dixit and Stiglitz (1977) C.E.S aggregator

$$C_t = \left(\int_{\omega \in \Omega_t} [q_t(\omega) c_t(\omega)]^{1-\frac{1}{\sigma}} d\omega \right)^{\frac{1}{1-\frac{1}{\sigma}}} \tag{2}$$

where $\sigma > 1$ is the elasticity of substitution between any couple of varieties,[6] $c_t(\omega)$ is

[6]When a C.E.S utility function in consumption represents preferences, the price elasticity of demand equals the elasticity of substitution. In monopolistic competition, the firm is interested in fixing its price in the region where the demand function is elastic. So, it is reasonable to assume that $\sigma > 1$.

individual demand for the variety ω, and $q_t(\omega) > 1$, is the time t quality of the variety ω.

Equation (2) posits that preferences over the differentiated varieties are additively separable, with weights proportional to product qualities. Thus, all varieties of the same quality and price are consumed at the same rate. When $q_t(\omega) = 1$, the model is reduced into the homogenous quality model à la HZ17.

The periodic price index that minimizes the consumption expenditure is

$$P_t = \left[\int_{\omega \in \Omega_t} \left(\frac{p_t(\omega)}{q_t(\omega)} \right)^{1-\sigma} d\omega \right]^{\frac{1}{1-\sigma}} \tag{3}$$

where $p_t(\omega)$ is the physical unit price of variety ω. Thus, $p_t(\omega)/q_t(\omega)$ is the quality-adjusted individual price of variety ω.

Finally, the optimal demand for ω that minimizes the cost of acquiring the aggregate consumption bundle C_t is :

$$c_t(\omega) = q_t(\omega)^{\sigma-1} \left[\frac{p_t(\omega)}{P_t} \right]^{-\sigma} C_t \tag{4}$$

Equation (4) shows that product quality enters as a demand shifter in preferences so that, conditional on price and industry characteristics, the residual demand for a variety is increasing in its quality. Also, a high-quality variety can be sold at a higher price without reducing its relative demand.

2.2 Production modules

A production module is indexed by its idiosyncratic productivity level z, fixed over time. The idiosyncratic productivity can represent, for instance, the organizational structure of the production within a module or, more generally, the firm-level total productivity factor (TFP), defined as the overall effectiveness with which labor is used in a production process. Each module only produces a specific variety of goods so that the multitude of production modules results in horizontal differentiation of products in terms of varieties. In addition to horizontal differentiation, products are also differentiated vertically. Vertical differentiation, or product quality (brand), refers to any factor, tangible or not, that increases the consumer's willingness to pay (Verhoogen, 2008).[7]

[7]Consider for instance a large firm that produces different groups of electronic goods (telephones,

Product quality is endogenous to the production module. For simplicity and following Gervais (2015), I assume that every period, each module has access to two types of technology: a high-technology, h, to produce a high-quality level, q_h, and a low-technology, l, to produce a low-quality level, q_l. Thus, $q_t(\omega) \in \{q_h, q_l\} \forall t$, with $q_h > q_l$. The use of high-technology is equivalent to investing in R&D or increasing innovation efforts. Although modules have access to two types of technology every period, only one technology is used by a module in a specific period. Thus, each module produces a single quality of its variety in a specific period.

2.2.1 Technology and pricing

Production requires only labor as input.[8] During each period t, the total labor demand, $l_{s,t}$, and the corresponding nominal costs of production, $\Gamma_{s,t} \equiv W_t l_{s,t}$, of a module z operating on the technological segment $s \in \{h, l\}$ are given by:

$$l_{s,t} = \begin{cases} \dfrac{y_t(\omega)}{Z_t z} + \dfrac{f_{h,t}}{Z_t} \text{ if } s = h \\ \dfrac{\alpha y_t(\omega)}{z} + f_{l,t} \text{ if } s = l \end{cases} \quad \text{and } \Gamma_{s,t} \equiv \begin{cases} \dfrac{W_t y_t(\omega)}{Z_t z} + \dfrac{W_t f_{h,t}}{Z_t} \text{ if } s = h \\ \dfrac{\alpha W_t y_t(\omega)}{z} + W_t f_{l,t} \text{ if } s = l \end{cases} \tag{5}$$

In equation (5), W_t stands for the nominal wage, $y_t(\omega)$ is the scale of production of module z and $f_{s,t}$ represents the exogenous fixed costs of operation on segment $s \in \{h, l\}$. Z_t is the aggregate labor productivity level of the economy, which follows an $AR(1)$ process in logs. The parameter α represents the relative marginal cost of a low-quality product at the steady state (i.e. when $Z = 1$). Furthermore, I take the simplest step and assume that only fixed and marginal costs of high-technology products fluctuate with aggregate productivity shocks. This assumption is made to accomodate the empirical findings of Andrews et al. (2016) who show that, on average, the labor productivity growth performance of frontier-plants in the manufacturing industry, i.e. plants in the top decile in terms of labor productivity, is similar to that of the industry overall in OECD countries. The authors also report that frontier plants are leaders in scientific research and development. For this reason, in my model, the aggregate productivity

televisions, computers, etc.). Let us assume that each good has only one variety which is produced in a specific module. The iPhone (a variety of telephones), LG (a variety of televisions) and HP (a variety of computers) can represent available varieties of these electronic goods: this is horizontal differentiation. However, an iPhone can be produced with a high-technology (iPhone 12 pro max for instance) or a low-technology (iPhone 7 for instance), as well as the LG and the HP: this is vertical differentiation.

[8] I make this choice to stay as close as possible to HZ17. Thus, any difference between my results and HZ17 can be attributed to the additional assumption of vertical product differentiation.

shocks are specific to the high-technology market. Thus, only the fixed and marginal production costs of high-quality producers decrease with the aggregate labor productivity. Moreover, I assume that $0 < \alpha < \frac{1}{Z_t}$ and $f_{l,t} < f_{h,t}/Z_t$ so that both the fixed and the marginal costs of production are increasing in product quality, regardless of the aggregate state of the economy (Spence, 1976; Grossman and Helpman, 1991). Finally, I allow the marginal cost to decrease in the idiosyncratic productivity level on each quality segment.

Thereafter, I choose the composite consumption good, C_t, as numeraire and I define the real price of a variety as $\rho_t(\omega) \equiv \frac{p_t(\omega)}{P_t}$ and the real wage (value of labor in unit of consumption goods C_t) as $w_t \equiv W_t/P_t$.

A module using the technology $s \in \{h, l\}$ in period t will maximize the real profits function

$$d_{s,t} = \begin{cases} \rho_{h,t} y_t(\omega) - w_t \dfrac{y_t(\omega)}{Z_t z} - w_t \dfrac{f_{h,t}}{Z_t} & \text{if } s = h \\ \rho_{l,t} y_t(\omega) - w_t \dfrac{\alpha y_t(\omega)}{z} - w_t f_{l,t} & \text{if } s = l \end{cases} \tag{6}$$

Because of the monopolistic competition, each module chooses the unit price of production that maximizes its profits function, leading to the following pricing rule:

$$\rho_{s,t}(z) = \begin{cases} \dfrac{\sigma}{\sigma - 1} \dfrac{w_t}{Z_t z} & \text{if } s = h \\ \dfrac{\sigma}{\sigma - 1} \dfrac{\alpha w_t}{z} & \text{if } s = l \end{cases} \tag{7}$$

Equation (7) shows that within each technology segment, the real price of a variety is a constant markup over the marginal cost. At the module-level, the difference in marginal costs between a high-technology and a low- technology fully explains the difference in prices between a high-quality and a low-quality variety. Thus, for a given level of productivity, a high-quality product is more expensive than a low-quality product because of the corresponding high marginal cost ($0 < \alpha < \frac{1}{Z_t}$).

2.2.2 Production decision

Due to fixed costs, a product of productivity z and quality $q \in \{q_l, q_h\}$ may or may not be produced because the associate profits of the producer can be positive, negative or zero. Substituting (7) into (6), if production materializes, the real profits of a production module z operating on the technological segment $s \in \{h, l\}$ is given by:

$$d_{s,t}(z) = \begin{cases} \frac{1}{\sigma}\left(\frac{\sigma}{\sigma-1}\frac{w_t}{Z_t}\right)^{1-\sigma} q_h^{\sigma-1} z^{\sigma-1} C_t - w_t \frac{f_{h,t}}{Z_t} & \text{if } s = h \\ \frac{1}{\sigma}\left(\frac{\sigma}{\sigma-1}\alpha w_t\right)^{1-\sigma} q_l^{\sigma-1} z^{\sigma-1} C_t - w_t f_{l,t} & \text{if } s = l \end{cases} \tag{8}$$

From (7) and (8), one can easily notice that, on each technology segment, the most productive modules are more likely to charge lower prices and earn higher profits, *ceteris paribus*. I now determine the conditions under which a production module selects its production segment.

Given its productivity level, a module will produce a specific quality of variety in a period if its profits are non-negative. The zero-profit efficiency threshold on the low-quality segment, $z^*_{l,t}$, is such that $d_{l,t}(z^*_{l,t}) = 0$. The zero-profit efficiency threshold on the high-quality segment, $z_{h,t}$, is such that $d_{f,t}(z_{h,t}) = 0$. Thus, at period t, a module z gains a positive profit on the low-quality segment if and only if its productivity level is above $z^*_{l,t}$ and gains positive profits on the high-quality segment if and only if its productivity level is above $z_{h,t}$. Since profit functions are monotonically increasing with productivity on each segment, without additional restrictions, $z^*_{l,t}$ might be superior, inferior or equal to $z_{h,t}$ as depicted in Figure 1 inspired from Gervais (2015).

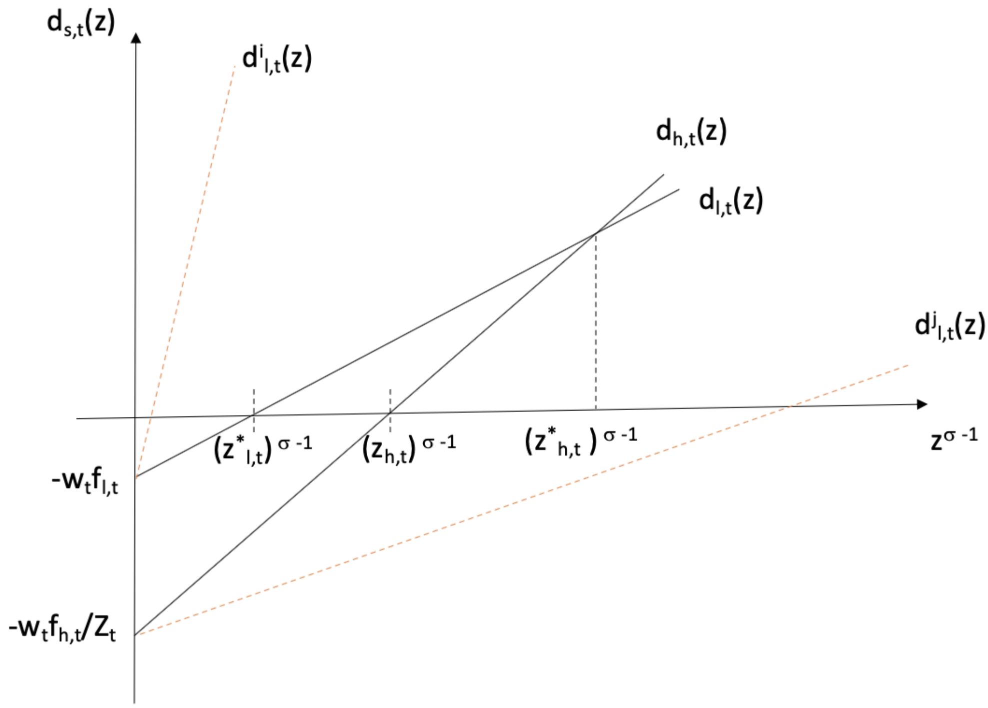


Figure 1: Profit functions

Indeed, according to Figure 1, it might be optimal for a module to produce a low-quality variety, whatever is its productivity level. This situation is represented by the functions $\{d^i_{l,t}(z), d_{h,t}(z)\}$ which show that the profits associated with a low-quality production are above the profits associated with a high-quality product for all z. Also, it might be optimal for modules to produce high-quality varieties, regardless of the productivity distribution. This situation is represented by the functions $\{d_{h,t}(z), d^j_{l,t}(z)\}$ which show that the profits associated with a high-quality production are above those associated with a low-quality production for all z. Based on figure 1, both low-quality and high-quality varieties are produced within the firm every period if and only if $d_{l,t}$ and $d_{h,t}$ intersect at a single point $(z^*_{h,t})^{\sigma-1}$. The intersection $z^*_{h,t}$ represents the productivity level above which modules find it optimal to switch from a low-quality to a high-quality product.

The cutoff productivity level on the low-quality segment and the high-quality segment are respectively:

$$z^*_{l,t} = w_t^{\frac{1}{\sigma-1}} f_{l,t}^{\frac{1}{\sigma-1}} \left(\frac{1}{\sigma}\right)^{-\frac{1}{\sigma-1}} \left(\frac{\sigma}{\sigma-1}\alpha w_t\right) q_l^{-1} C_t^{-\frac{1}{\sigma-1}} \tag{9}$$

and

$$z^*_{h,t} = \left[w_t\left(\frac{f_{h,t}}{Z_t} - f_{l,t}\right)\right]^{\frac{1}{\sigma-1}} \left[\frac{1}{\sigma}\left(\frac{\sigma}{\sigma-1}w_t\right)^{1-\sigma} C_t\left(Z_t^{\sigma-1}q_h^{\sigma-1} - \alpha^{1-\sigma}q_l^{\sigma-1}\right)\right]^{-\frac{1}{\sigma-1}} \tag{10}$$

Equations (9) and (10) show that the cutoff productivity levels on the two technology segments are both increasing in wages, w_t, and decreasing in aggregate demand, C_t. Recall that the aggregate labor productivity is only specific to the high technology segment. Thus, aggregate productivity shocks have a direct impact on $z^*_{h,t}$. However, $z^*_{l,t}$ also fluctuates over the business cycle due to general equilibrium effects working through wages and aggregate consumption.

From (9) and (10), I can express $z^*_{h,t}$ in terms of $z^*_{l,t}$ as follows:

$$z^*_{h,t} = A_t z^*_{l,t} \quad \text{with} \quad A_t \equiv \frac{q_l}{\alpha}\left[\frac{\left(\frac{f_{h,t}}{Z_t} - f_{l,t}\right)/f_{l,t}}{(Z_t q_h)^{\sigma-1} - (q_l/\alpha)^{\sigma-1}}\right]^{\frac{1}{\sigma-1}} = \left[\frac{\frac{f_{h,t}/Z_t}{f_{l,t}} - 1}{\left(\frac{\frac{Z_t q_h}{1}}{\frac{q_l}{\alpha}}\right)^{\sigma-1} - 1}\right]^{\frac{1}{\sigma-1}} \tag{11}$$

Equation (11) shows that the locus A_t will increase when there is a negative shock on the aggregate labor productivity of high-quality producers. Thus, the share of high-quality production modules decreases during recessions and increases during expansions.

Recall that a necessary condition for the profit functions $d_{h,t}(z)$ and $d_{l,t}(z)$ to intercect at a single point $z^*_{h,t}$ as in Figure 1 is that the slope of the profit function on the high-quality market is steeper than the slope of the profit function on the low-quality market, i.e. $\frac{\partial d_{h,t}(z)}{\partial z^{\sigma-1}} > \frac{\partial d_{l,t}(z)}{\partial z^{\sigma-1}}$. This happens under the following assumption:

Proposition 1.

$$\left(\frac{\frac{Z_t q_h}{1}}{\frac{q_l}{\alpha}}\right)^{\sigma-1} > 1 \Rightarrow \frac{q_h}{q_l} > \frac{1/Z_t}{\alpha}$$

According to assumption 1, the slope condition is satisfied when the transition from a low-technology to a high-technology leads to an increase in quality greater than the corresponding increase in the marginal cost of production.

Also, based on Figure 1, in equilibrium, the highest-productivity modules must produce high-quality varieties (Crespi and Zuniga, 2012; Klette and Kortum, 2004). This happens when A_t is greater than one as summarized by the following assumption:

Proposition 2.

$$\left[\frac{\frac{f_{h,t}/Z_t}{f_{l,t}}-1}{\left(\frac{\frac{Z_t q_h}{1}}{\frac{q_l}{\alpha}}\right)^{\sigma-1}-1}\right]^{\frac{1}{\sigma-1}} > 1 \Rightarrow \left[\frac{f_{h,t}/Z_t}{f_{l,t}}\right] > \left[\frac{\frac{q_h}{\left(\frac{1}{Z_t}\right)}}{\frac{q_l}{\alpha}}\right]^{\sigma-1}$$

Thus, the share of high-quality producers within the firm is driven by the relative fixed and marginal costs, the relative quality of products between modules and household preferences.

Under assumptions 1 and 2, in equilibrium, a candidate of productivity z immediately exits if $z < z^*_{l,t}$, produces a low-quality variety if $z \in [z^*_{l,t}, z^*_{h,t})$ and produces a high-quality variety if $z \geq z^*_{h,t}$. Similar to Hamano and Zanetti (2017, 2018), inefficient production lines that have drawn a lower productivity level than the cutoff-productivity necessary to ensure positive profits on the low-quality market are discontinued and stay idle (endogenous destruction) until a positive shock makes them profitable. The sorting of modules over the productivity spectrum partitions the productivity distribution of modules into three groups: exiting modules (productivities between z_{min} and $z^*_{l,t}$), low-quality modules (productivities between $z^*_{l,t}$ and $z^*_{h,t}$) and high-quality modules (productivities over $z^*_{h,t}$).

2.2.3 Product reallocation and business cycles

In this subsection, I illustrate the dynamics of product reallocation over the business cycles. Due to non-linearity, it may be challenging to examine how the productivity cutoffs behave over the business cycle. Therefore, in Figure 2, I present hypothetical impacts of aggregate labor productivity shocks on the economy cutoffs.

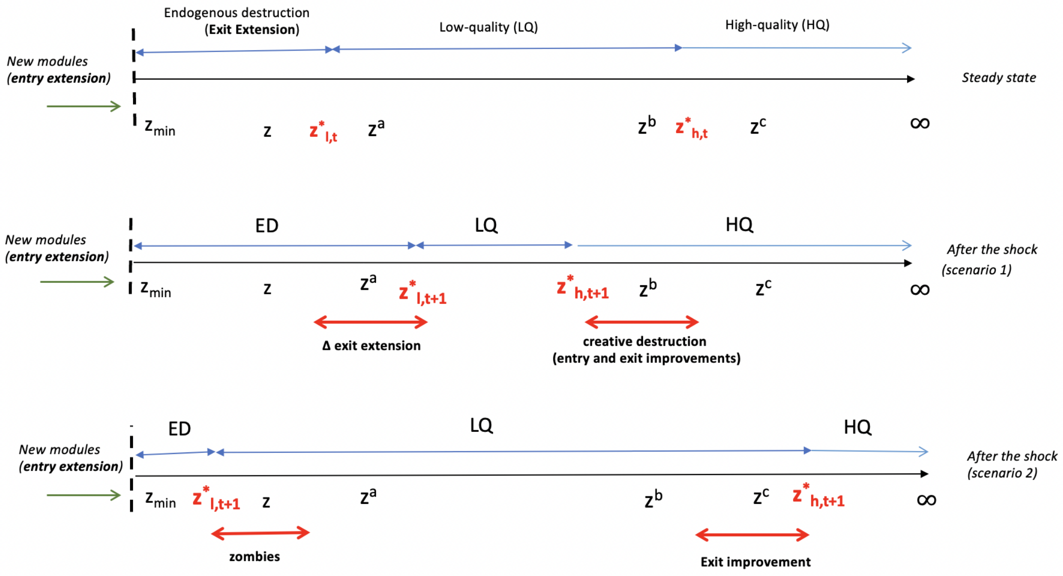


Figure 2: Product reallocation and aggregate productivity shocks

Let us consider the sorting of modules across the quality segments before an aggregate productivity shock as in Figure 2. Every period, new modules enter the firm. Argente et al. (2018) refer to the situation where product creation is due to the entry of new production modules within the firm as "entry extension". At the steady state, the module z is too unproductive to be profitable and optimally decides to discontinue production. Argente et al. (2018) refer to the situation where product destruction is due to a cessation of activity of the module as "exit extension". Modules z^a and z^b are producing a low-quality of their respective varieties, while module z^c is producing a high-quality variety.

Now, let us consider a situation where, after the shock, the cutoff-productivity level on the low-quality segment, $z^*_{l,t}$, increases while the cutoff-productivity level on the high-quality segment, $z^*_{h,t}$, decreases (scenario 1). Given this initial distribution of productivity levels, with an increase in $z^*_{l,t}$, the lowest-productivity varieties on the low-quality market (for instance z^a) disappear so that endogenous destruction increases. On the contrary, when $z^*_{h,t}$ decreases, some initially low-quality modules become more profitable on the high-quality market. Thus, they abandon the production of lower-quality varieties for high-quality varieties, as is the case with the module z^c. The quality-upgrading behaviour of modules illustrates the idea of Schumpeterian creative destruction as innovative products replace the old-fashioned ones. This represents a situation of product reallocation due to changes in the quality of products within production modules that the firm already has in its portfolio (entry and exit improvements).

In the second scenario, the shock increases $z^*_{h,t}$ and decreases $z^*_{l,t}$. Thus, some idle modules before the shock (for instance, the module z) become productive again (the "zombies") and enter the low-quality segment of the market with relatively low-productivity levels. The increase in $z^*_{h,t}$ induces a quality switching effect: some high-quality modules before the recession (for instance, the module z^c) abandon a high-quality production and switch to low-quality varieties to maximize profits. Thus, product quality behaves like a spring, which may stretch or retract, depending on the direction of economic turbulence, to protect a module from destruction.

Remark that by construction, creative destruction per se has no impact on the average productivity of the firm since changes in product quality within incumbent modules do not affect the average productivity of modules. Only module flows affect the average productivity of the firm.

2.2.4 Module averages

In this subsection, I present the average value of the main economic variables within the firm, taking into account horizontal and vertical differentiation of products.

I first derive the average productivity of potential producers. Suppose a mass M_t of candidates for low-quality and high-quality production at t. At period t, each candidate draws a productivity level z from a common and fixed distribution $g(z)$ that has a positive support over $[z_{min}, \infty)$ and a continuous cumulative distribution $G(z)$. Thus, $G(z)$ also represents the cumulative productivity distribution of all modules with production potential.

Given the definition of productivity cutoffs, only a number $N_t = N_{l,t} + N_{h,t} = [1 - G(z^*_{l,t})]M_t$ of varieties of different qualities is produced every period, where $N_{l,t}$ represents the total number of low-quality products and $N_{h,t}$, the total number of high-quality products. The average productivity of all the M_t modules, $\tilde{z}_{M_t}$, is determined as in Melitz (2003):

$$\tilde{z}_M = \left(\int_{z_{min}}^{\infty} z^{\sigma-1} \, dG(z) \right)^{\frac{1}{\sigma-1}} \tag{12}$$

where z_{min} is the minimum of productivity of the distribution. When varieties of both qualities are produced in equilibrium, the average productivity of high-quality products is given by :

$$\tilde{z}_{h,t} = \left(\frac{1}{1 - G(z^*_{h,t})} \int_{z^*_{h,t}}^{\infty} z^{\sigma-1} \, dG(z) \right)^{\frac{1}{\sigma-1}} \tag{13}$$

and the average productivity of low-quality products is given by:

$$\tilde{z}_{l,t} = \left(\frac{1}{G(z^*_{h,t}) - G(z^*_{l,t})} \int_{z^*_{l,t}}^{z^*_{h,t}} z^{\sigma-1} \, dG(z) \right)^{\frac{1}{\sigma-1}} \tag{14}$$

The terms $\tilde{z}_{h,t}$ and $\tilde{z}_{l,t}$ contains all the information about the distribution of productivities respectively on the high-quality segment and the low-quality segment. Thus, the average real price on each technology segment is given by:

$$\tilde{\rho}_{s,t} \equiv \rho_{s,t}(\tilde{z}_{s,t}) = \begin{cases} \dfrac{\sigma}{\sigma-1} \dfrac{w_t}{Z_t \tilde{z}_{h,t}} & \text{if } s = h \\ \dfrac{\sigma}{\sigma-1} \dfrac{\alpha w_t}{\tilde{z}_{l,t}} & \text{if } s = l \end{cases} \tag{15}$$

and the average real profits of producing modules on each technology-segment is:

$$\tilde{d}_{s,t} \equiv d_{s,t}(\tilde{z}_{s,t}) = \begin{cases} \dfrac{1}{\sigma} \tilde{\rho}_{h,t}^{1-\sigma} q_h^{\sigma-1} C_t - w_t \dfrac{f_{h,t}}{Z_t} & \text{if } s = h \\ \dfrac{1}{\sigma} \tilde{\rho}_{l,t}^{1-\sigma} q_l^{\sigma-1} C_t - w_t f_{l,t} & \text{if } s = l \end{cases} \tag{16}$$

Therefore, the weighted average real profit of producing modules is:

$$\tilde{d}_t = \frac{N_{l,t}}{N_t} \tilde{d}_{l,t} + \frac{N_{h,t}}{N_t} \tilde{d}_{h,t} \tag{17}$$

Finally, the average operational profits among potential producers is defined as

$$\tilde{d}^o_t = \frac{N_t}{M_t} \tilde{d}_t \tag{18}$$

2.2.5 Entry decision

Prior to entry, modules are homogenous and have no signal about the segment they are likely to produce on. Module creation and the productivity draw are conditioned by the payment of fixed and sunk costs expressed in terms of effective labor, f_{E_t}/Z_t. Thus, the sunk cost in unit of consumption good is $w_t f_{E_t}/Z_t$. Furthermore, modules are subject to an exogenous death shock at the end of each period with a probability $\gamma \in (0, 1)$. The shock forces the affected modules to leave the firm regardless of their level of productivity. As standard in the literature, I assume that new modules need one period to build. Thus, the expected post-entry value of any candidate to production is given by:

$$v_t = \mathbb{E}_t \sum_{j=t+1}^{\infty} [\beta(1-\gamma)]^{j-t} \left(\frac{C_j}{C_t}\right)^{-1} \tilde{d}_j^o \tag{19}$$

Equation (19) shows that the value of any candidate to production is determined by the stream of the expected profits $\{\tilde{d}_j^o\}_{j=t+1}^{\infty}$, discounted using the stochastic discount factor of households adjusted by exogenous exit shock. Entry occurs until the expected value of a module is equal to the entry costs so that the following free entry condition holds:

$$v_t = w_t f_{E_t}/Z_t \tag{20}$$

2.2.6 Entry and exit extensions

A module is only identified by its productivity level, while a product is identified by the productivity level of the producer and a specific quality level (high or low). Since a product can change quality over the business cycles while being produced in the same module, i.e. with the same productivity level, product reallocation does not necessarily result in module reallocation (see Figure 2).[9] Since module entry is endogenous and limited by the entry costs, I refer to the situation where the creation of products is the result of the creation of new modules in the firm as entry extension (or endogenous creation), and the situation where the destruction of products corresponds to a cessation of activity of unproductive modules as exit extension or (endogenous destruction).

Every period, H_t new modules are created. Modules entered at time t only start

[9]In Figure 2, following a shock as depicted in scenario 1, some low-quality products are destroyed, causing the death of their respective modules (for instance, the module z^a). However, in scenario 2, the module z^c is maintained in the firm despite the destruction of one of its products (switching from a high-quality to a low-quality product).

production at $t+1$ before which they can be destroyed by the exogenous death shock. Thus, the dynamics of the mass of modules is given by:

$$M_t = (M_{t-1} + H_{t-1})(1-\gamma) \tag{21}$$

Endogenous destruction in each period is given by:

$$D_t^{endo} \equiv M_t - (N_{l,t} + N_{h,t}) \tag{22}$$

and the number of varieties destroyed exogenously in each period is :

$$D_t^{exo} \equiv \gamma(M_t + H_t) \tag{23}$$

2.2.7 Parameterization of productivity draws

I assume a Pareto distribution for the productivity levels at entry. The cumulative distribution $G(z)$ is given by:

$$G(z) = 1 - \left(\frac{z_{min}}{z}\right)^{\kappa} \tag{24}$$

where $\kappa > (\sigma - 1)$ determines the shape of the productivity distribution. As κ increases, productivity levels are clustered toward the lower bound, z_{min}, and products become homogenous in productivity and quality. Substituting successively (24) into (13) and (14), the period t average productivity of high-quality producing modules becomes

$$\tilde{z}_{h,t} = z^*_{h,t}\left[\frac{\kappa}{\kappa - (\sigma - 1)}\right]^{\frac{1}{\sigma-1}}. \tag{25}$$

and the period t average productivity of low-quality producing modules becomes (see proofs in appendix A):

$$\tilde{z}_{l,t} = \left[\frac{\kappa}{\kappa - (\sigma - 1)}\right]^{\frac{1}{\sigma-1}} \left(\frac{z_{l,t}^{*-(\kappa-(\sigma-1))} - z_{h,t}^{*-(\kappa-(\sigma-1))}}{z_{l,t}^{*-\kappa} - z_{h,t}^{*-\kappa}}\right)^{\frac{1}{\sigma-1}}. \tag{26}$$

In particular, $\lim_{z^*_{h,t}\to+\infty} \tilde{z}_{l,t} = z^*_{l,t}\left[\frac{\kappa}{\kappa-(\sigma-1)}\right]^{\frac{1}{\sigma-1}}$. Thus, imposing infinite cutoff productivity for high-quality modules excludes the high-quality market, and the model is reduced to one quality-segment model. Also, equations (25) and (26) show that the average productivity of products on each quality segment depends on the cutoffs, given the shape of the distribution, κ, and household preferences, σ.

Recall that the zero-profit condition (ZPC) of the marginal low-quality module is given by

$$d_{l,t}(z^*_{l,t}) = 0 \Leftrightarrow \frac{1}{\sigma} q_l^{\sigma-1} \left[\frac{\sigma}{\sigma-1} \frac{\alpha w_t}{z^*_{l,t}} \right]^{1-\sigma} C_t - w_t f_{l,t} = 0 \tag{27}$$

Let $\tilde{z}_t \equiv \left(\frac{1}{1-G(z^*_{l,t})} \int_{z^*_{l,t}}^{\infty} z^{\sigma-1}\, dG(z) \right)^{\frac{1}{\sigma-1}}$ represent the average productivity of all producing modules regardless of their quality of production.

$$\tilde{z}_t = z^*_{l,t} \left[\frac{\kappa}{\kappa - (\sigma-1)} \right]^{\frac{1}{\sigma-1}} \Rightarrow z^*_{l,t} = \tilde{z}_t \left[\frac{\kappa}{\kappa - (\sigma-1)} \right]^{\frac{1}{1-\sigma}} \tag{28}$$

Equation (28) shows that the cutoff productivity of the marginal low-quality module is proportional to the average productivity of the economy.

Substitute (28) in (27) provides the ZPC of the average low-quality module:

$$\frac{1}{\sigma} q_l^{\sigma-1} \left[\frac{\kappa}{\kappa - (\sigma-1)} \right]^{-1} \left[\frac{\sigma}{\sigma-1} \frac{\alpha w_t}{\tilde{z}_t} \right]^{1-\sigma} C_t - w_t f_{l,t} = 0 \tag{29}$$

On the other hand, the equal profit condition (EPC) on which modules base their decision to enter the high-quality segment is $d_{l,t}(z^*_{h,t}) = d_{h,t}(z^*_{h,t})$, which is equivalent to

$$\frac{1}{\sigma} q_l^{\sigma-1} \left[\frac{\sigma}{\sigma-1} \frac{\alpha w_t}{z^*_{h,t}} \right]^{1-\sigma} C_t - w_t f_{l,t} = \frac{1}{\sigma} q_h^{\sigma-1} \left[\frac{\sigma}{\sigma-1} \frac{w_t}{Z_t z^*_{h,t}} \right]^{1-\sigma} C_t - w_t \frac{f_{h,t}}{Z_t}$$

$$\Leftrightarrow \frac{1}{\sigma} C_t \left[q_h^{\sigma-1} \left(\frac{\sigma}{\sigma-1} \frac{w_t}{Z_t z^*_{h,t}} \right)^{1-\sigma} - q_l^{\sigma-1} \left(\frac{\sigma}{\sigma-1} \frac{\alpha w_t}{z^*_{h,t}} \right)^{1-\sigma} \right] = w_t \left(\frac{f_{h,t}}{Z_t} - f_{l,t} \right)$$

$$\Leftrightarrow \frac{1}{\sigma} C_t \left[\left(\frac{\sigma}{\sigma-1} \frac{w_t}{z^*_{h,t}} \right)^{1-\sigma} \left((Z_t q_h)^{\sigma-1} - (q_l/\alpha)^{\sigma-1} \right) \right] = w_t \left(\frac{f_{h,t}}{Z_t} - f_{l,t} \right) \tag{30}$$

Isolating $z^*_{h,t}$ from equation (25) and substituting it into equation (30) leads to the following equal profits condition for the average high-quality module:

$$\frac{1}{\sigma} C_t \left[\left(\frac{\sigma}{\sigma-1} \frac{w_t}{\tilde{z}_{h,t}} \right)^{1-\sigma} \left[\frac{\kappa}{\kappa - (\sigma-1)} \right]^{-1} \left((Z_t q_h)^{\sigma-1} - (q_l/\alpha)^{\sigma-1} \right) \right] = w_t \left(\frac{f_{h,t}}{Z_t} - f_{l,t} \right) \tag{31}$$

2.2.8 Surviving rates

Each period, the fraction of high-quality modules that survive endogenous destruction is given by :

$$\frac{N_{h,t}}{M_t} = [1 - G(z^*_{h,t})] = z^{\kappa}_{min}\tilde{z}^{-\kappa}_{h,t}\left[\frac{\kappa}{\kappa - (\sigma - 1)}\right]^{\frac{\kappa}{\sigma-1}} \tag{32}$$

and the fraction of low-quality producers firms that survive endogenous destruction among potential producers is given by :

$$\frac{N_{l,t}}{M_t} = [G(z^*_{h,t}) - G(z^*_{l,t})] = \left(\frac{z_{min}}{z^*_{l,t}}\right)^{\kappa} - \left(\frac{z_{min}}{z^*_{h,t}}\right)^{\kappa} = z^{\kappa}_{min}\left(z^{*-\kappa}_{l,t} - z^{*-\kappa}_{h,t}\right).$$

Using the expression of $\left(z^{*-\kappa}_{l,t} - z^{*-\kappa}_{h,t}\right)$ from equation (26), the ratio $\frac{N_{l,t}}{M_t}$ becomes

$$\frac{N_{l,t}}{M_t} = z^{\kappa}_{min}\tilde{z}^{1-\sigma}_{l,t}\left[\frac{\kappa}{\kappa - (\sigma - 1)}\right]\left[z^{*-(\kappa-(\sigma-1))}_{l,t} - z^{*-(\kappa-(\sigma-1))}_{h,t}\right].$$

Once I substitute $z^*_{l,t}$ and $z^*_{h,t}$ in $\frac{N_{l,t}}{M_t}$, I get

$$\frac{N_{l,t}}{M_t} = z^{\kappa}_{min}\tilde{z}^{1-\sigma}_{l,t}\left[\frac{\kappa}{\kappa - (\sigma - 1)}\right]\left[\left(\tilde{z}_t\left(\frac{\kappa}{\kappa - (\sigma - 1)}\right)^{\frac{1}{1-\sigma}}\right)^{-(\kappa-(\sigma-1))} - \left(\tilde{z}_{h,t}\left(\frac{\kappa}{\kappa - (\sigma - 1)}\right)^{\frac{1}{1-\sigma}}\right)^{-(\kappa-(\sigma-1))}\right] \tag{33}$$

Finally, the proportion of all firms that survive endogenous destruction at t is:

$$\frac{N_t}{M_t} = [1 - G(z^*_{i,t})] = z^{\kappa}_{min}\tilde{z}^{-\kappa}_{t}\left[\frac{\kappa}{\kappa - (\sigma - 1)}\right]^{\frac{\kappa}{\sigma-1}} \tag{34}$$

2.3 Household budget constraint and intertemporal problem

The budget constraint of the representative household is defined on a real basis (i.e. in a unit of consumption goods). The household enters the period t with a shareholding x_t in a mutual fund of incumbent modules at time t, M_t. The mutual fund pays a total real dividend in each period that is equal to the total profit of all modules that produce in that period, $N_{l,t}\tilde{d}_{l,t} + N_{h,t}\tilde{d}_{h,t}$. Every period, the firm must issue equities to finance the next period's production of modules. Thus, during each period t, the representative household buys x_{t+1} shares in a mutual fund of incumbents, M_t, and new entrants, H_t, even though only modules that survive exogenous and endogenous destruction at t will pay back dividends to the household at period $t+1$. The date t real price of a claim to the future profit stream of the mutual fund of $M_t + H_t$ modules is equal to the average real price of claims to future profits of home firms, v_t. Every period, the representative household income consists of labor income, $L_t w_t$, dividend income, $x_t(N_{l,t}\tilde{d}_{l,t} + N_{h,t}\tilde{d}_{h,t})$, and the revenue of selling its initial share position, $x_t M_t v_t$. The household spends its total income on consumption, C_t, and future shares of existing and new modules for an amount $x_{t+1}v_t(M_t + H_t)$. Thus, it transfers resources intertemporally by investing in newly created modules so that the dynamics of H_t stands for the dynamics of investment. The budget constraint is given by:[10]

$$L_t w_t + x_t M_t \left[v_t + \tilde{d}^o_t\right] = C_t + x_{t+1} v_t (M_t + H_t) \tag{35}$$

During each period t, the representative household chooses consumption, C_t, labor supply, L_t, and shareholding, x_{t+1}, to maximize the lifetime utility (1) subject to the budget constraint (35). The first-order conditions with respect to consumption and labor supply yield the standard labor supply equation:

$$\chi (L_t)^{1/\varphi} = w_t C_t^{-1} \tag{36}$$

The first-order condition with respect to shareholdings once combined with the products law of motion (21) and the first-order condition for consumption yields the Euler equation

$$v_t = \beta(1-\gamma) E_t \left[\frac{C_{t+1}}{C_t}\right]^{-1} \left[v_{t+1} + \tilde{d}^o_{t+1}\right] \tag{37}$$

Equation (37), once iterated forward, shows that share prices are the expected discounted sum of future dividends, as in expression (19).

2.4 Closing the model

Labor and good markets must clear in general equilibrium. I assume that aggregate labor supply, L_t, is employed in either the production of consumption goods or the creation of new modules. Thus,

$$L_t = N_{l,t}\tilde{l}_{l,t}(\tilde{z}_{l,t}) + N_{h,t}\tilde{l}_{h,t}(\tilde{z}_{h,t}) + H_t \frac{v_t}{w_t} \tag{38}$$

[10] Explicitly, the household's budget constraint is given by: $L_t w_t + x_t \left[M_t v_t + N_{l,t}\tilde{d}_{l,t} + N_{h,t}\tilde{d}_{h,t}\right] = C_t + x_{t+1} v_t (M_t + H_t)$.
Substituting equation (17) in (18), we have $N_{l,t}\tilde{d}_{l,t} + N_{h,t}\tilde{d}_{h,t} = M_t \tilde{d}^o_t$. Thus, we can rewrite the budget constraint as $L_t w_t + x_t M_t \left[v_t + \tilde{d}^o_t\right] = C_t + x_{t+1} v_t (M_t + H_t)$.

$$\Leftrightarrow L_t = N_{l,t}\left[(\sigma-1)\frac{\tilde{d}_{l,t}}{w_t} + \sigma f_{l,t}\right] + N_{h,t}\left[(\sigma-1)\frac{\tilde{d}_{h,t}}{w_t} + \sigma\frac{f_{h,t}}{Z_t}\right] + H_t\frac{v_t}{w_t} \tag{39}$$

The transition from (38) to (39) is demonstrated in appendix B.

Finally, aggregating budget constraints among symmetric households of mass one provides the aggregated accounting identity of GDP:

$$Y_t \equiv C_t + v_t H_t = L_t w_t + N_t \tilde{d}_t \tag{40}$$

where Y_t is real GDP measured in the welfare basis of expenditure and income. Note that the model is composed of two markets: the market of goods and services and the labor market. According to Walras' law, one market equilibrium must ensure the other market equilibrium. Then, optimal allocations from (39) must equalize optimal allocations from (40) in general equilibrium.

Table 1 summarizes the main equilibrium conditions of the model. The baseline model consists of 18 equations and 18 endogenous variables, ($\tilde{z}_h$, $\tilde{z}_l$, $\tilde{z}_t$, $\tilde{\rho}_{h,t}$, $\tilde{\rho}_{l,t}$, w_t, $N_{h,t}$, $N_{l,t}$, N_t, $\tilde{d}_{h,t}$, $\tilde{d}_{l,t}$, $\tilde{d}_t$, $\tilde{d}^o_t$, M_t, v_t, C_t, H_t, L_t) among which the number of potential producers, M_t is predetermined. Additionally, the model has four exogenous variables: the aggregate productivity Z_t, the low-segment fixed cost $f_{l,t}$, the high- segment fixed cost $f_{h,t}$ and the entry cost $f_{E,t}$.

Table 1:
The model equations

Average pricings:	$\tilde{\rho}_{h,t} = \frac{\sigma}{\sigma-1}\frac{w_t}{Z_t\tilde{z}_{h,t}}; \tilde{\rho}_{l,t} = \frac{\sigma}{\sigma-1}\frac{\alpha w_t}{\tilde{z}_{l,t}}$
Price index:	$\tilde{\rho}_{l,t}^{1-\sigma} q_l^{\sigma-1} N_{l,t} + \tilde{\rho}_{h,t}^{1-\sigma} q_h^{\sigma-1} N_{h,t} = 1$
module profits:	$\tilde{d}_{l,t} = \frac{1}{\sigma}\tilde{\rho}_{l,t}^{1-\sigma} q_l^{\sigma-1} C_t - w_t f_{l,t}; \tilde{d}_{h,t} = \frac{1}{\sigma}\tilde{\rho}_{h,t}^{1-\sigma} q_h^{\sigma-1} C_t - w_t \frac{f_{h,t}}{Z_t}$
	$\tilde{d}_t = \frac{N_{l,t}}{N_t}\tilde{d}_{l,t} + \frac{N_{h,t}}{N_t}\tilde{d}_{h,t}; \tilde{d}_t^o = \frac{N_t}{M_t}\tilde{d}_t$
Producing modules:	$N_t = N_{l,t} + N_{h,t}$
Free entry condition:	$v_t = \frac{w_t f_{Et}}{Z_t}$
Motion of modules:	$M_t = (M_{t-1} + H_{t-1})(1-\gamma)$
Surviving rates:	$\frac{N_{h,t}}{M_t} = z_{min}^{\kappa} \tilde{z}_{h,t}^{-\kappa} \left[\frac{\kappa}{\kappa-(\sigma-1)}\right]^{\frac{\kappa}{\sigma-1}}; \frac{N_t}{M_t} = z_{min}^{\kappa} \tilde{z}_t^{-\kappa} \left[\frac{\kappa}{\kappa-(\sigma-1)}\right]^{\frac{\kappa}{\sigma-1}}$
	$\frac{N_{l,t}}{M_t} = z_{min}^{\kappa} \tilde{z}_{l,t}^{1-\sigma} \left[\frac{\kappa}{\kappa-(\sigma-1)}\right].$
	$\left[\left(\tilde{z}_t \left(\frac{\kappa}{\kappa-(\sigma-1)}\right)^{\frac{1}{1-\sigma}}\right)^{-(\kappa-(\sigma-1))} - \left(\tilde{z}_{h,t} \left(\frac{\kappa}{\kappa-(\sigma-1)}\right)^{\frac{1}{1-\sigma}}\right)^{-(\kappa-(\sigma-1))}\right]$
Labor supply:	$\chi L_t^{\frac{1}{\varphi}} = w_t C_t^{-1}$
Euler equation:	$v_t = \beta(1-\gamma)\mathbb{E}_t \left[\frac{C_{t+1}}{C_t}\right]^{-1} \left[v_{t+1} + \tilde{d}_{t+1}^o\right]$
Labor market clearing:	$L_t = N_{l,t}\left[(\sigma-1)\frac{\tilde{d}_{l,t}}{w_t} + \sigma f_{l,t}\right] + N_{h,t}\left[(\sigma-1)\frac{\tilde{d}_{h,t}}{w_t} + \sigma \frac{f_{h,t}}{Z_t}\right] + H_t \frac{v_t}{w_t}$
ZPC:	$\frac{1}{\sigma} q_l^{\sigma-1} \left[\frac{\kappa}{\kappa-(\sigma-1)}\right]^{-1} \left[\frac{\sigma}{\sigma-1}\frac{\alpha w_t}{\tilde{z}_t}\right]^{1-\sigma} C_t = w_t f_{l,t}$
EPC:	$\frac{1}{\sigma} C_t \left[\left(\frac{\sigma}{\sigma-1}\frac{w_t}{\tilde{z}_{h,t}}\right)^{1-\sigma} \left[\frac{\kappa}{\kappa-(\sigma-1)}\right]^{-1} \left((Z_t q_h)^{\sigma-1} - (q_l/\alpha)^{\sigma-1}\right)\right] = w_t \left(\frac{f_{h,t}}{Z_t} - f_{l,t}\right)$

Note : see appendix C for the proof of the price index (row 2 in Table 1).

I assume that Z_t, $f_{l,t}$, $f_{h,t}$ and $f_{E,t}$ follow an AR(1) process in logarithm, i.e.

$$\begin{bmatrix} \log Z_t \\ \log f_{h,t} \\ \log f_{l,t} \\ \log f_{E,t} \end{bmatrix} = \begin{bmatrix} \rho_Z & 0 & 0 & 0 \\ 0 & \rho_{f_h} & 0 & 0 \\ 0 & 0 & \rho_{f_l} & 0 \\ 0 & 0 & 0 & \rho_{f_E} \end{bmatrix} \begin{bmatrix} \log Z_{t-1} \\ \log f_{h,t-1} \\ \log f_{l,t-1} \\ \log f_{E,t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{Z,t} \\ \varepsilon_{h,t} \\ \varepsilon_{l,t} \\ \varepsilon_{f_E,t} \end{bmatrix}$$

where $\rho_v, \forall v \in \{Z, f_E, f_l, f_h\}$, represents the persistence parameter that corresponds to each exogenous variable. Shocks, $\varepsilon_{v,t}$ are normally distributed with zero mean and variance equal to σ_v^2. If the process of exogenous variables is stationary, this will imply that all the model variables will be stationary as well.

2.5 The reduced model

To facilitate the calculation of the steady state of the system analytically, I derive a reduced version of the model, expressing most of the baseline system equations in terms of the high-quality modules averages only.

From (11),

$$z^*_{l,t} = z^*_{h,t} A_t^{-1}. \tag{41}$$

Substitute (41) in (26) yields

$$z^*_{h,t} = \tilde{z}_{l,t} \left[\frac{\kappa}{\kappa - (\sigma - 1)} \right]^{\frac{1}{1-\sigma}} \left[\frac{A_t^{\kappa-(\sigma-1)} - 1}{A_t^{\kappa} - 1} \right]^{\frac{1}{1-\sigma}} \tag{42}$$

and substitute (42) in (41) yields

$$z^*_{l,t} = \tilde{z}_{l,t} \left[\frac{\kappa}{\kappa - (\sigma - 1)} \right]^{\frac{1}{1-\sigma}} \left[\frac{A_t^{\kappa-(\sigma-1)} - 1}{A_t^{\kappa} - 1} \right]^{\frac{1}{1-\sigma}} A_t^{-1} \tag{43}$$

Using (25) and (43), $\tilde{z}_h$ is related to $\tilde{z}_l$ as follows:

$$\tilde{z}_{l,t} = B^{-1} \tilde{z}_{h,t} \qquad \text{with} \qquad B_t \equiv \left[\frac{A_t^{\kappa-(\sigma-1)} - 1}{A_t^{\kappa} - 1} \right]^{\frac{1}{1-\sigma}} \tag{44}$$

Thus, prices in equation (15) are related as follows:

$$\tilde{\rho}_{l,t} = (\alpha Z_t B_t) \tilde{\rho}_{h,t} \tag{45}$$

The fraction of low-quality producers, $\frac{N_{l,t}}{M_t} = z_{min}^{\kappa} (z_{l,t}^{*-\kappa} - z_{h,t}^{*-\kappa})$, becomes:

$$\frac{N_{l,t}}{M_t} = z_{min}^{\kappa} \left[\tilde{z}_{h,t}^{-\kappa} \left(\frac{\kappa}{\kappa - (\sigma - 1)} \right)^{\frac{\kappa}{\sigma - 1}} \left(A_t^{\kappa} - 1 \right) \right] \tag{46}$$

Taking the ratio (32)/(46) ,

$$N_{l,t} = \left(A_t^{\kappa} - 1 \right) N_{h,t} \tag{47}$$

Thus, the total number of producers is given by [11]

$$N_t = A_t^{\kappa} N_{h,t} \tag{48}$$

and the operational profit of modules is

$$\tilde{d}_t^o = \frac{A_t^{\kappa} N_{h,t}}{M_t} \tilde{d}_t \tag{49}$$

Using the aggregate price index, $\tilde{\rho}_{l,t}^{1-\sigma} q_l^{\sigma-1} N_{l,t} + \tilde{\rho}_{h,t}^{1-\sigma} q_h^{\sigma-1} N_{h,t} = 1$, and equation (45), the average price of high-quality varieties becomes:

$$\tilde{\rho}_{h,t} = N_{h,t}^{\frac{1}{\sigma-1}} D_t^{\frac{1}{\sigma-1}} \qquad \text{with} \qquad D_t \equiv (\alpha Z_t B_t)^{1-\sigma} q_l^{\sigma-1} \left(A_t^{\kappa} - 1\right) + q_h^{\sigma-1} \tag{50}$$

Finally, based on the previous results, the average profit of producers, $\tilde{d}_t$, the ZPC and the labor market clearing of the model are successively equivalent to:

$$\begin{aligned} \tilde{d} = \tfrac{1}{\sigma} \tfrac{C_t}{D_t N_{h,t}} E_t - w_t G_t \quad \text{with} \quad & E_t \equiv \tfrac{A_t^{\kappa}-1}{A_t^{\kappa}} (\alpha Z_t B_t)^{1-\sigma} q_l^{\sigma-1} + \tfrac{1}{A_t^{\kappa}} q_h^{\sigma-1} \\ \text{and} \quad & G_t \equiv \tfrac{f_{h,t}}{A_t^{\kappa} Z_t} + \tfrac{A_t^{\kappa}-1}{A_t^{\kappa}} f_{l,t} \end{aligned} \tag{51}$$

$$\frac{1}{\sigma} \frac{C_t}{D_t N_{h,t}} = q_l^{1-\sigma} (\alpha Z_t B_t)^{\sigma-1} \left[\frac{\kappa}{\kappa - (\sigma-1)}\right] \left[\frac{A_t^{\kappa-(\sigma-1)} - 1}{A_t^{\kappa} - 1}\right] A_t^{\sigma-1} w_t f_{l,t} \tag{52}$$

$$L_t = (\sigma - 1) A_t^{\kappa} N_{h,t} \frac{\tilde{d}_t}{w_t} + \sigma N_{h,t} \left[\left(A_t^{\kappa} - 1\right) f_{l,t} + \frac{f_{h,t}}{Z_t}\right] + H_t \frac{v_t}{w_t} \tag{53}$$

The reduced model is summarized in Table 2 and consists of 16 equations and 16 endogenous variables. The steady state of the reduced model is presented in appendix D. Note that the good market clearing condition $\left(C_t + v_t H_t = L_t w_t + N_t \tilde{d}_t\right)$ and the equal profit condition $\left(\frac{1}{\sigma} C_t \left[\left(\frac{\sigma}{\sigma-1} \frac{w_t}{\tilde{z}_{h,t}}\right)^{1-\sigma} \left[\frac{\kappa}{\kappa-(\sigma-1)}\right]^{-1} \left((Z_t q_h)^{\sigma-1} - (q_l/\alpha)^{\sigma-1}\right)\right] = w_t \left(\frac{f_{h,t}}{Z_t} - f_{l,t}\right)\right)$ are automatically satisfied once we solve for the reduced system.

[11] Substituting (43) into (28) yields the average productivity of all producing modules, which is given by: $\tilde{z}_t = \tilde{z}_{l,t} \left[\frac{A_t^{\kappa-(\sigma-1)}-1}{A_t^{\kappa}-1}\right]^{\frac{1}{1-\sigma}} A_t^{-1} = \tilde{z}_{l,t} B_t A_t^{-1}$.

Table 2:
The reduced system

Average pricing:	$\tilde{\rho}_{h,t} = \frac{\sigma}{\sigma-1}\frac{w_t}{Z_t\tilde{z}_{h,t}}$
Real price:	$\tilde{\rho}_{h,t} = N_{h,t}^{\frac{1}{\sigma-1}} D_t^{\frac{1}{\sigma-1}}$
Average profit of survivors:	$\tilde{d}_t = \frac{1}{\sigma}\frac{C_t}{D_t N_{h,t}} E_t - w_t G_t$
Operational profit:	$\tilde{d}_t^o = \frac{A_t^\kappa N_{h,t}}{M_t}\tilde{d}_t$
Free entry condition:	$v_t = \frac{w_t f_{Et}}{Z_t}$
Motion of modules:	$M_t = (M_{t-1} + H_{t-1})(1-\gamma)$
Euler equation:	$v_t = \beta(1-\gamma)\mathbb{E}_t\left[\frac{C_{t+1}}{C_t}\right]^{-1}\left[v_{t+1} + \tilde{d}^o_{t+1}\right]$
Optimal labor supply:	$\chi L_t^{\frac{1}{\varphi}} = w_t C_t^{-1}$
ZPC:	$\frac{1}{\sigma}\frac{C_t}{D_t N_{h,t}} = q_l^{1-\sigma}(\alpha Z_t B_t)^{\sigma-1}\left[\frac{\kappa}{\kappa-(\sigma-1)}\right]\left[\frac{A_t^{\kappa-(\sigma-1)}-1}{A_t^\kappa-1}\right]A_t^{\sigma-1} w_t f_{l,t}$
Surviving rate:	$\frac{N_{h,t}}{M_t} = z_{min}^\kappa \tilde{z}_{h,t}^{-\kappa}\left[\frac{\kappa}{\kappa-(\sigma-1)}\right]^{\frac{\kappa}{\sigma-1}}$
Labor market clearing:	$L_t = (\sigma-1)A_t^\kappa N_{h,t}\frac{\tilde{d}_t}{w_t} + \sigma N_{h,t}\left[\left(A_t^\kappa - 1\right) f_{l,t} + \frac{f_{h,t}}{Z_t}\right] + H_t\frac{v_t}{w_t}$
Definitions:	$A_t \equiv \left[\frac{\frac{f_{h,t}/Z_t}{f_{l,t}}-1}{\left(\frac{Z_t q_h}{\frac{1}{\frac{q_l}{\alpha}}}\right)^{\sigma-1}-1}\right]^{\frac{1}{\sigma-1}}$
	$B_t \equiv \left[\frac{A_t^{\kappa-(\sigma-1)}-1}{A_t^\kappa-1}\right]^{\frac{1}{1-\sigma}}$
	$D_t \equiv (\alpha Z_t B_t)^{1-\sigma} q_l^{\sigma-1}\left(A_t^\kappa - 1\right) + q_h^{\sigma-1}$
	$E_t \equiv \frac{A_t^\kappa-1}{A_t^\kappa}(\alpha Z_t B_t)^{1-\sigma} q_l^{\sigma-1} + \frac{1}{A_t^\kappa} q_h^{\sigma-1}$
	$G_t \equiv \frac{f_{h,t}}{A_t^\kappa Z_t} + \frac{A_t^\kappa-1}{A_t^\kappa} f_{l,t}$

3 Calibration

Table 3:
Parameterization

Parameter	Definition	Value	Source
β	Discount factor	0.99	Literature
φ	Frish elasticity of labor supply	2	HZ17
σ	Elasticity of substitution among varieties	7.5	Across brand module elasticity (Broda and Weinstein, 2010).
κ	Shape of the Pareto distribution parameter	6.6249	HZ17 with $(1 - N_t/M_t) = 0.016$ and revenue share=0.5(3.79%).
γ	Exogenous destruction rate	0.0089	To match the quaterly rate of "Entry Extensions" of 0.9% in Argente et al. (2018).
χ	Marginal disutility of labor supply	0.93053	To deliver a labor supply equals to one at the steady state.
z_{min}	Minimum idiosyncratic productivity level	1	Assumption
α	Relative marginal cost of low-quality modules	0.24	Calibrated so that $\log(\frac{\rho_h}{\rho^{median}})/\log(\frac{\rho_l}{\rho^{median}}) = \frac{0.78}{-0.64} \Rightarrow \frac{\rho_h}{\rho_l} = \frac{exp(0.78)}{exp(-0.64)}$.
q_l	Low-quality value level	1	Assumption.
q_h	High-quality value level	4.2	Calibrated so that $\frac{q_h}{1} > \frac{1}{0.24}$.
Z	Aggregate productivity	1	Assumption
f_E	Entry cost	1	Assumption
f_l	fixed cost on the low-segment	1	Assumption
f_h	fixed cost on the high-segment	37.179	calibrated to deliver $B_t = 3.79$.
ρ_Z	Persistence of aggregate productivity	0.979	King and Rebelo (1999)
ρ_{f_E}	Persistence of entry cost	1	Permanent shock
ρ_{f_h}	Persistence of f_h	1	Permanent shock
ρ_{f_l}	Persistence of f_l	1	Permanent shock
σ_Z	Standard deviation of productivity shocks	0.0072	King and Rebelo (1999)
σ_{f_E}	Standard deviation of f_E shocks	0.0072	HZ17
σ_{f_h}	Standard deviation of f_h shocks	0.0072	HZ17
σ_{f_h}	Standard deviation of f_l shocks	0.0072	HZ17

The calibration of the model is summarized in Table 3. The discount factor, β, and the Frisch elasticity of labor supply, φ, are set to 0.99 and 2 respectively following Ghironi and Melitz (2005) and Hamano and Zanetti (2017, 2018). The elasticity of substitution among varieties, σ, is set to 7.5, the across brand modules median elasticity of substitution estimated by Broda and Weinstein (2010). HZ17 calibrate the shape of the productivity distribution, κ, of multiple production lines operating within a large firm and producing homogenous quality goods. They found $\kappa = 11.5070$, which corresponds to an endogenous destruction rate of 6% and a mean relative sales of exiting products to those of the average products of 0.09. In their model, product destruction induces the destruction of production lines. However, their data on product dynamics are taken from Broda and Weinstein (2010) who do not separate the reallocation of products that induce a reallocation of production lines from a reallocation of product that does not induce a reallocation of production lines. This separation is done in Argente et al. (2018). Indeed, Argente et al. (2018) classify the exit of products within a firm due to the destruction of modules as "exit extensions". They found an exit extensions rate of 0.016. They also classify the exit of products within a firm due to changes in the characteristics of products without exit of producing modules as "exit improvement" and found an "exit improvement" rate of 0.020. Thus, I recalculate the HZ17's shape of the productivity distribution using the "exit extensions" rate of 0.016 and a within-firm revenue share of exiting products that correspond to half of the one calculated by Argente et al. (2018), i.e. 0.5(3.79%). Indeed, the revenue share of exiting products calculated by Argente et al. (2018) is not separated by type of exiters (extension exiters or improvement exiters). Since "exit extensions" represents about half of total exits of products, I assume that the revenue share of exiting products due to a temporary stop of the production of modules is half of the revenue share of all exiting products, i.e. exits that induce cessation of activity of modules or not. This procedure gives $\kappa = 6.6249$.

The value of the parameter of disutility in labor supply, χ, is calibrated to deliver a labor supply equal to one at the steady state (Hamano and Zanetti, 2017, 2018; Mumtaz and Zanetti, 2015). This gives $\chi = 0.93053$. Argente et al. (2018) classify entries of new products within a firm due to the creation of new modules as "entry extensions ". Accordingly, in our model, the creation of new modules, H_t, corresponds to the creation of products for extension motives. Thus, I calibrate the exogenous destruction rate, γ, to match the quarterly rate of "Entry Extensions" of 0.9%, as calculated in Argente et al. (2018). This calibration is performed using the steady state equation (64), which provides γ equal to 0.0089.

I normalize the lower bound of the distribution of production, z_{min}, to one since it does not affect the dynamics of the system. I set the steady state value of the aggregate labor productivity, Z, to one. Argente et al. (2018) calculate the relative price of products with the median prices of products in their categories. They find that the highest average log relative price (the 90th percentile) is equal to 0.78 and the lowest average log relative price (the 10th percentile) is equal to -0.64. In my model, this corresponds to $\log(\frac{\rho_h}{\rho^{median}})/\log(\frac{\rho_l}{\rho^{median}}) = \frac{0.78}{-0.64} \Rightarrow \frac{\rho_h}{\rho_l} = \frac{exp(0.78)}{exp(-0.64)}$, given the median price of products within a module, ρ^{median}. The procedure gives the relative marginal cost, α equal to 0.24. An important assumption that determines the coexistence of high-tech and low-tech modules within the large representative firm is that the relative quality, $\frac{q_h}{q_l}$, exceeds the inverse of the relative marginal costs, $1/\alpha$ (see assumption 1). Since only the gap between the fixed costs, the marginal costs and the quality levels matters for the sorting of modules across the technological segments (and not the individual values), I, therefore, normalize the steady state value of f_l, and q_l to one. Then, I choose a value of q_h so that

$\frac{q_h}{1} > \frac{1}{0.24}$. I begin the sensitivity analysis on parameter q_h by the value $q_h = 4.2$.

Since I don't know the dispersion of productivity among production modules within firms in the US, I use the plant-level statistics of Canada's distribution of labor productivity. Gu et al. (2018) show that in the manufacturing sector of Canada, frontier plants (i.e. plants in the top decile in terms of labor productivity levels, which are also innovation leaders) are 3.79 times more productive than non-frontier plants from 2002 to 2009. Thus, I assume that in my model, high-technology modules are 3.79 times more productive than low-technology modules, which leads to a steady state value of B_t equal to 3.79, using equation (44). Then, I pin down the steady state value of A which gives $A = 2.7279$ and $f_h = 37.179$ when q_h= 4.2. Finally, I follow HZ17 and set the persistence parameter on the aggregate productivity ρ_z, and the standard deviation of productivity shock, σ_Z, to 0.979 and 0.0072, respectively, as in King and Rebelo (1999).

3.1 Shutting down the vertical differentiation

To understand the role of entry and exit improvements in product reallocation, I derive an alternative model which abstracts from the possibility for modules to adjust their product quality over the business cycles. Afterward, I compare the performance of the two models in explaining the procyclicality of module creation and destruction and that of the firm-level TFP, as in Argente et al. (2018).

In the alternative model, all the production modules use the same technology and produce varieties of the same quality. Thus, they face the same variable and fixed costs of production, i.e. $\alpha = 1/Z_t$ and $f_{h,t} = f_{l,t} = f_t$. Therefore, product quality becomes irrelevant for the dynamics of the model so that, without loss of generality, one can assume that $q_h = q_l = 1$. Furthermore, modules now have a unique pricing rule, ρ_t, and profit function, d_t, which leads to a single cutoff productivity level, z_t^*, above which N_t modules among a mass of M_t candidates are profitable. The impossibility of a quality switching over the business cycles induces that the destruction of a specific product always leads to a cessation of activity of the module offering that product. Finally, successful entrants have an average productivity, $\tilde{z}_t = \left(\frac{1}{1-G(z_t^*)}\int_{z_t^*}^{\infty} z^{\sigma-1}\, dG(z)\right)^{\frac{1}{\sigma-1}}$, which determines the aggregate variables of the economy.

The system that characterizes the model's equilibrium is the same as in HZ17. It consists of 11 equations and 11 endogenous variables, as in Table 4 (see appendix E).

All the variables in Table 4 have the same meaning as in the baseline model, the only difference being that they refer to modules that produce varieties of the same quality. The model without vertical differentiation has three sources of shocks, each following an $AR(1)$ process in logs: the aggregate labor productivity, Z_t, the entry cost, $f_{E,t}$, and the fixed costs of production, f_t.

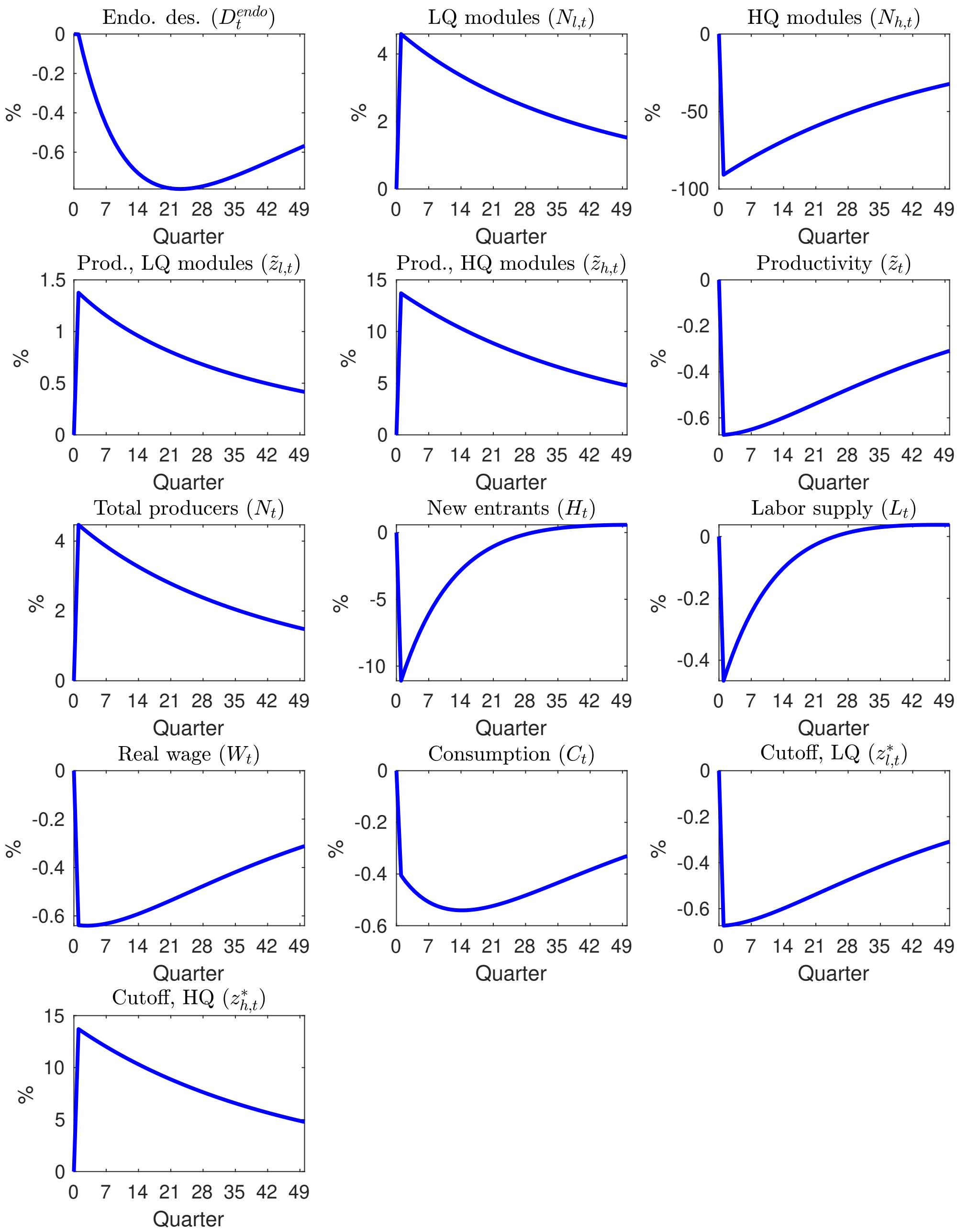
The steady state of the model is presented in HZ17. Regarding the calibration of the model without vertical differentiation, most parameters take the same value as in the baseline model with quality differentiation. These parameters are the minimum productivity level, z_{min}, the discount factor, β, the Frisch elasticity of labor supply, φ, the elasticity of substitution among varieties, σ, the shape of the distribution Pareto, κ, the exogenous destruction rate, γ, the parameters related to the exogenous shocks, and the steady state value of aggregate productivity, Z_t and entry costs, f_E. Two parameters change value compared to the baseline model. First, the steady state value of the fixed operational costs, f, is calculated to deliver a mean quarterly exit extension rate of 0.016 using equation (23) in HZ17. Second, the parameter of disutility in labor supply, χ, is calibrated to deliver a labor supply equal to one. I find $f = 0.00037$ and $\chi = 0.93053$.

4 Results

The model has no analytical solution due to its stochastic and non-linear properties. Therefore, I solve it numerically using perturbation (Maih, 2015; Judd and Guu, 1993; Juillard et al., 2003). The perturbation method uses a first-order Taylor approximation of a non-linear system around the steady-state. In this way, the system's dynamics are determined by the deviation of the endogenous variables from their steady state value in reaction to exogenous shocks.

4.1 Impulse response functions to a recessionary productivity shock

The dynamics of selected variables of the system in response to a negative shock on Z_t is represented in Figure 3. A recession increases entry costs for modules. Thus, product creation (H_t) decreases. This result is consistent with the empirical findings of Argente et al. (2018) who show that the entry of products due to the introduction of new production modules within firms ("entry extension") decreases during the US Great Recession. Furthermore, the fall in entries leads to a drop in aggregate labor demand, which in turn decreases wages (W_t). Therefore, households optimally decide to consume less and consumption (C_t) falls.



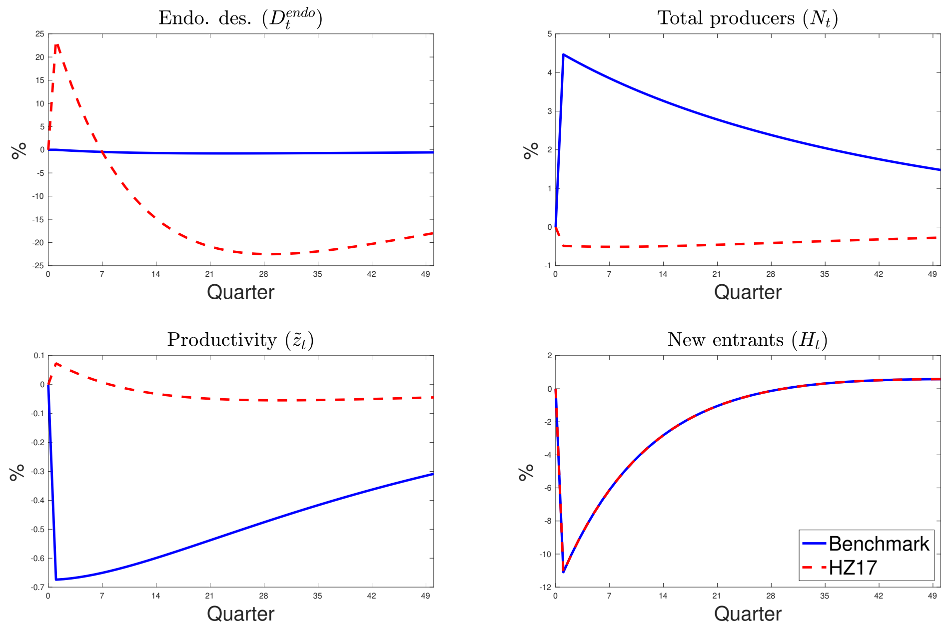
Note : each entry shows the percentage-point response of a variable to a one-percent deviation of a negative productivity shock.

Figure 3: Impulse responses to a recessionary productivity shock

Recall that the cutoff-productivity level necessary to produce low-quality products is increasing in wage and decreasing in consumption. Thus, the significant drop in wage relative to that of consumption decreases the productivity cutoff on the low-quality segment ($z^*_{l,t}$), which allows the survival of relatively low-productivity modules (the "zombies") so that endogenous destruction (D^{endo}_t) decreases. Such a procyclical pattern of endogenous destruction is consistent with the empirical findings of Argente et al. (2018) who show that exits of products due to a cessation of the activity of modules (exit extension) decreases during the Great Recession. The decrease in endogenous destruction implies that, compared to the steady state, the average productivity of all producing modules ($\tilde{z}_t$) decreases, and the total number of producers (N_t) increases. The moderate growth in the destruction of low-quality and low-productivity products (i.e. the lowest productivity modules), which took so long to return to pre-recession levels, leads to a slow recovery in the firm-level productivity in the aftermath of the shock, as empirically documented by Davis and Haltiwanger (2014) and Decker et al. (2014) after the Great Recession. Note that the value of the parameter κ, which determines the shape of the distribution of productivity levels within the firm, is critical for the system's dynamics. Indeed, for high values of κ, it becomes difficult for modules to remain profitable in recession as the cutoff productivity level increases more on the high-quality segment and decreases less on the low-quality segment. Thus, the number of modules in operation within the firm during economic downturns increases less as κ increases. High values of κ dampen the procyclicality of endogenous destruction of modules and the firm-level TFP (see Figure 7 in appendix F).

I now investigate what happens on each quality market. The recessionary shock increases the fixed operational and marginal costs required to produce high-quality varieties. Thus, the cutoff productivity level necessary to produce high-quality products ($z^*_{h,t}$) increases. As a result, the less efficient high-quality products are destroyed, decreasing thereby the number of high-quality producers ($N_{h,t}$) as well as the average productivity level on the high-quality segment ($\tilde{z}_{h,t}$). However, to survive destruction, the less efficient production modules producing high-quality products revert to a low-quality production. Thus, the number of low-quality products (NL) increases due to the decrease in endogenous destruction and the quality-switching behaviour of modules. The increase in the average productivity level on the low-quality segment ($\tilde{z}_{l,t}$) suggests that the average productivity of modules that switch to low-quality production outweighs the average productivity of "zombies". Appendix F shows that reasonable changes in the high-quality value, q_h, do not significantly affect the dynamics of the system.

Figure 4 compares the impulse response functions of the benchmark model with those implied by HZ17 in response to a negative shock on Z_t. The results show that the dynamics of endogenous creation (H_t) are the same in the two models. This happens because there is no difference in entry costs between the two models. However, in the HZ17 model, a recessionary technology shock raises fixed operational costs, requiring a higher module-specific productivity level for modules to survive the destruction. Thus, without the quality switching mechanism, unproductive modules leave the economy, i.e. endogenous destruction (D_t^{endo}) increases, reducing the number of producing modules (N_t). Exits of the lowest-productivity modules generate a "cleansing effect" of recessions à la Caballero and Hammour (1994) which increases the average productivity of the economy. Extending the HZ17 model to include endogenous quality helps explain the procyclicality of endogenous destruction and the firm-level TFP, as in Argente et al. (2018). Indeed, the ease for modules to produce low-quality goods during recession allows the survival of low-productivity modules that would not survive in a fixed quality model.

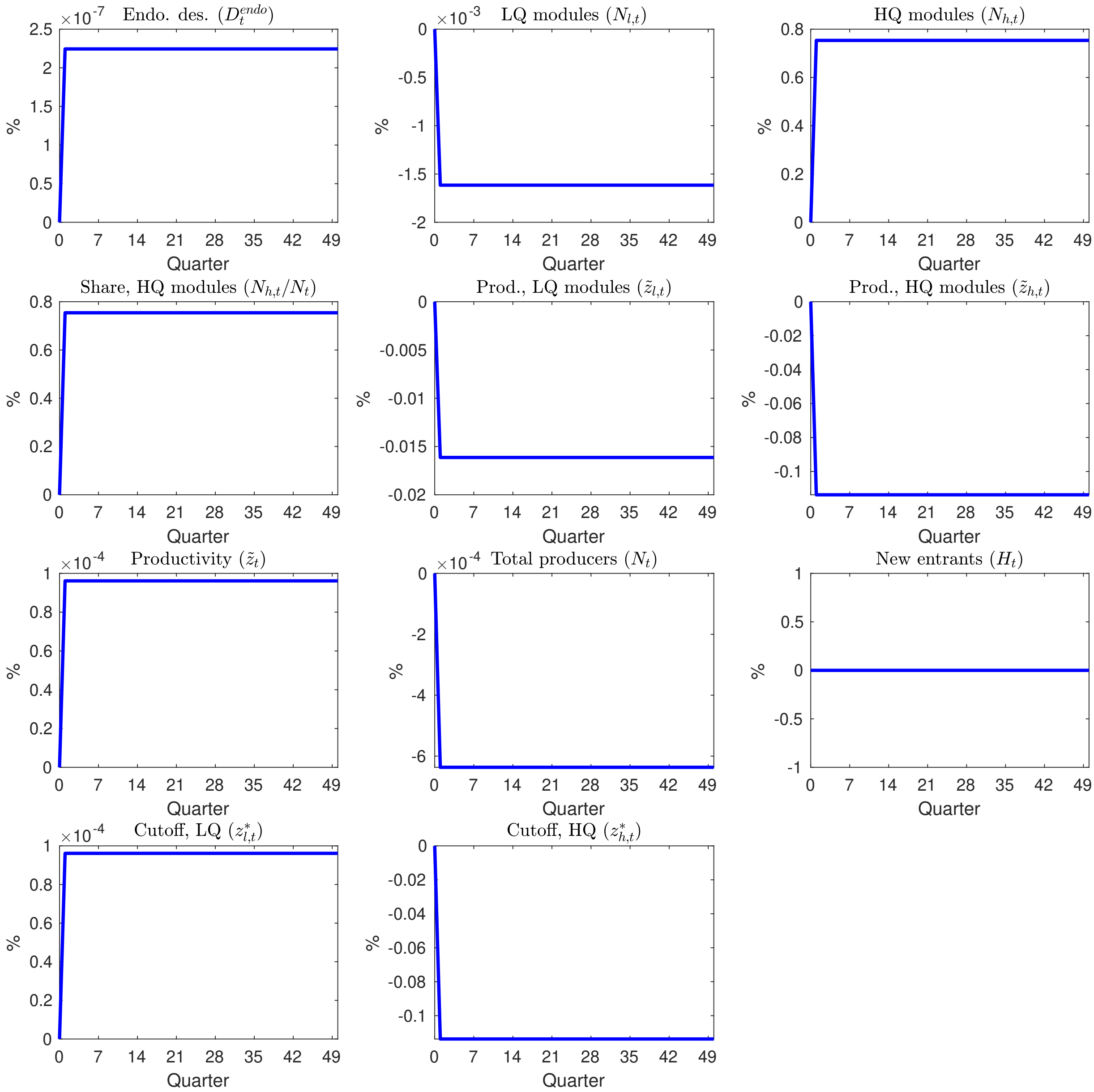


Note: each entry shows the percentage-point response of a variable to a one-percent deviation of a negative productivity shock for the benchmark economy (solid line) and the HZ17 model (dashed line).

Figure 4: Impulse responses to a recessionary productivity shock: the benchmark versus HZ17

4.2 Impulse response functions to a permanent increase in subsidies to high-quality modules

Figure 5 presents the models' dynamics to a permanent increase in subsidies to high-quality modules, proxied by a one-percent reduction in the high-segment fixed costs, $f_{h,t}$. The decrease in $f_{h,t}$ allows modules of relatively low-productivity levels to now produce high-quality goods. Therefore, the average productivity of high-quality products, ($\tilde{z}_{h,t}$), decreases. Since the most productive modules among low-quality producers now produce on the high-quality segment due to the subsidy, the average productivity of low-quality products ($\tilde{z}_{l,t}$) decreases. Endogenous creation (H_t) and destruction (D_t^{endo}) of modules are unchanged. Thus, the number of products (N_t) and the average productivity of the economy ($\tilde{z}_t$) remain practically unchanged, as do the labor supply (L_t), wage (W_t), and the aggregate demand (C_t). However, the share of high-quality products increases because the number of high-quality producers ($N_{h,t}$) increases while the number of low-quality producers ($N_{l,t}$) decreases.



Note: each entry shows the percentage-point response of a variable to a permanent subsidy shock to high-quality modules.

Figure 5: Permanent subsidy shock to high-quality modules

5 Conclusion

This paper revisits the impact of product reallocation on macroeconomic dynamics, featuring recent stylized facts on product dynamics in Argente et al. (2018). Product reallocation happens through two margins: the creation and destruction of production modules within firms and changes in the quality of products within modules over the business cycles.

The paper develops an extension of Hamano and Zanetti (2017) to include quality differentiation among heterogeneous productivity modules that compete monopolistically within a large firm. Upon entry, each module chooses its production quality to maximize profits. Producing a high-quality variety increases production costs. Thus, every period, modules that cannot afford production costs terminate production or reduce their product quality. The endogenous quality switching adds a new channel through which products adjust to shocks and helps better explain observed features of the data, in particular, the procyclicality of firm-level module creation and destruction and the procyclicality of the firm TFP.

To simplify the analysis, I assume that each module can only produce one type of quality among two available, each period: a low-quality or a high-quality. However, Argente et al. (2018) and Broda and Weinstein (2010) document that products of different qualities (or brands) are manufactured within production modules. Extending the model to include the possibility of continuous product quality within production modules would certainly be a useful extension that will help to better understand the reallocation of products and modules within firms. Furthermore, Argente et al. (2018) show that the rate of product reallocation within less diversified firms is more sensitive to aggregate conditions than that of more diversified firms. Therefore, an important topic for future research is understanding how heterogenous firms endogenously determine their number of production modules and product quality over the business cycles.

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Appendix

A Aggregate productivity levels

Determination of $\tilde{z}_{h,t}$:

$$\tilde{z}_{h,t} = \left[\frac{1}{1-G(z^*_{h,t})}\int_{z^*_{h,t}}^{\infty} z^{\sigma-1}\, dG(z)\right]^{\frac{1}{\sigma-1}} = \left[\frac{1}{\left(\frac{z_{min}}{z^*_{h,t}}\right)^{\kappa}}\int_{z^*_{h,t}}^{\infty} z^{\sigma-1}\, g(z)dz\right]^{\frac{1}{\sigma-1}}$$

where $g(z) = \frac{dG(z)}{dz} = \kappa z_{min}^{\kappa} z^{-\kappa-1}$. Thus,

$$\begin{aligned}
\tilde{z}_{h,t} &= \left[\left(\frac{z^*_{h,t}}{z_{min}}\right)^{\kappa}\int_{z^*_{h,t}}^{\infty} \kappa z_{min}^{\kappa} z^{\sigma-1-\kappa-1}dz\right]^{\frac{1}{\sigma-1}} \\
&= \left[\left(\frac{z^*_{h,t}}{z_{min}}\right)^{\kappa} \kappa z_{min}^{\kappa}\int_{z^*_{h,t}}^{\infty} z^{\sigma-1-\kappa-1}dz\right]^{\frac{1}{\sigma-1}} \\
&= \left[\left(\frac{z^*_{h,t}}{z_{min}}\right)^{\kappa} \kappa z_{min}^{\kappa}\left[\frac{1}{\sigma-1-\kappa} z^{\sigma-1-\kappa}\right]_{z^*_{h,t}}^{\infty}\right]^{\frac{1}{\sigma-1}} \\
&= \left[\left(\frac{z^*_{h,t}}{z_{min}}\right)^{\kappa} \kappa z_{min}^{\kappa}\left[-\frac{1}{\sigma-1-\kappa} z^{*\sigma-1-\kappa}_{h,t}\right]\right]^{\frac{1}{\sigma-1}} \\
&= \left[\left(\frac{z^*_{h,t}}{z_{min}}\right)^{\kappa}\left[\frac{\kappa z_{min}^{\kappa}}{\kappa-(\sigma-1)} z^{*\sigma-1-\kappa}_{h,t}\right]\right]^{\frac{1}{\sigma-1}} \\
&= z^*_{h,t}\left[\frac{\kappa}{\kappa-(\sigma-1)}\right]^{\frac{1}{\sigma-1}}
\end{aligned} \tag{54}$$

Determination of $\tilde{z}_{l,t}$:

$$
\begin{aligned}
\tilde{z}_{l,t} &= \left(\frac{1}{G(z^*_{h,t})-G(z^*_{l,t})}\int_{z^*_{l,t}}^{z^*_{h,t}} z^{\sigma-1}\, dG(z)\right)^{\frac{1}{\sigma-1}} \\
&= \left[\frac{1}{\left(\frac{z_{min}}{z^*_{l,t}}\right)^{\kappa}-\left(\frac{z_{min}}{z^*_{h,t}}\right)^{\kappa}}\int_{z^*_{l,t}}^{z^*_{h,t}} \kappa z_{min}^{\kappa} z^{\sigma-1-\kappa-1} dz\right]^{\frac{1}{\sigma-1}} \\
&= \left[\frac{1}{\left(\frac{z_{min}}{z^*_{l,t}}\right)^{\kappa}-\left(\frac{z_{min}}{z^*_{h,t}}\right)^{\kappa}}\kappa z_{min}^{\kappa}\left[\frac{1}{\sigma-1-\kappa} z^{\sigma-1-\kappa}\right]_{z^*_{l,t}}^{z^*_{h,t}}\right]^{\frac{1}{\sigma-1}} \\
&= \left[\frac{1}{\left(\frac{1}{z^{*\kappa}_{l,t}}\right)-\left(\frac{1}{z^{*\kappa}_{h,t}}\right)}\left[\frac{\kappa}{\sigma-1-\kappa} z^{\sigma-1-\kappa}\right]_{z^*_{l,t}}^{z^*_{h,t}}\right]^{\frac{1}{\sigma-1}} \\
&= \left[\frac{\kappa}{\kappa-(\sigma-1)}\right]^{\frac{1}{\sigma-1}}\left(\frac{z_{l,t}^{*-(\kappa-(\sigma-1))}-z_{h,t}^{*-(\kappa-(\sigma-1))}}{z_{l,t}^{*-\kappa}-z_{h,t}^{*-\kappa}}\right)^{\frac{1}{\sigma-1}}
\end{aligned}
\tag{55}
$$

B Labor market clearing

$$
L_t = N_{l,t} l_{l,t}(\tilde{z}_{l,t}) + N_{h,t} l_{f,t}(\tilde{z}_{h,t}) + H_t \frac{v_t}{w_t}
$$

I first derivate an expression for $l_{l,t}(\tilde{z}_{l,t})$. We know that:

$$
\begin{aligned}
d_{l,t} &= \rho_{l,t} y_t - w_t \frac{\alpha y_t}{z} - w_t f_{l,t} \\
&= y_t \left[\rho_{l,t} - \frac{\alpha w_t}{z}\right] - w_t f_{l,t} \\
&= y_t \left[\frac{\sigma}{\sigma-1}\frac{\alpha w_t}{z} - \frac{\alpha w_t}{z}\right] - w_t f_{l,t} \\
&= y_t \left[\frac{\alpha w_t}{z}\left(\frac{\sigma}{\sigma-1} - 1\right)\right] - w_t f_{l,t} \\
&= y_t \left[\frac{\alpha w_t}{z}\left(\frac{1}{\sigma-1}\right)\right] - w_t f_{l,t} \\
&= y_t \left[\frac{1}{\sigma}\rho_{l,t}\right] - w_t f_{l,t} \\
&= \frac{1}{\sigma}\rho_{l,t} y_t - w_t f_{l,t}
\end{aligned}
$$

Thus,

$$
\tilde{d}_{l,t} = \frac{1}{\sigma}\tilde{\rho}_{l,t}\tilde{y}_{l,t} - w_t f_{l,t}
$$

where $\tilde{y}_{l,t}$ is the average scale of production of low-quality products.

Using the labor demand function on the low-quality segment, the average scale of production of low-quality varieties is given by:

$$\tilde{y}_{l,t} = \left[l_{l,t}(\tilde{z}_{l,t}) - f_{l,t}\right]\frac{\tilde{z}_{l,t}}{\alpha}$$

Substituting $\tilde{y}_{l,t}$ into the average profit function of low-quality modules, we get:

$$\tilde{d}_{l,t} = \frac{1}{\sigma}\tilde{\rho}_{l,t}\left[l_{l,t}(\tilde{z}_{l,t}) - f_{l,t}\right]\frac{\tilde{z}_{l,t}}{\alpha} - w_t f_{l,t}$$

Thus, the average labor demand on the low-quality segment is given by:

$$\begin{aligned} l_{l,t}(\tilde{z}_{l,t}) &= \left[\tilde{d}_{l,t} + w_t f_{l,t}\right]\frac{\alpha\sigma}{\tilde{z}_{l,t}}\frac{\sigma-1}{\alpha\sigma}\frac{\tilde{z}_{l,t}}{w_t} + f_{l,t} \\ &= (\sigma-1)\frac{\tilde{d}_{l,t}}{w_t} + \sigma f_{l,t} \end{aligned}$$

Equivalently, I derive the expression of the average labor demand on the high-quality segment as follows:

$$l_{h,t}(\tilde{z}_{h,t}) = (\sigma-1)\frac{\tilde{d}_{h,t}}{w_t} + \sigma\frac{f_{h,t}}{Z_t}$$

Substituting $l_{l,t}(\tilde{z}_{l,t})$ and $l_{h,t}(\tilde{z}_{h,t})$ into the aggregate labor demand function, L_t, leads to the following equivalent labor market clearing:

$$L_t = N_{l,t}\left[(\sigma-1)\frac{\tilde{d}_{l,t}}{w_t} + \sigma f_{l,t}\right] + N_{h,t}\left[(\sigma-1)\frac{\tilde{d}_{h,t}}{w_t} + \sigma\frac{f_{h,t}}{Z_t}\right] + H_t\frac{v_t}{w_t}$$

C Price index

$$\begin{aligned} P_t \equiv & \left[\int_{\omega\in\Omega_t}\left(\frac{p_t(\omega)}{q_t(\omega)}\right)^{1-\sigma} d\omega\right]^{\frac{1}{1-\sigma}} \\ = & \left[\int_{z^*_{l,t}}^{z^*_{h,t}}\left(\frac{p_{l,t}(z)}{q_l}\right)^{1-\sigma} dz + \int_{z^*_{h,t}}^{\infty}\left(\frac{p_{h,t}(z)}{q_h}\right)^{1-\sigma} dz\right]^{\frac{1}{1-\sigma}} \\ = & \left[\int_{z^*_{l,t}}^{z^*_{h,t}}\left(\frac{p_{l,t}(z)}{q_l}\right)^{1-\sigma}\frac{1}{G(z^*_{h,t})-G(z^*_{l,t})}N_{l,t}g(z)\,dz + \int_{z^*_{h,t}}^{\infty}\left(\frac{p_{h,t}(z)}{q_h}\right)^{1-\sigma}\frac{1}{1-G(z^*_{h,t})}N_{h,t}g(z)\,dz\right]^{\frac{1}{1-\sigma}} \end{aligned}$$

Thus,

$$\begin{aligned} P_t^{1-\sigma} = & \left(\frac{1}{q_l}\frac{\sigma}{\sigma-1}\alpha W_t\right)^{1-\sigma} N_{l,t}\frac{1}{G(z^*_{h,t})-G(z^*_{l,t})}\int_{z^*_{l,t}}^{z^*_{h,t}} z^{\sigma-1}g(z)\,dz + \\ & \left(\frac{1}{q_h}\frac{\sigma}{\sigma-1}\frac{W_t}{Z_t}\right)^{1-\sigma} N_{h,t}\frac{1}{1-G(z^*_{h,t})}\int_{z^*_{h,t}}^{\infty} z^{\sigma-1}g(z)\,dz. \end{aligned}$$

We know that $p_{l,t}(z) = \frac{\sigma}{\sigma-1}\alpha W_t z^{-1}$ and $p_{h,t}(z) = \frac{\sigma}{\sigma-1}\frac{W_t}{Z_t}z^{-1}$. Thus,

$$p_{l,t}\left\{\left[\frac{1}{G(z^*_{h,t})-G(z^*_{l,t})}\int_{z^*_{l,t}}^{z^*_{h,t}} z^{\sigma-1}g(z)\,dz\right]^{-1}\right\} = \frac{\sigma}{\sigma-1}\alpha W_t\left[\frac{1}{G(z^*_{h,t})-G(z^*_{l,t})}\int_{z^*_{l,t}}^{z^*_{h,t}} z^{\sigma-1}g(z)\,dz\right] = p_{l,t}\left(\tilde{z}_{l,t}^{1-\sigma}\right)$$

and

$$p_{h,t}\left\{\left[\frac{1}{1-G(z^*_{h,t})}\int_{z^*_{h,t}}^{\infty} z^{\sigma-1}g(z)\,dz\right]^{-1}\right\} = \frac{\sigma}{\sigma-1}\frac{W_t}{Z_t}\left[\frac{1}{1-G(z^*_{h,t})}\int_{z^*_{h,t}}^{\infty} z^{\sigma-1}g(z)\,dz\right] = p_{h,t}\left(\tilde{z}_{h,t}^{1-\sigma}\right)$$

Substituting $p_{l,t}\left(\tilde{z}_{l,t}^{1-\sigma}\right)$ and $p_{h,t}\left(\tilde{z}_{h,t}^{1-\sigma}\right)$ into the expression of $P_t^{1-\sigma}$ leads to:

$$P_t^{1-\sigma} = p_{l,t}\left(\tilde{z}_{l,t}^{1-\sigma}\right)\left(\frac{\sigma}{\sigma-1}\alpha W_t\right)^{-\sigma}\left(\frac{1}{q_l}\right)^{1-\sigma}N_{l,t} + p_{h,t}\left(\tilde{z}_{h,t}^{1-\sigma}\right)\left(\frac{\sigma}{\sigma-1}\frac{W_t}{Z_t}\right)^{-\sigma}\left(\frac{1}{q_h}\right)^{1-\sigma}N_{h,t}$$

$$\begin{aligned}
\Rightarrow \frac{P_t^{1-\sigma}}{P_t^{1-\sigma}} &= \frac{p_{l,t}\left(\tilde{z}_{l,t}^{1-\sigma}\right)}{P_t}\frac{\left(\frac{\sigma}{\sigma-1}\alpha W_t\right)^{-\sigma}}{P_t^{-\sigma}}\left(\frac{1}{q_l}\right)^{1-\sigma}N_{l,t} + \frac{p_{h,t}\left(\tilde{z}_{h,t}^{1-\sigma}\right)}{P_t}\frac{\left(\frac{\sigma}{\sigma-1}\frac{W_t}{Z_t}\right)^{-\sigma}}{P_t^{-\sigma}}\left(\frac{1}{q_h}\right)^{1-\sigma}N_{h,t} \\
\Rightarrow 1 &= \rho_{l,t}\left(\tilde{z}_{l,t}^{1-\sigma}\right)\left(\frac{\sigma}{\sigma-1}\alpha w_t\right)^{-\sigma}q_l^{\sigma-1}N_{l,t} + \rho_{h,t}\left(\tilde{z}_{h,t}^{1-\sigma}\right)\left(\frac{\sigma}{\sigma-1}\frac{w_t}{Z_t}\right)^{-\sigma}q_h^{\sigma-1}N_{h,t} \\
\Rightarrow 1 &= \left(\frac{\sigma}{\sigma-1}\frac{\alpha w_t}{\tilde{z}_{l,t}}\right)^{1-\sigma}q_l^{\sigma-1}N_{l,t} + \left(\frac{\sigma}{\sigma-1}\frac{w_t}{Z_t\tilde{z}_{h,t}}\right)^{1-\sigma}q_h^{\sigma-1}N_{h,t} \\
\Rightarrow 1 &= \tilde{\rho}_{l,t}^{1-\sigma}q_l^{\sigma-1}N_{l,t} + \tilde{\rho}_{h,t}^{1-\sigma}q_h^{\sigma-1}N_{h,t}
\end{aligned} \tag{56}$$

D Steady state

At the steady state, (37) leads to

$$\frac{1}{\beta} = (1-\gamma)\left(1+\frac{\tilde{d}^o}{v}\right) \tag{57}$$

Plugging (52) in (51) yields

$$\tilde{d} = q_l^{1-\sigma}(\alpha Z B)^{\sigma-1}\left[\frac{\kappa}{\kappa-(\sigma-1)}\right]\left[\frac{A^{\kappa-(\sigma-1)}-1}{A^\kappa-1}\right]A^{\sigma-1}w f_l E - wG \tag{58}$$

$$\Rightarrow \frac{\tilde{d}}{w} = q_l^{1-\sigma}(\alpha Z B)^{\sigma-1}\left[\frac{\kappa}{\kappa-(\sigma-1)}\right]\left[\frac{A^{\kappa-(\sigma-1)}-1}{A^\kappa-1}\right]A^{\sigma-1}f_l E - G \tag{59}$$

$$\frac{\tilde{d}}{w} = \Xi \quad \text{with} \quad \Xi = q_l^{1-\sigma}(\alpha Z B)^{\sigma-1}\left[\frac{\kappa}{\kappa-(\sigma-1)}\right]\left[\frac{A^{\kappa-(\sigma-1)}-1}{A^\kappa-1}\right]A^{\sigma-1}f_l E - G \tag{60}$$

Remark that Ξ is only a function of parameters and exogenous variables.

Furthermore, I assume that $f_E = Z = 1$ at the steady state. Thus, the free entry condition (20) becomes

$$v = w \tag{61}$$

Substituting (49) and (61) into (57) leads to

$$\frac{1}{\beta} = (1-\gamma)\left(1 + A^{\kappa}\frac{N_h}{M}\frac{\tilde{d}}{w}\right) \tag{62}$$

Finally, Substituting (61) into (62) yields

$$\frac{1}{\beta} = (1-\gamma)\left(1 + A^{\kappa}\frac{N_h}{M}\Xi\right) \tag{63}$$

Equation (63) provides the steady state value of $\frac{N_h}{M}$ given the parameter values and the steady-state value of Ξ.

At the steady state, equation (21) implies that

$$H = \frac{\gamma}{1-\gamma}M \tag{64}$$

The parameter χ is calibrated so that the labor supply is equal to one at the steady state. Thus

$$L = 1 \qquad \text{and} \qquad \chi = wC^{-1} \tag{65}$$

Substituting (61), (64) and (65) into the labor market clearing condition (53) and rewriting N in terms of N_h, I obtain the following equation, which provides the unique solution for M:

$$\frac{1}{M} = (\sigma-1)A^{\kappa}\frac{N_h}{M}\Xi + \sigma\frac{N_h}{M}\left[\left(A_t^{\kappa}-1\right)f_{l,t} + \frac{f_{h,t}}{Z_t}\right] + \frac{\gamma}{1-\gamma} \tag{66}$$

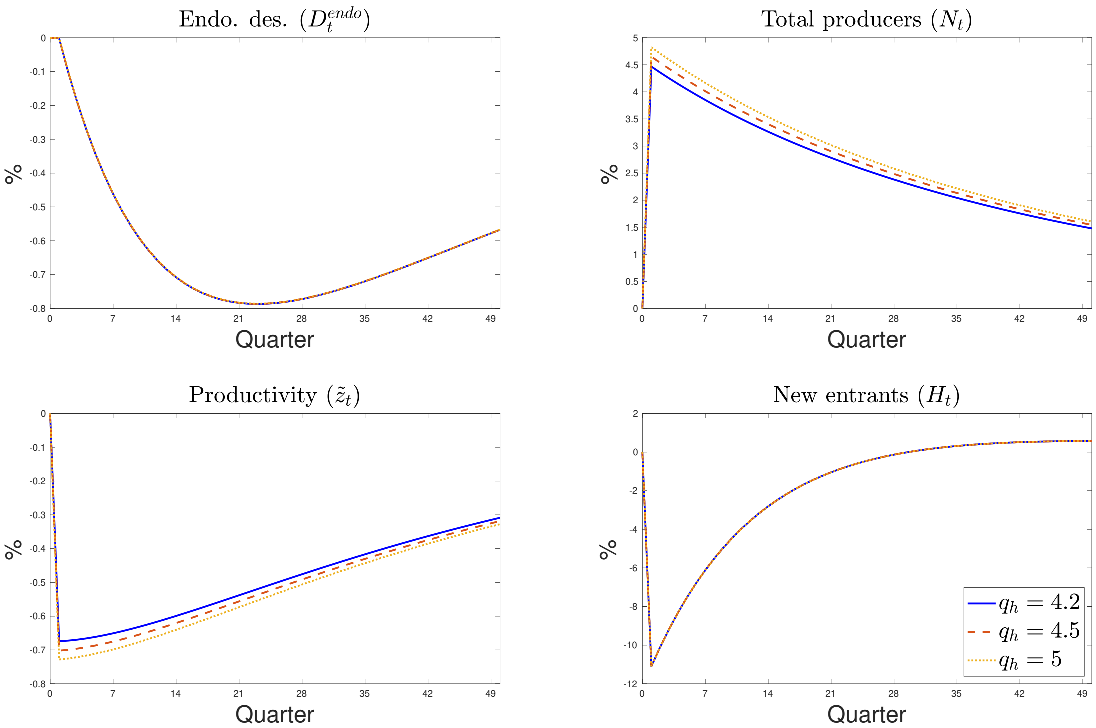
Once I solve for M, the remainder of the unknowns is easy to find.

E The model wihout vertical differentiation

Table 4:
The model HZ17

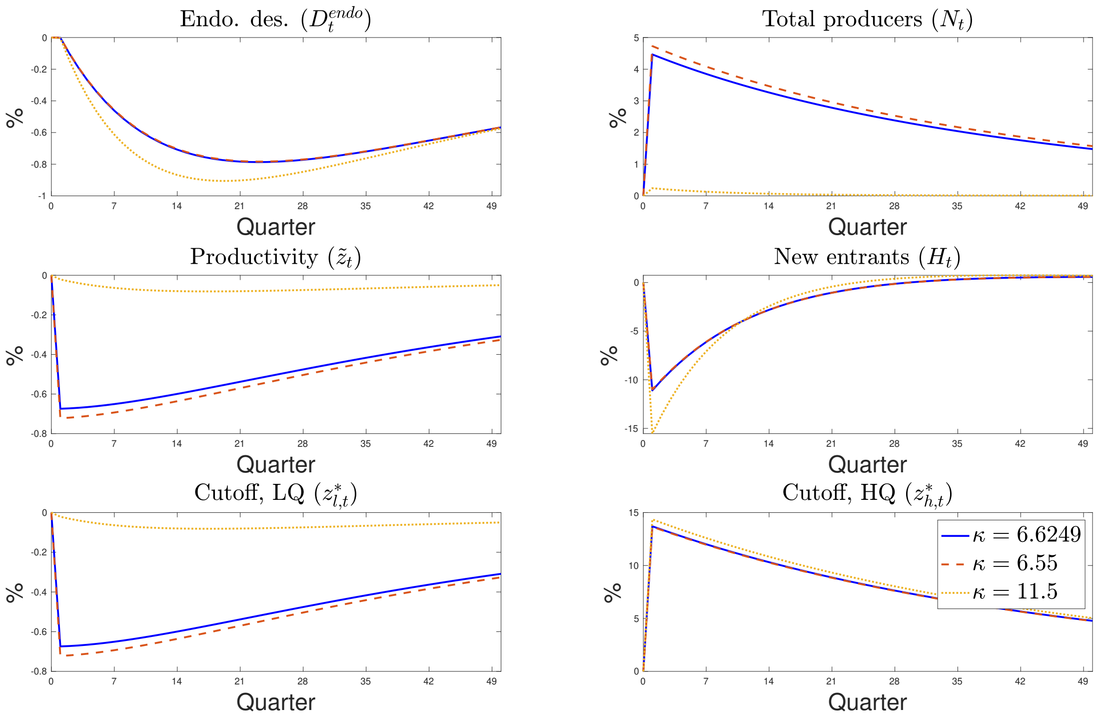
Average pricing:	$\tilde{\rho}_t = \frac{\sigma}{\sigma-1}\frac{w_t}{Z_t\tilde{z}_t}$
Real price:	$\tilde{\rho}_t = N_t^{\frac{1}{\sigma-1}}$
Average profit of survivors:	$\tilde{d}_t = \frac{1}{\sigma}\frac{C_t}{N_t} - w_t\frac{f_t}{Z_t}$
Operational profit:	$\tilde{d}_t^o = \frac{N_t}{M_t}\tilde{d}_t$
Free entry condition:	$v_t = \frac{w_t f_{Et}}{Z_t}$
Motion of products:	$M_t = (M_{t-1} + H_{t-1})(1-\gamma)$
Euler equation:	$v_t = \beta(1-\gamma)\mathbb{E}_t\left[\frac{C_{t+1}}{C_t}\right]^{-1}\left[v_{t+1} + \tilde{d}_{t+1}^o\right]$
Optimal labor supply:	$\chi L_t^{\frac{1}{\varphi}} = w_t C_t^{-1}$
ZPC:	$\frac{1}{\sigma}\frac{C_t}{N_t} = \frac{\kappa}{\kappa-(\sigma-1)}\frac{w_t f_t}{Z_t}$
Surviving rate:	$\frac{N_t}{M_t} = z_{min}^{\kappa}\tilde{z}_t^{-\kappa}\left[\frac{\kappa}{\kappa-(\sigma-1)}\right]^{\frac{\kappa}{\sigma-1}}$
Labor market clearing:	$L_t = N_t\left[(\sigma-1)\frac{\tilde{d}_t}{w_t} + \sigma\frac{f_t}{Z_t}\right] + H_t\frac{v_t}{w_t}$

F Sensitivity analysis



Note: each entry shows the percentage-point response of a variable to a one-percent deviation of a negative productivity shock

Figure 6: Impulse response functions to a recessionary shock for different values of q_h



Note: each entry shows the percentage-point response of a variable to a one-percent deviation of a negative productivity shock

Figure 7: Impulse response functions to a recessionary shock for different values of κ